Chapter 10: The Other Side of Zero

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Exercise 10.1 — Addition to Keep Track of Movement

Q1 You start from Floor +2 and press –3 in the lift. Where will you reach? Write an expression for this movement.

SOLUTION

Starting Floor + Movement = Target Floor

(+2) + (–3) = –1

Floor –1 is the Toy Store.

Answer: Floor –1 (Toy Store). Expression: (+2) + (–3) = –1

Q2 Evaluate these expressions (you may think of them as Starting Floor + Movement by referring to the Building of Fun).

a. (+1) + (+4) = _____

b. (+4) + (+1) = _____

c. (+4) + (–3) = _____

d. (–1) + (+2) = _____

e. (–1) + (+1) = _____

f. 0 + (+2) = _____

g. 0 + (–2) = _____

SOLUTION

a. Starting at +1 (Food Court), move +4 floors up: 1 + 4 = +5 (Ice Cream floor)

b. Starting at +4, move +1 up: 4 + 1 = +5

c. Starting at +4, move –3 down: 4 – 3 = +1 (Food Court)

d. Starting at –1 (Toy Store), move +2 up: –1 + 2 = +1

e. Starting at –1, move +1 up: –1 + 1 = 0 (Welcome Hall)

f. Starting at 0, move +2 up: 0 + 2 = +2

g. Starting at 0, move –2 down: 0 + (–2) = –2

Answer: a. +5 b. +5 c. +1 d. +1 e. 0 f. +2 g. –2

Q3 Starting from different floors, find the movements required to reach Floor –5. For example, if I start at Floor +2, I must press –7 to reach Floor –5. The expression is (+2) + (–7) = –5. Find more such starting positions and the movements needed to reach Floor –5 and write the expressions.

SOLUTION

Use: Starting Floor + Movement = –5, so Movement = –5 – Starting Floor.
Starting at +1: (+1) + (–6) = –5
Starting at 0: (0) + (–5) = –5
Starting at –2: (–2) + (–3) = –5
Starting at –3: (–3) + (–2) = –5
(Many other starting positions are possible.)

Answer: Sample expressions: (+1)+(–6)=–5; (0)+(–5)=–5; (–2)+(–3)=–5; (–3)+(–2)=–5

Exercise 10.2 — Combining Button Presses is Also Addition

Q1 Evaluate these expressions by thinking of them as the resulting movement of combining button presses:

a. (+1) + (+4) = _____

b. (+4) + (+1) = _____

c. (+4) + (–3) + (–2) = _____

d. (–1) + (+2) + (–3) = _____

SOLUTION

a. Press + once, then + four more times: net +5. (+1) + (+4) = +5

b. Press + four times, then + once more: net +5. (+4) + (+1) = +5

c. Press +4, –3, –2: net = 4 – 3 – 2 = –1. (+4) + (–3) + (–2) = –1

d. Press –1, +2, –3: net = –1 + 2 – 3 = –2. (–1) + (+2) + (–3) = –2

Answer: a. +5 b. +5 c. –1 d. –2

Exercise 10.3 — Comparing Numbers Using Floors

Q1 Compare the following numbers using the Building of Fun and fill in the boxes with < or >.

a. –2 [ ] +5
b. –5 [ ] +4
c. –5 [ ] –3
d. +6 [ ] –6
e. 0 [ ] –4
f. 0 [ ] +4

SOLUTION

Negative floors are always below Floor 0, positive floors always above Floor 0.

a. –2 is below +5, so –2 < +5
b. –5 is below +4, so –5 < +4
c. –5 is lower than –3 (farther from 0 in the negative direction), so –5 < –3
d. +6 is above –6, so +6 > –6
e. 0 is above –4, so 0 > –4
f. 0 is below +4, so 0 < +4

Answer: a. < b. < c. < d. > e. > f. <

Q2 Imagine the Building of Fun with more floors. Compare the numbers and fill in the boxes with < or >:

a. –10 [ ] –12
b. +17 [ ] –10
c. 0 [ ] –20
d. +9 [ ] –9
e. –25 [ ] –7
f. +15 [ ] –17

SOLUTION

a. –10 is higher than –12 (less negative), so –10 > –12
b. +17 is above –10, so +17 > –10
c. 0 is above –20, so 0 > –20
d. +9 is above –9, so +9 > –9
e. –25 is lower than –7 (more negative), so –25 < –7
f. +15 is above –17, so +15 > –17

Answer: a. > b. > c. > d. > e. < f. >

Q3 If Floor A = –12, Floor D = –1 and Floor E = +1 in the building shown on the right as a line, find the numbers of Floors B, C, F, G, and H.

SOLUTION

From the figure, D = –1 and E = +1, so each gap between adjacent labeled floors can be read off the diagram.

Reading the scale from the building diagram:

B = –9, C = –6 (below ground), F = +2, G = +6, H = +11 (above ground)

Answer: B = –9, C = –6, F = +2, G = +6, H = +11

Q4 Mark the following floors of the building shown on the right.

a. –7 b. –4 c. +3 d. –10

SOLUTION

Using the scale where D = –1 and E = +1 (each gap = 1 floor):

a. –7 lies between C(–6) and B(–9), 1 below C

b. –4 lies between D(–1) and C(–6), 3 below D

c. +3 lies between F(+2) and G(+6), 1 above F

d. –10 lies between B(–9) and A(–12), 1 below B

Mark each point on the building diagram at its respective position.

Answer: –7 is 1 floor below C; –4 is 3 floors below D; +3 is 1 floor above F; –10 is 1 floor below B

Exercise 10.4 — Subtraction to Find Which Button to Press

Q1 Complete these expressions. You may think of them as finding the movement needed to reach the Target Floor from the Starting Floor.

a. (+1) – (+4) = _____
b. (0) – (+2) = _____
c. (+4) – (+1) = _____
d. (0) – (–2) = _____
e. (+4) – (–3) = _____
f. (–4) – (–3) = _____
g. (–1) – (+2) = _____
h. (–2) – (–2) = _____
i. (–1) – (+1) = _____
j. (+3) – (–3) = _____

SOLUTION

Target Floor – Starting Floor = Movement needed.

a. (+1) – (+4): to go from +4 to +1, move –3. Answer: –3
b. (0) – (+2): to go from +2 to 0, move –2. Answer: –2
c. (+4) – (+1): to go from +1 to +4, move +3. Answer: +3
d. (0) – (–2): to go from –2 to 0, move +2. Answer: +2
e. (+4) – (–3): to go from –3 to +4, move +7 (3 to reach 0, then 4 more). Answer: +7
f. (–4) – (–3): to go from –3 to –4, move –1. Answer: –1
g. (–1) – (+2): to go from +2 to –1, move –3. Answer: –3
h. (–2) – (–2): to go from –2 to –2, no movement needed. Answer: 0
i. (–1) – (+1): to go from +1 to –1, move –2. Answer: –2
j. (+3) – (–3): to go from –3 to +3, move +6 (3 to reach 0, then 3 more). Answer: +6

Answer: a. –3 b. –2 c. +3 d. +2 e. +7 f. –1 g. –3 h. 0 i. –2 j. +6

Exercise 10.5 — Adding and Subtracting Larger Numbers (Mineshaft)

Q1 Complete these expressions.

a. (+40) + _____ = +200
b. (+40) + _____ = –200
c. (–50) + _____ = +200
d. (–50) + _____ = –200
e. (–200) – (–40) = _____
f. (+200) – (+40) = _____
g. (–200) – (+40) = _____

SOLUTION

a. +200 – 40 = +160. So (+40) + (+160) = +200
b. –200 – 40 = –240. So (+40) + (–240) = –200
c. +200 – (–50) = +250. So (–50) + (+250) = +200
d. –200 – (–50) = –150. So (–50) + (–150) = –200
e. (–200) – (–40) = –200 + 40 = –160
f. (+200) – (+40) = 200 – 40 = +160
g. (–200) – (+40) = –200 – 40 = –240

Answer: a. +160 b. –240 c. +250 d. –150 e. –160 f. +160 g. –240

Exercise 10.6 — Adding, Subtracting, and Comparing Any Numbers (Infinite Lift)

Q1 Try evaluating the following expressions by similarly drawing or imagining a suitable lift:

a. –125 + (–30)
b. +105 – (–55)
c. +105 + (+55)
d. +80 – (–150)
e. +80 + (+150)
f. –99 – (–200)
g. –99 + (+200)
h. +1500 – (–1500)

SOLUTION

Recall: subtracting a negative = adding the corresponding positive.

a. –125 + (–30) = –155
b. +105 – (–55) = +105 + 55 = +160
c. +105 + (+55) = +160
d. +80 – (–150) = +80 + 150 = +230
e. +80 + (+150) = +230
f. –99 – (–200) = –99 + 200 = +101
g. –99 + (+200) = +101
h. +1500 – (–1500) = +1500 + 1500 = +3000

Answer: a. –155 b. +160 c. +160 d. +230 e. +230 f. +101 g. +101 h. +3000

Exercise 10.7 — Back to the Number Line

Q1 Mark 3 positive numbers and 3 negative numbers on the number line above (from –10 to 10).

SOLUTION

Any three positive integers and three negative integers can be marked.
Example: positive numbers 2, 5, 8; negative numbers –1, –3, –7.
Mark them as points on the number line at their respective positions.

Answer: Sample: positive: 2, 5, 8; negative: –1, –3, –7 (other choices are valid)

Q2 Write down the above 3 marked negative numbers in the following boxes (in increasing order).

SOLUTION

Using the sample from Q1: the three negative numbers are –1, –3, –7.
In increasing order (smallest first): –7 < –3 < –1.

Answer: –7, –3, –1 (using sample values; order your own chosen negatives from least to greatest)

Q3 Is 2 > –3? Why? Is –2 < 3? Why?

SOLUTION

On the number line, 2 is to the right of –3, so 2 > –3.
On the number line, –2 is to the left of 3, so –2 < 3.
In general, any positive number is greater than any negative number.

Answer: Yes, 2 > –3 (2 is to the right of –3 on the number line). Yes, –2 < 3 (–2 is to the left of 3).

Q4 What are:
a. –5 + 0
b. 7 + (–7)
c. –10 + 20
d. 10 – 20
e. 7 – (–7)
f. –8 – (–10)?

SOLUTION

a. –5 + 0 = –5 (adding zero leaves the number unchanged)

b. 7 + (–7) = 0 (a number plus its inverse = 0)

c. –10 + 20 = 10

d. 10 – 20 = –10

e. 7 – (–7) = 7 + 7 = 14

f. –8 – (–10) = –8 + 10 = 2

Answer: a. –5 b. 0 c. 10 d. –10 e. 14 f. 2

Exercise 10.8 — Using the Unmarked Number Line to Add and Subtract

Q1 Use unmarked number lines to evaluate these expressions:

a. –125 + (–30) = _____

b. +105 – (–55) = _____

c. +80 – (–150) = _____

d. –99 – (–200) = _____

SOLUTION

a. –125 + (–30): start at –125, move 30 further left: –125 – 30 = –155

b. +105 – (–55) = +105 + 55 = +160

c. +80 – (–150) = +80 + 150 = +230

d. –99 – (–200) = –99 + 200 = +101

Answer: a. –155 b. +160 c. +230 d. +101

Exercise 10.9 — The Token Model — Using Tokens for Addition

Q1 Complete the additions using tokens.

a. (+6) + (+4)

b. (–3) + (–2)

c. (+5) + (–7)

d. (–2) + (+6)

SOLUTION

a. 6 positive tokens + 4 positive tokens = 10 positive tokens: (+6) + (+4) = +10

b. 3 negative tokens + 2 negative tokens = 5 negative tokens: (–3) + (–2) = –5

c. 5 positive + 7 negative: form 5 zero pairs, leaving 2 negatives: (+5) + (–7) = –2

d. 2 negative + 6 positive: form 2 zero pairs, leaving 4 positives: (–2) + (+6) = +4

Answer: a. +10 b. –5 c. –2 d. +4

Q2 Cancel the zero pairs in the following two sets of tokens. On what floor is the lift attendant in each case? What is the corresponding addition statement in each case? (a. shows 3 positive and 5 negative tokens; b. shows 6 positive and 3 negative tokens)

SOLUTION

a. 3 positive (+) and 5 negative (–) tokens.
Cancel 3 zero pairs, leaving 2 negative tokens.
Addition statement: (+3) + (–5) = –2. Attendant is at Floor –2.

b. 6 positive (+) and 3 negative (–) tokens.
Cancel 3 zero pairs, leaving 3 positive tokens.
Addition statement: (+6) + (–3) = +3. Attendant is at Floor +3.

Answer: a. Floor –2; statement: (+3) + (–5) = –2. b. Floor +3; statement: (+6) + (–3) = +3.

Exercise 10.10 — Using Tokens for Subtraction — Same-Sign Cases

Q1 Evaluate the following differences using tokens. Check that you get the same result as with other methods you now know:

a. (+10) – (+7)
b. (–8) – (–4)
c. (–9) – (–4)
d. (+9) – (+12)
e. (–5) – (–7)
f. (–2) – (–6)

SOLUTION

a. (+10) – (+7): start with 10 positive tokens, remove 7: left with 3 positive. = +3
b. (–8) – (–4): start with 8 negative tokens, remove 4 negatives: left with 4 negative. = –4
c. (–9) – (–4): start with 9 negative tokens, remove 4 negatives: left with 5 negative. = –5
d. (+9) – (+12): start with 9 positive tokens, need to remove 12. Add 3 zero pairs (3+,3–), remove 12 positives: left with 3 negative. = –3
e. (–5) – (–7): start with 5 negative tokens, need to remove 7. Add 2 zero pairs (2+,2–), remove 7 negatives: left with 2 positive. = +2
f. (–2) – (–6): start with 2 negative tokens, need to remove 6. Add 4 zero pairs (4+,4–), remove 6 negatives: left with 4 positive. = +4

Answer: a. +3 b. –4 c. –5 d. –3 e. +2 f. +4

Q2 Complete the subtractions:

a. (–5) – (–7)
b. (+10) – (+13)
c. (–7) – (–9)
d. (+3) – (+8)
e. (–2) – (–7)
f. (+3) – (+15)

SOLUTION

a. (–5) – (–7) = –5 + 7 = +2
b. (+10) – (+13) = 10 – 13 = –3
c. (–7) – (–9) = –7 + 9 = +2

d. (+3) – (+8) = 3 – 8 = –5

e. (–2) – (–7) = –2 + 7 = +5

f. (+3) – (+15) = 3 – 15 = –12

Answer: a. +2 b. –3 c. +2 d. –5 e. +5 f. –12

Exercise 10.11 — Using Tokens for Subtraction — Mixed-Sign Cases

Q1 Try to subtract: –3 – (+5). How many zero pairs will you have to put in? What is the result?

SOLUTION

Start with 3 negative tokens.
We need to remove 5 positive tokens, but there are none.
Add 5 zero pairs (5 positive + 5 negative tokens). Now we have 5+ and 8– tokens.
Remove 5 positive tokens: left with 8 negative tokens.
Result: –3 – (+5) = –8
We put in 5 zero pairs.

Answer: 5 zero pairs must be added. Result: –3 – (+5) = –8

Q2 Evaluate the following using tokens.
a. (–3) – (+10)
b. (+8) – (–7)
c. (–5) – (+9)
d. (–9) – (+10)
e. (+6) – (–4)
f. (–2) – (+7)

SOLUTION

a. (–3) – (+10): add 10 zero pairs, remove 10 positives, left with 13 negatives. = –13
b. (+8) – (–7): add 7 zero pairs (8+7 positive, 7 negative), remove 7 negatives: 15 positive. = +15
c. (–5) – (+9): add 9 zero pairs, remove 9 positives: 14 negatives. = –14
d. (–9) – (+10): add 10 zero pairs, remove 10 positives: 19 negatives. = –19
e. (+6) – (–4): add 4 zero pairs (10 positive, 4 negative), remove 4 negatives: 10 positive. = +10
f. (–2) – (+7): add 7 zero pairs, remove 7 positives: 9 negatives. = –9

Answer: a. –13 b. +15 c. –14 d. –19 e. +10 f. –9

Exercise 10.12 — Integers in Other Places — Credits and Debits

Q1 Suppose you start with 0 in your bank account, and then you have credits of 30, 40, and 50, and debits of 40, 50, and 60. What is your bank account balance now?

SOLUTION

Total credits = 30 + 40 + 50 = 120
Total debits = 40 + 50 + 60 = 150
Balance = Credits – Debits = 120 – 150 = –30
Equivalently: 0 + 30 + 40 + 50 – 40 – 50 – 60 = –30

Answer: –30 (the account is 30 in the negative)

Q2 Suppose you start with 0 in your bank account, and then you have debits of 1, 2, 4, 8, 16, 32, 64, and 128, and then a single credit of 256. What is your bank account balance now?

SOLUTION

Total debits = 1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 = 255
Credit = 256
Balance = 0 – 255 + 256 = +1

Answer: +1

Exercise 10.13 — Integers in Other Places — Geographical Cross Sections

Q1 Looking at the geographical cross section, fill in the respective heights:
a. b. c. d. e. f. g.

SOLUTION

Reading heights from the cross-section graph (sea level = 0):
a. A ≈ +1500 m (highest peak, left side)
b. B ≈ –500 m (below sea level, between A and C)
c. C ≈ +300 m (above sea level)
d. D ≈ –1200 m (deepest trough, right-centre)
e. E ≈ +1200 m (tall peak, right side)
f. F ≈ –200 m (shallow trough)
g. G ≈ +100 m (small hill just above sea level)

Answer: A ≈ +1500 m, B ≈ –500 m, C ≈ +300 m, D ≈ –1200 m, E ≈ +1200 m, F ≈ –200 m, G ≈ +100 m

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Q2 Which is the highest point in this geographical cross section? Which is the lowest point?

SOLUTION

Highest point: A at approximately +1500 m (tallest peak above sea level).
Lowest point: D at approximately –1200 m (deepest below sea level).

Answer: Highest: A (+1500 m); Lowest: D (–1200 m)

Q3 Can you write the points A, B, …, G in a sequence of decreasing order of heights? Can you write the points in a sequence of increasing order of heights?

SOLUTION

Using the heights: A(+1500), E(+1200), C(+300), G(+100), F(–200), B(–500), D(–1200).
Decreasing order (highest first): A, E, C, G, F, B, D
Increasing order (lowest first): D, B, F, G, C, E, A

Answer: Decreasing: A, E, C, G, F, B, D. Increasing: D, B, F, G, C, E, A.

Q4 What is the highest point above sea level on Earth? What is its height?

SOLUTION

Mount Everest is the highest point above sea level on Earth.
Its height is approximately 8848 m above sea level.

Answer: Mount Everest, approximately 8848 m above sea level

Q5 What is the lowest point with respect to sea level on land or on the ocean floor? What is its height? (This height should be negative).

SOLUTION

The lowest known point on Earth is Challenger Deep, located in the Mariana Trench in the Pacific Ocean.
Its depth is approximately –10,994 m (about 11 km below sea level).

Answer: Challenger Deep, Mariana Trench; approximately –10,994 m

Exercise 10.14 — Integers in Other Places — Temperature

Q1 Leh in Ladakh gets very cold during the winter. The following is a table of temperature readings taken during different times of the day and night in Leh on a day in November. Match the temperature with the appropriate time of the day and night.

Temperatures: 14°C, 8°C, –2°C, –4°C
Times: 02:00 a.m., 11:00 p.m., 02:00 p.m., 11:00 a.m.

SOLUTION

02:00 p.m. (afternoon) is warmest: 14°C
11:00 a.m. (late morning) is warm: 8°C
11:00 p.m. (late night) is cool: –2°C
02:00 a.m. (early morning, coldest part of night) is coldest: –4°C

Answer: 14°C → 02:00 p.m. | 8°C → 11:00 a.m. | –2°C → 11:00 p.m. | –4°C → 02:00 a.m.

Exercise 10.15 — Explorations with Integers — A Hollow Integer Grid

Q1 Do the calculations for the second grid and find the border sum.

The second grid is:

5 –3 –5

0 [ ] –5

–8 –2 7

SOLUTION

Top row: 5 + (–3) + (–5) = –3

Bottom row: (–8) + (–2) + 7 = –3

Left column: 5 + 0 + (–8) = –3

Right column: (–5) + (–5) + 7 = –3

All four border lines give the same sum.

Answer: Border sum = –3

Q2 Complete the grids to make the required border sum:

Grid 1 (given –10, –5, 9; border sum +4)

Grid 2 (given 6, 8, –5, –2; border sum –2)

Grid 3 (given 7, –5; border sum –4)

SOLUTION

Grid 1 (border sum +4): one solution:

Top row: –10, 10, 4 (–10+10+4 = +4)

Middle row: 5, [centre], –5

Bottom row: 9, –10, 5 (9+(–10)+5 = +4)

Left col: –10+5+9 = +4 ✓; Right col: 4+(–5)+5 = +4 ✓

Grid 2 (border sum –2): one solution:

Top row: 6, 8, –16 (6+8+(–16) = –2)

Middle row: 11, [centre], –5

Bottom row: –19, –2, 19 ((–19)+(–2)+19 = –2)

Left col: 6+11+(–19) = –2 ✓; Right col: –16+(–5)+19 = –2 ✓

Grid 3 (border sum –4): one solution:

Top row: 7, –2, –9 (7+(–2)+(–9) = –4)

Middle row: –3, [centre], –5

Bottom row: –8, –6, 10 ((–8)+(–6)+10 = –4)

Left col: 7+(–3)+(–8) = –4 ✓; Right col: –9+(–5)+10 = –4 ✓

Answer: Grid 1: top –10,10,4; mid 5,_,–5; bot 9,–10,5. Grid 2: top 6,8,–16; mid 11,_,–5; bot –19,–2,19. Grid 3: top 7,–2,–9; mid –3,_,–5; bot –8,–6,10.

Exercise 10.16 — Explorations with Integers — An Amazing Grid of Numbers

Q1 Play the same game with the grids below. What answer did you get?

Grid A:

7	10	13	16
–2	1	4	7
–11	–8	–5	–2
–20	–17	–14	–11

Grid B:

–11	–10	–9	–8
–7	–6	–5	–4
–3	–2	–1	0
1	2	3	4

SOLUTION

In each grid, circle one number, then strike its entire row and column; repeat until all numbers are struck; add all circled numbers.

Grid A: No matter which numbers you choose (one per row, one per column), the sum is always –8.

Example: circle 7(r0c0), 1(r1c1), –5(r2c2), –11(r3c3): 7+1+(–5)+(–11) = –8

Another: circle 16(r0c3), –2(r1c0), –8(r2c1), –14(r3c2): 16+(–2)+(–8)+(–14) = –8

Grid B: No matter which numbers you choose, the sum is always –14.

Example: circle –11(r0c0), –6(r1c1), –1(r2c2), 4(r3c3): –11+(–6)+(–1)+4 = –14

Another: circle –8(r0c3), –5(r1c2), –2(r2c1), 1(r3c0): –8+(–5)+(–2)+1 = –14

Answer: Grid A: always –8. Grid B: always –14.

Exercise 10.17 — Explorations with Integers — Mixed Problems

Q1 Write all the integers between the given pairs, in increasing order.

a. 0 and –7

b. –4 and 4

c. –8 and –15

d. –30 and –23

SOLUTION

a. Integers strictly between 0 and –7 (i.e., between –7 and 0): –6, –5, –4, –3, –2, –1

b. Integers strictly between –4 and 4: –3, –2, –1, 0, 1, 2, 3

c. Integers strictly between –8 and –15 (between –15 and –8): –14, –13, –12, –11, –10, –9

d. Integers strictly between –30 and –23: –29, –28, –27, –26, –25, –24

Answer: a. –6,–5,–4,–3,–2,–1 b. –3,–2,–1,0,1,2,3 c. –14,–13,–12,–11,–10,–9 d. –29,–28,–27,–26,–25,–24

Q2 Give three numbers such that their sum is –8.

SOLUTION

Many answers are possible.

One example: –5 + 7 + (–10) = –8.

Another: –1 + (–3) + (–4) = –8.

Answer: Sample: –5, 7, –10 (their sum is –8). Other combinations are also valid.

Q3 There are two dice whose faces have these numbers: –1, 2, –3, 4, –5, 6. The smallest possible sum upon rolling these dice is –10 = (–5) + (–5) and the largest possible sum is 12 = (6) + (6). Some numbers between (–10) and (+12) are not possible to get by adding numbers on these two dice. Find those numbers.

SOLUTION

List all possible sums from adding any face of die 1 with any face of die 2 (faces: –1, 2, –3, 4, –5, 6).

Possible sums: –10,–8,–6,–4,–3,–2,–1,1,3,4,5,6,8,10,12

Integers from –10 to +12 that do NOT appear: –9, –7, –5, 0, 2, 7, 9, 11

Answer: Impossible sums: –9, –7, –5, 0, 2, 7, 9, 11

Q4 Solve these:

8 – 13 | (–8) – (13) | (–13) – (–8) | (–13) + (–8)

8 + (–13) | (–8) – (–13) | (13) – 8 | 13 – (–8)

SOLUTION

8 – 13 = –5

(–8) – (13) = –8 – 13 = –21

(–13) – (–8) = –13 + 8 = –5

(–13) + (–8) = –21

8 + (–13) = –5

(–8) – (–13) = –8 + 13 = +5

(13) – 8 = +5

13 – (–8) = 13 + 8 = +21

Answer: –5 | –21 | –5 | –21 (top row); –5 | +5 | +5 | +21 (bottom row)

Q5 Find the years below.

a. From the present year, which year was it 150 years ago?

b. From the present year, which year was it 2200 years ago? (Recall that there was no year 0.)

c. What will be the year 320 years after 680 BCE?

SOLUTION

a. Present year = 2026. 150 years ago: 2026 – 150 = 1876 CE

b. 2026 – 2200 = –174. Since there is no year 0, year –174 corresponds to 175 BCE.

c. 680 BCE is –680 on a timeline. Adding 320: –680 + 320 = –360, i.e., 360 BCE.

Answer: a. 1876 CE b. 175 BCE c. 360 BCE

Q6 Complete the following sequences:

a. (–40), (–34), (–28), (–22), _____, _____, _____

b. 3, 4, 2, 5, 1, 6, 0, 7, _____, _____, _____

c. _____, _____, 12, 6, 1, (–3), (–6), _____, _____, _____

SOLUTION

a. Each term increases by +6: –40, –34, –28, –22, –16, –10, –4

Next three: –16, –10, –4

b. Two interleaved sequences: 3,2,1,0,–1,… (decreasing by 1) and 4,5,6,7,8,… (increasing by 1).

Pattern: 3,4,2,5,1,6,0,7, –1, 8, –2

Next three: –1, 8, –2

c. Differences: 12–6=6, 6–1=5, 1–(–3)=4, –3–(–6)=3; differences decrease by 1.

Before 12: previous difference was 7, so term before 12 is 12+7=19; before 19: 19+8=27.

After –6: next differences are 2,1,0: –6–2=–8, –8–1=–9, –9–0=–9.

Sequence: 27, 19, 12, 6, 1, –3, –6, –8, –9, –9

Answer: a. –16, –10, –4 b. –1, 8, –2 c. 27, 19 (before); –8, –9, –9 (after)

Q7 Here are six integer cards: (+1), (+7), (+18), (–5), (–2), (–9). You can pick any of these and make an expression using addition(s) and subtraction(s). Here is an expression: (+18) + (+1) – (+7) – (–2) which gives a value (+14). Now, pick cards and make an expression such that its value is closer to (–30).

SOLUTION

We need a combination that gives a value near –30.

Try: (–2) + (–9) – (+18) – (+1)

= –2 – 9 – 18 – 1 = –30

This gives exactly –30.

Answer: (–2) + (–9) – (+18) – (+1) = –30

Q8 The sum of two positive integers is always positive but a (positive integer) – (positive integer) can be positive or negative. What about:
a. (positive) – (negative)
b. (positive) + (negative)
c. (negative) + (negative)
d. (negative) – (negative)
e. (negative) – (positive)
f. (negative) + (positive)

SOLUTION

a. (positive) – (negative) = positive + positive = always positive
b. (positive) + (negative): can be positive or negative (depends on magnitudes)
c. (negative) + (negative) = always negative
d. (negative) – (negative) = negative + positive: can be positive or negative
e. (negative) – (positive) = negative + negative = always negative
f. (negative) + (positive): can be positive or negative (same as b)

Answer: a. Always positive b. Can be positive or negative c. Always negative d. Can be positive or negative e. Always negative f. Can be positive or negative

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Q9 This string has a total of 100 tokens arranged in a particular pattern. What is the value of the string? (The pattern shown is: +,+,+,–,–,+,+,+,–,–, … repeating)

SOLUTION

The pattern repeats every 5 tokens: 3 positive (+) and 2 negative (–).
100 ÷ 5 = 20 complete cycles.
Net value per cycle = (+3) + (–2) = +1.
Total value = 20 × (+1) = +20.

Answer: +20

Exercise 10.18 — A Pinch of History — Brahmagupta's Rules

Q1 Can you explain each of Brahmagupta's rules in terms of Bela's Building of Fun, or in terms of a number line?

SOLUTION

Addition rules:

Rule 1 (positive + positive = positive): Moving up from a positive floor by a positive amount stays above ground. E.g., Floor +2, press +3 → Floor +5.

Rule 2 (negative + negative = negative): Moving down from a below-ground floor by more floors stays below ground. E.g., Floor –2, press –3 → Floor –5.

Rule 3 (positive + negative or negative + positive): The result depends on which is larger. E.g., Floor –5, press +3 → Floor –2 (still below ground since |–5| > |+3|).

Rule 4 (number + inverse = 0): Pressing +n and then –n returns you to the ground floor (zero).

Rule 5 (number + 0 = number): Pressing 0 (not pressing any button) leaves you on the same floor.

Subtraction rules explained similarly using Target – Starting = Movement on the number line.

Answer: Each rule corresponds to movements in the Building of Fun or positions on the number line (see solution for full explanation).

Q2 Give your own examples of each rule.

SOLUTION

Addition Rule 1: 5 + 3 = 8 (positive + positive)

Addition Rule 2: (–4) + (–6) = –10 (negative + negative)

Addition Rule 3: (–7) + 4 = –3 (negative larger than positive in magnitude)

Addition Rule 4: 9 + (–9) = 0 (number + inverse)

Addition Rule 5: –11 + 0 = –11 (number + zero)

Subtraction Rule 1: 7 – 3 = 4 (smaller positive from larger positive)

Subtraction Rule 2: 3 – 7 = –4 (larger positive from smaller positive)

Subtraction Rule 3: 5 – (–2) = 5 + 2 = 7 (subtracting negative = adding positive)

Subtraction Rule 4: 6 – 6 = 0; (–3) – (–3) = 0

Subtraction Rule 5: –5 – 0 = –5; 0 – (–4) = 4

Answer: See solution for one valid set of examples. Any correct examples following each rule are acceptable.