Chapter 4: Expressions using Letter Numbers

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Exercise 4.1 — The Notion of Letter-Numbers (Page 84)

Q1 Write formulas for the perimeter of: (a) a triangle with all sides equal, (b) a regular pentagon, (c) a regular hexagon.

SOLUTION

Let the length of each side be 'a' units.

(a) Equilateral triangle has 3 equal sides.
Perimeter = a + a + a = 3a

(b) A regular pentagon has 5 equal sides.
Perimeter = a + a + a + a + a = 5a

(c) A regular hexagon has 6 equal sides.
Perimeter = a + a + a + a + a + a = 6a

Answer: (a) 3a (b) 5a (c) 6a

Q2 Munirathna has a 20 m long pipe. He joins another pipe of some length to this one. Give the expression for the combined length of the pipe. Use the letter-number 'k' to denote the length in metres of the other pipe.

SOLUTION

The first pipe has length 20 m.
The other pipe has length k m.
Combined length = 20 + k

Answer: (20 + k) metres

Q3 What is the total amount Krithika has if she has the following numbers of notes of Rs 100, Rs 20, and Rs 5? Complete the table: (i) 3 notes of Rs 100, 5 of Rs 20, 6 of Rs 5; (ii) 6 of Rs 100, 4 of Rs 20, 3 of Rs 5 (row given: expression = 6×100 + 4×20 + 3×5 = 695); (iii) 8 of Rs 100, 4 of Rs 20, z of Rs 5; (iv) x of Rs 100, y of Rs 20, z of Rs 5.

SOLUTION

(i) 3 notes of Rs 100, 5 of Rs 20, 6 of Rs 5:
Expression = 3 × 100 + 5 × 20 + 6 × 5 = 300 + 100 + 30 = 430

(ii) 6 of Rs 100, 4 of Rs 20, 3 of Rs 5 (given in table):
Expression = 6 × 100 + 4 × 20 + 3 × 5 = 600 + 80 + 15 = 695

(iii) 8 of Rs 100, 4 of Rs 20, z of Rs 5:
Expression = 8 × 100 + 4 × 20 + z × 5 = 800 + 80 + 5z = 880 + 5z

(iv) x of Rs 100, y of Rs 20, z of Rs 5:
Expression = x × 100 + y × 20 + z × 5 = 100x + 20y + 5z

Answer: (i) 3×100 + 5×20 + 6×5 = Rs 430 (ii) Rs 695 (given) (iii) 880 + 5z rupees (iv) 100x + 20y + 5z rupees

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Q4 Venkatalakshmi owns a flour mill. It takes 10 seconds for the roller mill to start running. Once it is running, each kg of grain takes 8 seconds to grind. Which expression describes the time taken to grind 'y' kg of grain (machine is off initially)? (a) 10 + 8 + y (b) (10 + 8) × y (c) 10 × 8 × y (d) 10 + 8 × y (e) 10 × y + 8

SOLUTION

The machine needs 10 seconds to start.
After starting, each of the y kg takes 8 seconds, so grinding time = 8 × y seconds.
Total time = startup time + grinding time = 10 + 8 × y.

Check option (a): 10 + 8 + y just adds y to 18, which is wrong.
Check option (b): (10 + 8) × y = 18y charges startup time per kg, which is wrong.
Check option (d): 10 + 8y = 10 + 8 × y. For y = 5: 10 + 40 = 50 seconds. Correct interpretation.

Answer: (d) 10 + 8 × y

Q5 Write algebraic expressions using letters of your choice: (a) 5 more than a number (b) 4 less than a number (c) 2 less than 13 times a number (d) 13 less than 2 times a number.

SOLUTION

Let 'd' represent the number.

(a) 5 more than d: d + 5
(b) 4 less than d: d − 4
(c) 13 times d is 13 × d, then 2 less: 13d − 2
(d) 2 times d is 2d, then 13 less: 2d − 13

Answer: (a) d + 5 (b) d − 4 (c) 13d − 2 (d) 2d − 13

Q6 Describe situations corresponding to the following algebraic expressions: (a) 8 × x + 3 × y (b) 15 × j − 2 × k.

SOLUTION

(a) 8 × x + 3 × y:
A shopkeeper sells pens at Rs x each and notebooks at Rs y each.
A customer buys 8 pens and 3 notebooks.
Total cost = 8x + 3y rupees.

(b) 15 × j − 2 × k:
A factory produces 15 chairs each day for j days, making 15j chairs in total.
Over k days, 2 chairs per day break, so 2k chairs are broken.
Number of good chairs remaining = 15j − 2k.

Answer: (a) Cost of 8 pens at Rs x each and 3 notebooks at Rs y each (b) Chairs produced in j days minus chairs broken in k days

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Q7 In a calendar month, if any 2 × 3 grid full of dates is chosen, write expressions for the dates in the blank cells if the bottom middle cell has date 'w'. (The grid shows the bottom row with w−1 and w pre-filled.)

SOLUTION

The 2 × 3 grid has 2 rows and 3 columns.
The bottom middle cell = w (given); the bottom left cell = w − 1 (given).

In a calendar, moving one column right adds 1 to the date.
Moving one row up subtracts 7 from the date (one week).

Bottom row: w − 1	w	w + 1
Top row: w − 8	w − 7	w − 6

Check: w − 1 is one column left of w ✓; w + 1 is one column right of w ✓
Top row is 7 less than the bottom row in each column ✓

Answer: Bottom row: w−1, w (given), w+1 | Top row: w−8, w−7, w−6

Exercise 4.2 — Simplification of Algebraic Expressions — Figure it Out (Page 93)

Q1 Add the numbers in each picture below. Write their corresponding expressions and simplify them. (i) Grid with 5y, −6, x (top row) and x, 2, 5y (bottom row). (ii) Grid 4×4 with 2p,3q,−2,3 / 3q,2p,3,−2 / 2p,3q / 3q,2p. (iii) Grid 4×4 with −5g,5k,5k,−5g / 5k,5k,5k,5k / 5k,5k,5k,5k / −5g,5k,5k,−5g.

SOLUTION

(i) Grid (2 rows × 3 columns): 5y, −6, x (top) and x, 2, 5y (bottom).
Method 1 — group by like terms:
Sum = 5y + (−6) + x + x + 2 + 5y
= (5y + 5y) + (x + x) + (−6 + 2)
= 10y + 2x − 4

(ii) 4×4 grid with 2p and 3q and constants.
2p appears 4 times: 4 × 2p = 8p
3q appears 4 times: 4 × 3q = 12q
Constants: −2 + 3 + 3 + (−2) = 2
Sum = 8p + 12q + 2

(iii) 4×4 grid with −5g and 5k.
−5g appears 4 times: 4 × (−5g) = −20g
5k appears 12 times: 12 × 5k = 60k
Sum = −20g + 60k

Answer: (i) 10y + 2x − 4 (ii) 8p + 12q + 2 (iii) 60k − 20g

Q2 Simplify each of the following expressions: (a) p+p+p+p; p+p+p+q (b) p+q+p−q (c) p−q+p−q (d) p+q−p+q (e) p+q−(p+q) (f) p−q−p−q (g) 2d−d−d−d (h) 2d−d−d−c (i) 2d−d−(d−c) (j) 2d−(d−d)−c (k) 2d−d−c−c.

SOLUTION

(a) p + p + p + p = 4p
p + p + p + q = 3p + q

(b) p + q + p − q = (p + p) + (q − q) = 2p

(c) p − q + p − q = (p + p) + (−q − q) = 2p − 2q

(d) p + q − p + q = (p − p) + (q + q) = 2q

(e) p + q − (p + q) = p + q − p − q = 0

(f) $p - q - p - q = (p - p) + (-q - q) = -2q$

(g) $2d - d - d - d = (2d - d - d) - d = 0 - d = -d$

(h) $2d - d - d - c = (2d - d - d) - c = 0 - c = -c$

(i) $2d - d - (d - c) = 2d - d - d + c = (2d - d - d) + c = 0 + c = c$

(j) $2d - (d - d) - c = 2d - 0 - c = 2d - c$

(k) $2d - d - c - c = (2d - d) - 2c = d - 2c$

Answer: (a) 4p; 3p+q (b) 2p (c) 2p−2q (d) 2q (e) 0 (f) −2q (g) −d (h) −c (i) c (j) 2d−c (k) d−2c

Exercise 4.3 — Mind the Mistake, Mend the Mistake — Algebraic Simplification (Page 94

Q1 Some simplifications of algebraic expressions are given with claimed simplest forms. Identify mistakes and give the correct simplest form for each: (1) 3a+2b = 5 (2) 3b−2b−b = 0 (3) 6(p+2) = 6p+8 (4) (4x+3y)−(3x+4y) = x+y (5) 5−(2−6z) = 3−6z (6) 2+(x+3) = 2x−6 (7) 2y+(3y−6) = −y+6 (8) 7p−p+5q−2q = 7p+3q (9) 5(2w+3x+4w) = 10w+15x+20w (10) 3j+6k+9h+12 = 3(j+2k+3h+4) (11) 4(2r+3s+5) = −20−8r−12s.

SOLUTION

(1) 3a+2b: The terms 3a and 2b are unlike (different letter-numbers), so they cannot be added to give 5.
Correct: 3a + 2b (already simplest form).

(2) 3b − 2b − b = (3−2−1)b = 0b = 0. Correct as given.

(3) 6(p+2): Using distributive property: 6×p + 6×2 = 6p + 12, not 6p+8.
Mistake: 6×2 was computed as 8 instead of 12.
Correct: 6p + 12.

(4) (4x+3y) − (3x+4y) = 4x + 3y − 3x − 4y = (4x−3x) + (3y−4y) = x − y, not x+y.
Mistake: sign of 4y was not changed when removing the bracket.
Correct: x − y.

(5) 5 − (2 − 6z) = 5 − 2 + 6z = 3 + 6z, not 3−6z.
Mistake: sign of −6z inside bracket was not reversed.
Correct: 3 + 6z.

(6) 2 + (x + 3) = 2 + x + 3 = x + 5, not 2x−6.
Mistake: incorrectly applied operations inside and outside bracket.
Correct: x + 5.

(7) 2y + (3y − 6) = 2y + 3y − 6 = 5y − 6, not −y+6.
Mistake: 2y and 3y were subtracted instead of added.
Correct: 5y − 6.

(8) 7p − p + 5q − 2q = (7−1)p + (5−2)q = 6p + 3q, not 7p+3q.
Mistake: −p was not subtracted from 7p.
Correct: 6p + 3q.

(9) 5(2w + 3x + 4w): First simplify inside brackets: 2w + 4w = 6w, so 5(6w + 3x) = 30w + 15x.
Mistake: like terms inside bracket were not combined first.

Correct: 30w + 15x.

(10) 3j + 6k + 9h + 12 = 3(j + 2k + 3h + 4). Check: 3×j+3×2k+3×3h+3×4 = 3j+6k+9h+12. Correct as given.

(11) 4(2r + 3s + 5) = 4×2r + 4×3s + 4×5 = 8r + 12s + 20, not −20−8r−12s.
Mistake: all signs were changed as if there were a negative outside, but there is a positive 4.
Correct: 8r + 12s + 20.

Answer: (1) 3a+2b (2) 0 ✓ (3) 6p+12 (4) x−y (5) 3+6z (6) x+5 (7) 5y−6 (8) 6p+3q (9) 30w+15x (10) 3(j+2k+3h+4) ✓ (11) 8r+12s+20

Exercise 4.4 — Pick Patterns and Reveal Relationships — Number Machines (Page 95)

Q1 Find the formulas for the two number machines and write the expression for each set of inputs. Machine 1 (blue): inputs (5,2)→5; (8,1)→7; (9,11)→18; (10,10)→18; (a,b)→? Machine 2 (purple): inputs (4,1)→5; (6,0)→1; (3,2)→7; (10,3)→?; (a,b)→?

SOLUTION

Machine 1 (blue):

Check: (5,2): 5+2−2=5 ✓; (8,1): 8+1−2=7 ✓; (9,11): 9+11−2=18 ✓; (10,10): 10+10−2=18 ✓

Formula: sum of the two inputs minus 2, i.e., a + b − 2.

Expressions: 5+2−2=5; 8+1−2=7; 9+11−2=18; 10+10−2=18; a+b−2

Machine 2 (purple):

Check: (4,1): 4×1+1=5 ✓; (6,0): 6×0+1=1 ✓; (3,2): 3×2+1=7 ✓

Formula: product of the two inputs plus 1, i.e., a × b + 1 = ab + 1.

Expressions: 4×1+1=5; 6×0+1=1; 3×2+1=7; 10×3+1=31; ab+1

Answer: Machine 1 formula: a + b − 2; (a,b) expression: a+b−2. Machine 2 formula: ab + 1; (10,3)→31; (a,b) expression: ab+1

Exercise 4.5 — Pick Patterns and Reveal Relationships — Figure it Out (Page 102)

Q1 One plate of Jowar roti costs Rs 30 and one plate of Pulao costs Rs 20. If x plates of Jowar roti and y plates of Pulao were ordered in a day, which expression(s) describe the total amount earned? (a) 30x+20y (b) (30+20)×(x+y) (c) 20x+30y (d) (30+20)×x+y (e) 30x−20y.

SOLUTION

Revenue from Jowar roti = Rs 30 per plate × x plates = 30x.
Revenue from Pulao = Rs 20 per plate × y plates = 20y.
Total = 30x + 20y.

Check (b): (30+20)×(x+y) = 50(x+y) = 50x+50y. This is wrong — it applies a uniform rate to everything.
Check (c): 20x+30y reverses the prices. Wrong.
Only (a) gives the correct total.

Answer: (a) 30x + 20y

Q2 Pushpita sells two types of flowers: champak and marigold. 'p' customers bought only champak, 'q' customers bought only marigold, and 'r' customers bought both. She gave a national flag to every customer. How many flags did she give away? (a) p+q+r (b) p+q+2r (c) 2×(p+q+r) (d) p+q+r+2 (e) p+q+r+1 (f) 2×(p+q).

SOLUTION

Each customer receives exactly one flag, regardless of how many types of flowers they bought.
Total customers = (only champak) + (only marigold) + (both) = p + q + r.
Flags given = total customers = p + q + r.

Note: customers who bought both are still counted once each (r customers, not 2r).

Answer: (a) p + q + r

Q3 A snail climbing a well: during the day it climbs u cm, during the night it slips down d cm. This happens for 10 days and 10 nights. (a) Write an expression for how far the snail is from its starting position. (b) What can we say about the snail's movement if d > u?

SOLUTION

(a) In each full day-night cycle:
Net movement = climbs up u − slips down d = u − d cm.
After 10 cycles: total displacement = 10 × (u − d) = 10(u − d) cm from starting position.
(Positive means above start; negative means below.)

(b) If d > u, then u − d < 0, so each cycle the snail moves downward.
After 10 cycles the snail is 10(u−d) cm below its starting position.
The snail moves away from the top of the well and will never reach the top.

Answer: (a) 10(u − d) cm from starting position (b) If d > u, the snail slips backward overall and will never reach the top

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Q4 Radha practices cycling daily. The first week she cycles 5 km every day. Every week she increases the daily distance by 'z' km. How many kilometres would Radha have cycled after 3 weeks?

SOLUTION

Week 1: daily distance = 5 km, for 7 days → 7 × 5 = 35 km.
Week 2: daily distance = 5 + z km, for 7 days → 7 × (5 + z) = 35 + 7z km.
Week 3: daily distance = 5 + 2z km, for 7 days → 7 × (5 + 2z) = 35 + 14z km.

Total distance = 35 + (35 + 7z) + (35 + 14z)
= 105 + 21z km.

Answer: 105 + 21z km

Q5 In the flow diagram, the expression w+2 is the starting point. One path (given): w+2 → (+3) → w+5 → (×4) → 4w+20. Another path ends at 3w−6. Fill in the missing blanks on all paths. The ovals contain expressions and the boxes contain operations.

SOLUTION

Given path (top-right): w+2 → +3 → w+5 → ×4 → 4w+20. ✓

Top-left path (going left from w+2):
w+2 → (−5) → w−3 → (×3) → 3(w−3) = 3w−9
Blank oval on the far left = 3w−9.

Bottom path (going through w+2):
Starting from the far left: ? → (−4) → ? → (−8) → w+2 → (−4) → w−2 → (×3) → 3(w−2) = 3w−6
Working backwards from w+2: +8 → w+10; then +4 → w+14.
So the bottom chain is: w+14 → −4 → w+10 → −8 → w+2 → −4 → w−2 → ×3 → 3w−6.
Blank ovals: w+14, w+10, w−2.

Answer: Top-left end: 3w−9. Bottom blanks (left to right): w+14, w+10, w−2.

Q6 A local train from Yahapur to Vahapur stops at three stations at equal distances. Travel time between any two consecutive stations is t minutes. The train stops for 2 minutes at each of the three stations. (a) If t = 4, what is the total travel time? (b) Write the algebraic expression for the total travel time.

SOLUTION

The route: Yahapur → Station 1 → Station 2 → Station 3 → Vahapur = 4 segments.
Each segment takes t minutes.
The train stops at 3 intermediate stations for 2 minutes each.

(b) Total time = 4t (travel) + 3 × 2 (stops) = 4t + 6 minutes.

(a) When t = 4: Total time = 4 × 4 + 6 = 16 + 6 = 22 minutes.

Answer: (a) 22 minutes (b) 4t + 6 minutes

Q7 Simplify the following expressions: (a) 3a+9b−6+8a−4b−7a+16 (b) 3(3a−3b)−8a−4b−16 (c) 2(2x−3)+8x+12 (d) 8x−(2x−3)+12 (e) 8h−(5+7h)+9 (f) 23+4(6m−3n)−8n−3m−18.

SOLUTION

(a) 3a + 9b − 6 + 8a − 4b − 7a + 16
= (3a + 8a − 7a) + (9b − 4b) + (−6 + 16)
= 4a + 5b + 10

(b) 3(3a − 3b) − 8a − 4b − 16
= 9a − 9b − 8a − 4b − 16
= (9a − 8a) + (−9b − 4b) − 16
= a − 13b − 16

(c) 2(2x − 3) + 8x + 12
= 4x − 6 + 8x + 12
= (4x + 8x) + (−6 + 12)
= 12x + 6

(d) 8x − (2x − 3) + 12
= 8x − 2x + 3 + 12
= 6x + 15

(e) 8h − (5 + 7h) + 9
= 8h − 5 − 7h + 9
= (8h − 7h) + (−5 + 9)
= h + 4

(f) 23 + 4(6m − 3n) − 8n − 3m − 18
= 23 + 24m − 12n − 8n − 3m − 18
= (24m − 3m) + (−12n − 8n) + (23 − 18)
= 21m − 20n + 5

Answer: (a) 4a+5b+10 (b) a−13b−16 (c) 12x+6 (d) 6x+15 (e) h+4 (f) 21m−20n+5

Q8 Add the expressions given below: (a) 4d−7c+9 and 8c−11+9d (b) −6f+19−8s and −23+13f+12s (c) 8d−14c+9 and 16c−(11+9d) (d) 6f−20+8s and 23−13f−12s (e) 13m−12n and 12n−13m (f) −26m+24n and 26m−24n.

SOLUTION

(a) (4d − 7c + 9) + (8c − 11 + 9d)
= (4d + 9d) + (−7c + 8c) + (9 − 11)
= 13d + c − 2

(b) (−6f + 19 − 8s) + (−23 + 13f + 12s)
= (−6f + 13f) + (−8s + 12s) + (19 − 23)
= 7f + 4s − 4

(c) (8d − 14c + 9) + (16c − (11 + 9d))
= (8d − 14c + 9) + (16c − 11 − 9d)
= (8d − 9d) + (−14c + 16c) + (9 − 11)
= −d + 2c − 2

(d) (6f − 20 + 8s) + (23 − 13f − 12s)
= (6f − 13f) + (8s − 12s) + (−20 + 23)
= −7f − 4s + 3

(e) (13m − 12n) + (12n − 13m)
= (13m − 13m) + (−12n + 12n)
= 0

(f) (−26m + 24n) + (26m − 24n)
= (−26m + 26m) + (24n − 24n)
= 0

Answer: (a) 13d+c−2 (b) 7f+4s−4 (c) 2c−d−2 (d) −7f−4s+3 (e) 0 (f) 0

Q9 Subtract the expressions given below: (a) 9a−6b+14 from 6a+9b−18 (b) −15x+13−9y from 7y−10+3x (c) 17g+9−7h from 11−10g+3h (d) 9a−6b+14 from 6a−(9b+18) (e) 10x+2+10y from −3y+8−3x (f) 8g+4h−10 from 7h−8g+20.

SOLUTION

(a) (6a + 9b − 18) − (9a − 6b + 14)
= 6a + 9b − 18 − 9a + 6b − 14
= (6a − 9a) + (9b + 6b) + (−18 − 14)
= −3a + 15b − 32

(b) (7y − 10 + 3x) − (−15x + 13 − 9y)
= 7y − 10 + 3x + 15x − 13 + 9y
= (3x + 15x) + (7y + 9y) + (−10 − 13)
= 18x + 16y − 23

(c) (11 − 10g + 3h) − (17g + 9 − 7h)
= 11 − 10g + 3h − 17g − 9 + 7h
= (−10g − 17g) + (3h + 7h) + (11 − 9)
= −27g + 10h + 2

(d) (6a − (9b + 18)) − (9a − 6b + 14)
= (6a − 9b − 18) − (9a − 6b + 14)
= 6a − 9b − 18 − 9a + 6b − 14
= (6a − 9a) + (−9b + 6b) + (−18 − 14)
= −3a − 3b − 32

(e) (−3y + 8 − 3x) − (10x + 2 + 10y)
= −3y + 8 − 3x − 10x − 2 − 10y
= (−3x − 10x) + (−3y − 10y) + (8 − 2)
= −13x − 13y + 6

(f) (7h − 8g + 20) − (8g + 4h − 10)
= 7h − 8g + 20 − 8g − 4h + 10
= (7h − 4h) + (−8g − 8g) + (20 + 10)
= 3h − 16g + 30

Answer: (a) −3a+15b−32 (b) 18x+16y−23 (c) −27g+10h+2 (d) −3a−3b−32 (e) −13x−13y+6 (f) 3h−16g+30

Q10 Describe situations corresponding to the following algebraic expressions: (a) 8x + 3y (b) 15x − 2x.

SOLUTION

(a) 8x + 3y:
A fruit seller sells mangoes at Rs x each and bananas at Rs y each.
A customer buys 8 mangoes and 3 bananas.
Total cost = 8x + 3y rupees.

(b) 15x − 2x:
A shopkeeper buys 15 items each at cost Rs x, totalling 15x.
He returns 2 items (worth 2x) and gets a refund.

Net amount spent = 15x − 2x rupees.
(This simplifies to 13x, i.e., the cost of 13 items.)

Answer: (a) Total cost of 8 mangoes at Rs x and 3 bananas at Rs y (b) Cost of 15 items at Rs x each after returning 2 items

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Q11 A straight rope: if cut once → 2 pieces; folded once and cut → 3 pieces. Observe the pattern and find: how many pieces when folded 10 times and cut? What is the expression for pieces when folded r times and cut?

SOLUTION

Observe the pattern:
0 folds, 1 cut → 2 pieces
1 fold, 1 cut → 3 pieces
2 folds, 1 cut → 4 pieces

Each additional fold adds 1 more piece.
Pattern: r folds → r + 2 pieces.

For r = 10: number of pieces = 10 + 2 = 12.

Answer: 10 folds → 12 pieces; general expression: r + 2 pieces

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Q12 Look at the matchstick pattern of squares placed in a row (each adjacent pair shares one side). How many matchsticks are required to make 10 such squares? How many are required to make w squares?

SOLUTION

For 1 square: 4 matchsticks.
For 2 squares: 4 + 3 = 7 matchsticks (second square shares 1 side with first, needs only 3 new).
For 3 squares: 4 + 3 + 3 = 10 matchsticks.

Pattern: first square needs 4; each additional square needs 3.
For w squares: 4 + 3(w − 1) = 4 + 3w − 3 = 3w + 1.

For 10 squares: 3 × 10 + 1 = 31 matchsticks.
Verify: 4 + 3 × 9 = 4 + 27 = 31. ✓

Answer: 10 squares: 31 matchsticks; w squares: 3w + 1 matchsticks

Q13 A traffic signal cycles through colours in the sequence: Red (pos 1), Yellow (pos 2), Green (pos 3), Yellow (pos 4), Red (pos 5), … Find the colour at positions 90, 190, and 343. Write expressions to describe positions for each colour.

SOLUTION

The sequence repeats every 4 positions: Red, Yellow, Green, Yellow.

To find the colour at position n, compute n mod 4:
Remainder 1 → Red; Remainder 2 → Yellow; Remainder 3 → Green; Remainder 0 → Yellow.

Position 90: 90 ÷ 4 = 22 remainder 2 → Yellow.
Position 190: 190 ÷ 4 = 47 remainder 2 → Yellow.
Position 343: 343 ÷ 4 = 85 remainder 3 → Green.

Expressions for each colour (n = 1, 2, 3, …):
Red positions: $4n - 3$ → 1, 5, 9, 13, …
Green positions: $4n - 1$ → 3, 7, 11, 15, …
Yellow positions: $2n$ → 2, 4, 6, 8, 10, 12, … (all even positions)

Answer: Position 90: Yellow; Position 190: Yellow; Position 343: Green. Expressions — Red: $4n-3$; Green: $4n-1$; Yellow: $2n$

Q14 Observe the growing cross/plus-shaped square pattern (Step 1 has 5 squares; each step adds 4 squares, one to each arm). How many squares in Step 4, Step 10, Step 50? Write a general formula. How does the formula change for counting vertices of all squares?

SOLUTION

Step 1: 5 squares. Step 2: 9 squares. Step 3: 13 squares.
Each step adds 4 squares, so the sequence is 5, 9, 13, 17, … (arithmetic sequence, common difference 4).

For step n: number of squares = $5 + 4(n - 1) = 4n + 1$.

Step 4: $4 \times 4 + 1 = 17$ squares.
Step 10: $4 \times 10 + 1 = 41$ squares.
Step 50: $4 \times 50 + 1 = 201$ squares.

For vertices: each square has 4 corners, so total vertices = 4 × (number of squares) = $4(4n + 1) = 16n + 4$.
(This counts all 4 vertices of each square, including shared ones.)

Answer: Step 4: 17 squares; Step 10: 41 squares; Step 50: 201 squares; Formula: 4n+1; Vertices formula: 16n+4

Q15 Numbers are written in a 4-column endless grid (rows: 1,2,3,4 / 5,6,7,8 / 9,10,11,12 …). (a) Give expressions for all numbers in each column (1, 2, 3, 4). (b) Find row and column for: (i) 124 (ii) 147 (iii) 201. (c) What number appears in row r and column c? (d) Observe the positions of multiples of 3.

SOLUTION

(a) Let r be the row number (starting from 1). In row r, the first number is 4(r−1)+1.
Column 1: 4(r − 1) + 1
Column 2: 4(r − 1) + 2
Column 3: 4(r − 1) + 3
Column 4: 4(r − 1) + 4
General: column c in row r → 4(r − 1) + c.

(b) To find row and column of a number N: divide (N − 1) by 4.
Quotient = r − 1, so row = quotient + 1. Column = remainder + 1.

(i) 124: (124 − 1) ÷ 4 = 123 ÷ 4 = 30 remainder 3. Row = 31, Column = 4.
Verify: 4(31 − 1) + 4 = 120 + 4 = 124. ✓

(ii) 147: (147 − 1) ÷ 4 = 146 ÷ 4 = 36 remainder 2. Row = 37, Column = 3.
Verify: 4(37 − 1) + 3 = 144 + 3 = 147. ✓

(iii) 201: (201 − 1) ÷ 4 = 200 ÷ 4 = 50 remainder 0. Row = 51, Column = 1.
Verify: 4(51 − 1) + 1 = 200 + 1 = 201. ✓

(c) Number in row r and column c = 4(r − 1) + c.

(d) Multiples of 3 appear in columns that follow a repeating pattern: 3, 2, 1, 4, 3, 2, 1, 4, …
Other patterns: all multiples of 4 appear in column 4; all even numbers appear in columns 2 and 4; all odd numbers appear in columns 1 and 3.

Answer: (a) Column c, row r: 4(r−1)+c (b)(i) Row 31, Col 4 (ii) Row 37, Col 3 (iii) Row 51, Col 1 (c) 4(r−1)+c (d) Multiples of 3 cycle through columns 3,2,1,4,3,2,1,4,…