Chapter 5: Parallel and Intersecting Lines

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Exercise 5.1 — Across the Line — Intersecting Lines (Page 108)

Q1 In Fig. 5.3, lines l and m intersect at a point forming four angles labelled a, b, c, d (a is top-left, b is top-right, c is bottom-right, d is bottom-left). List all the linear pairs and all the pairs of vertically opposite angles.

SOLUTION

A linear pair is a pair of adjacent angles whose non-common arms form a straight line (they add up to 180°).

Linear pairs (adjacent angles on a straight line):
∠a and ∠b (together they form the straight line m, from left to right above the intersection)
∠b and ∠c (together they form the straight line l, from top to bottom on the right)
∠c and ∠d (together they form the straight line m, from right to left below the intersection)
∠d and ∠a (together they form the straight line l, from bottom to top on the left)

Vertically opposite angles are the angles across from each other at an intersection. They are always equal.

Pairs of vertically opposite angles:
∠a and ∠c (opposite each other across the intersection)
∠b and ∠d (opposite each other across the intersection)

Answer: Linear pairs: (∠a, ∠b), (∠b, ∠c), (∠c, ∠d), (∠d, ∠a). Vertically opposite angle pairs: (∠a, ∠c) and (∠b, ∠d).

Exercise 5.2 — Between Lines — Parallel Lines (Page 110)

Q1 In Fig. 5.6, several line segments are drawn on a dot paper: a (diagonal, top-left), b (diagonal, left-lower), c (diagonal, crossing through the grid), d (short diagonal, top-centre), e (diagonal, top-right area), f (diagonal, lower-right area), g (diagonal, centre-right), h (short, lower-centre), i (short, lower-centre). Which pairs of lines appear to be parallel?

SOLUTION

Parallel lines are lines that lie on the same plane and never meet, no matter how far they are extended.

To spot parallel lines on the dot paper, we look for line segments that run in the same direction and are always the same distance apart.

From Fig. 5.6, the pairs that appear to be parallel are:
a, i and h — these three line segments have the same slope/direction and are equally spaced.
c and g — these two run in the same direction.
d and f — these two run in the same direction.
e and b — these two run in the same direction.

Answer: Lines a, i, h are parallel to each other; c and g are parallel; d and f are parallel; e and b are parallel.

Exercise 5.3 — Parallel and Perpendicular Lines in Paper Folding (Pages 113–114)

Q1 In Fig. 5.11, a grid-based figure is shown with various line segments forming shapes. Mark the parallel lines using arrow notation (single arrow, double arrow, etc.) and mark perpendicular lines with a square symbol. (a) How did you spot the perpendicular lines? (b) How did you spot the parallel lines?

SOLUTION

Step 1: Identify perpendicular lines.
Perpendicular lines meet at a right angle (90°). On a grid, any horizontal line and any vertical line are perpendicular to each other.
Mark a small square (□) at each point where a horizontal and vertical line meet.

Step 2: Identify parallel lines.
Parallel lines run in the same direction and are always the same distance apart.
On the grid: all horizontal lines are parallel to each other; all vertical lines are parallel to each other; all diagonal lines with the same slope are parallel.
Mark them with single arrows (→) for one set, double arrows (⇒) for another set, and so on.

(a) The perpendicular lines were spotted because they are the horizontal and vertical lines on the grid — they meet at exactly 90° (right angles), which can be confirmed by the square corners of each grid cell.

(b) The parallel lines were spotted by identifying lines that always remain the same distance apart and never meet. On the grid, all lines going in the same direction (e.g., all horizontals, all verticals, all diagonals with the same slope) are parallel.

Answer: (a) Perpendicular lines are spotted because horizontal and vertical lines meet at 90° (right angles) — visible from the square grid corners. (b) Parallel lines are spotted by finding lines that go in the same direction and are always the same distance apart.

Q2 In Fig. 5.13, three line segments a, b, and c are shown: a is at the bottom, c is just above and parallel-looking to a, and b is slightly above c. Which line is parallel to line a — line b or line c? How do you decide this?

SOLUTION

Parallel lines are lines that remain the same distance apart everywhere and never meet.

From Fig. 5.13: line c and line a run in the same direction (same slope/orientation) and are always the same distance apart. Line b, while close to a, has a slightly different direction.

Decision method: If we draw a transversal crossing all three lines and measure the corresponding angles, the corresponding angles between the transversal and line a must equal the corresponding angles between the transversal and the parallel line.

For line c: the angle it makes with any transversal is the same as the angle line a makes → so c ∥ a.
For line b: the angle it makes with a transversal is different from line a → so b is not parallel to a.

Answer: Line c is parallel to line a. They always remain the same distance apart and run in the same direction (their corresponding angles with any transversal are equal).

Exercise 5.4 — Drawing Parallel Lines (Page 119)

Q1 In Fig. 5.23, a line l and a point A (above the line) are given. Can you draw a line parallel to l that goes through point A? Describe your method using the tools from your geometry box.

SOLUTION

Yes, we can draw a line parallel to l through point A using a ruler and set square.

Method:

Step 1: Place the set square so that one of its sides lies exactly along line l.

Step 2: Place a ruler firmly against the other (perpendicular) side of the set square, so the ruler acts as a guide.

Step 3: Hold the ruler fixed and slide the set square along the ruler until one of its sides passes through point A.

Step 4: Draw a line along that side of the set square, passing through point A.

This new line is parallel to l and passes through A.

Why it works: Both lines (l and the new line) make the same angle with the set square's perpendicular edge. This means the corresponding angles formed by any transversal are equal, which guarantees the lines are parallel.

Answer: Draw a line through A parallel to l using a ruler and set square: align one side of the set square along l, hold the ruler against the perpendicular side, slide the set square until it reaches A, then draw the line.

Exercise 5.5 — Alternate Angles and Parallel Line Properties (Pages 123–125)

Q1 Find the angles marked a through j in Fig. 5.30. Each sub-figure shows either two parallel lines (marked with arrows) cut by a transversal, or lines meeting at a point. The given angles are: (a) 48°; (b) 52° with parallel lines; (c) 99° and 81° with parallel lines; (d) 81° and 99° at lower intersection with parallel lines, d° at upper; (e) 97°, 83°, and 69° at a multi-line intersection; (f) 132° with parallel lines; (g) 58°, 122°, 58°, 122° at a crossing of two parallel lines with transversal; (h) 75° at an intersection; (i) two parallel rays (arrow marks) rise from a base line and a transversal crosses them — at the base line the three angles 70°, 54°, 56° are shown side by side on one straight line, and i° is marked at the apex where a ray meets the transversal; (j) three lines form a triangle with two of them parallel (arrow marks); the given angles are 27° (top apex), 97° (left intersection) and 124° (right intersection on the base line), with j° at the lower intersection of the two transversals.

SOLUTION

SOLUTION

(a) a°: The two lines cut by a transversal have arrows (parallel lines). a° is vertically opposite to 48°.
Vertically opposite angles are equal.
a = 48°

(b) b°: Two parallel lines (arrow marks) cut by a transversal; 52° is at the upper intersection and b° is at the lower intersection, on the same side.
Corresponding angles (parallel lines, same transversal) are equal.
b = 52°

(c) c°: Two parallel lines cut by a transversal; 99° and 81° are at the upper intersection (linear pair: 99 + 81 = 180 ✓). c° is at the lower intersection corresponding to 81°.
Corresponding angles (parallel lines) are equal.
c = 81°

(d) d°: Two parallel lines cut by a transversal; 81° and 99° form a linear pair at the lower intersection. d° is at the upper intersection on the side corresponding to 99°.
Corresponding angles (parallel lines) are equal, so d corresponds to 99°.
d = 99°

(e) e°: Three lines meeting at one point; 97° and 83° form a linear pair (97 + 83 = 180°) from one straight line. 69° is from a second line; e° is vertically opposite to 69°.
Vertically opposite angles are equal.
e = 69°

(f) f°: Two parallel lines (arrow marks) cut by a transversal; 132° is at the lower intersection and f° is at the upper intersection on the same side of the transversal.
Co-interior angles (interior angles on the same side of a transversal between parallel lines) add up to 180°.
f + 132 = 180
f = 180 − 132 = 48°

(g) g°: Two parallel lines (nearly vertical, with arrow marks) cut by a diagonal transversal; the crossing shows four angles: 58°, 122°, 58°, 122° (alternating). g° is at the upper-left position, which corresponds to 122°.
Corresponding angles (parallel lines) are equal.
g = 122°

(h) h°: Two lines intersecting; 75° is given and h° is on the opposite side of the intersection.
Vertically opposite angles are equal.
h = 75°

(i) i°: Two rays rise from the base line and are parallel (arrow marks); a transversal cuts them. At the base, the three angles 70°, 54°, 56° lie on the straight base line (70 + 54 + 56 = 180 ✓). i° is at the apex, where the transversal meets a parallel ray.
The transversal makes 54° with the base line at the point where it crosses. i° is the alternate interior angle to that 54° between the two parallel rays.
Alternate interior angles between parallel lines are equal.
i = 54°

(j) j°: Three lines form a triangle; two of them are parallel (arrow marks) and are cut by two transversals. The 97° angle is at the left intersection and j° is at the lower intersection.
j° and 97° are alternate interior angles formed by the two parallel lines and the same transversal.
Alternate interior angles between parallel lines are equal.
j = 97°

Answer: a = 48°, b = 52°, c = 81°, d = 99°, e = 69°, f = 48°, g = 122°, h = 75°, i = 54°, j = 97°.

Q2 Find the angle represented by a in each sub-figure of Fig. 5.31. (i) Three parallel lines (with arrows) cut by transversals; 42° is at the upper-right and 100°, a° are at the lower crossing. (ii) Two parallel lines cut by transversal; 62° is at upper crossing, a° is at the lower crossing on the opposite side. (iii) Three parallel horizontal lines cut by a bent transversal; 110° is at the top crossing (upper-left angle), 35° is at the bend on the middle parallel, a° is at the lower parallel. (iv) A corner with a right angle (90°) and 67° inside; a° is the remaining angle on the straight line.

SOLUTION

SOLUTION

(i) The given angles include 42° at the upper intersection and 100°, a° at the lower intersection. The lines are parallel (arrow marks).
42° and a° are co-interior angles (on the same side of the transversal, between the parallel lines).
Co-interior angles between parallel lines add up to 180°.
a + 42 = 180
a = 180 − 42 = 138°

(ii) Two parallel lines; 62° is at the upper intersection (one side) and a° is at the lower intersection (opposite side).
62° and a° are co-interior angles (same side of the transversal, between the parallel lines).
a + 62 = 180
a = 180 − 62 = 118°

(iii) Three parallel horizontal lines with a bent transversal. The first segment makes 110° with the top parallel (upper-left angle = 110°, so the lower-left angle at the top parallel = 180° − 110° = 70°).
At the middle parallel, the transversal bends; 35° is the angle between the two segments of the bent transversal at the middle parallel.
The second segment of the transversal makes an angle of 70° + 35° = 105° with the middle parallel (on the upper-left side).
Since the middle parallel and the bottom parallel are parallel, a° (at the bottom parallel) is a corresponding angle to 105°.

a = 105°

(iv) The figure shows a right-angle corner (90°) and 67° inside, with a° as the remaining angle on the straight line at the base.
Angles on a straight line add up to 180°.
a + 90 + 67 = 180
a = 180 − 90 − 67 = 23°

Answer: (i) a = 138°, (ii) a = 118°, (iii) a = 105°, (iv) a = 23°.

Q3 In Fig. 5.32, what angles do x and y stand for? (i) Two parallel horizontal lines: a vertical transversal meets the upper parallel at a right angle (90°); a diagonal line meets the lower parallel at 65° on the right; x° is the angle between the diagonal and the vertical transversal below the lower parallel; y° is the supplement of x. (ii) Two parallel horizontal lines: two transversals meet the lower parallel at one point, making 53° (left) and 78° (right); they meet the upper parallel at a common point where x° is the angle between them.

SOLUTION

SOLUTION

(i) The vertical transversal is perpendicular to both parallel lines (corresponding angles = 90°). The diagonal line meets the lower parallel at 65°.
Below the lower parallel, x° is the angle between the downward vertical and the diagonal going to the lower-left.
The vertical makes 90° with the lower parallel. The diagonal makes 65° with the lower parallel.
x = 90° − 65° = 25°
y and x are supplementary (they form a straight line together).
y = 180° − 25° = 155°

(ii) Two transversals from a point T on the upper parallel go down to a single point O on the lower parallel. At O, one transversal makes 53° with the lower parallel and the other makes 78°.
The angle between the two transversals at O (above the lower parallel) = 78° − 53° = 25°.
Since the upper and lower lines are parallel, corresponding angles are equal. x° at T (angle between the two transversals) equals the angle between them at O.
x = 25°

Answer: (i) x = 25°, y = 155°. (ii) x = 25°.

Q4 In Fig. 5.33, two parallel horizontal lines are shown: the upper line passes through points I–K–B–A (left to right) and the lower line passes through G–E–D (left to right). One transversal runs J–K–E–F (J at top, then down through K on the upper line, through E on the lower line, ending at F below-right of E); the other transversal runs C–B–E–H (C at top-right, then down through B on the upper line, through E, ending at H below-left of E). Both transversals meet at the same point E on the lower parallel line. ∠ABC = 45° (at B) and ∠IKJ = 78° (at K). Find angles ∠GEH, ∠HEF, and ∠FED.

SOLUTION

SOLUTION

The upper and lower parallel lines are both cut by two transversals meeting at E. ∠GEH, ∠HEF, and ∠FED are three angles below the lower parallel line at E; they lie on a straight line so they add up to 180°.

Step 1: Find ∠GEH using transversal C–B–E–H.
The transversal C–B–E–H passes through B (upper parallel) and E (lower parallel); its lower end is H (below-left of E).
∠ABC = 45°: the angle at B between BA (rightward along the upper parallel) and BC (upward) = 45°.
G is to the left on the lower parallel and H lies on this same transversal below E.
∠GEH is the alternate interior angle to ∠ABC (same transversal C–B–E–H, parallel lines AB ∥ GD).
Alternate interior angles between parallel lines are equal.
∠GEH = ∠ABC = 45°

Step 2: Find ∠FED using transversal J–K–E–F.
The transversal J–K–E–F passes through K (upper parallel) and E (lower parallel); its lower end is F (below-right of E).
∠IKJ = 78°: the angle at K between KI (leftward along the upper parallel) and KJ (upward) = 78°.
D is to the right on the lower parallel and F lies on this same transversal below E.
∠FED is the alternate interior angle to ∠IKJ (same transversal J–K–E–F, parallel lines AB ∥ GD).
∠FED = ∠IKJ = 78°

Step 3: Find ∠HEF.
∠GEH + ∠HEF + ∠FED = 180° (angles on straight line GED below the parallel)
45° + ∠HEF + 78° = 180°
∠HEF = 180° − 45° − 78° = 57°

Verification: 45° + 57° + 78° = 180° ✓

Answer: ∠GEH = 45°, ∠HEF = 57°, ∠FED = 78°.

Q5 In Fig. 5.34, AB, CD, and EF are three parallel lines (vertical, with upward arrows). EA is a horizontal line perpendicular to all three. A diagonal line passes through B (on AB), D (on CD), and F (on EF), making ∠BEF = 55° at E (the angle between this diagonal and EF). x° is the angle the diagonal makes with AB at B; y° is the angle it makes with CD at D. Find x and y.

SOLUTION

SOLUTION

Given: AB ∥ CD ∥ EF, and the diagonal line cuts all three parallel lines.
∠BEF = 55° is the acute angle between the diagonal and line EF at E.

Since AB ∥ CD ∥ EF, any transversal makes equal corresponding angles with all three parallel lines.
So the diagonal makes the same angle with AB, CD, and EF.

However, x° and y° are the angles on the other side of 55°, since from the figure x° and y° are the obtuse angles at B and D respectively.
x° and 55° are co-interior angles (same side of the transversal, between the parallel lines AB and EF).
Co-interior angles between parallel lines add up to 180°.
x + 55 = 180
x = 180 − 55 = 125°

Similarly, y° and 55° are co-interior angles (between CD and EF).
y + 55 = 180
y = 180 − 55 = 125°

Answer: x = 125°, y = 125°.

Q6 In Fig. 5.35, L is at the top and M is at the bottom-left; LM is a straight line. N is a point on a horizontal line LN (to the right of L). O is below-right of N. P is to the upper-right and Q is below P; PQ is a straight line. ∠LNO = 96° (angle at N between NL leftward and NO going down-right) and ∠OPQ = 52° (angle at P between OP going up-left and PQ going down). ∠NLM = 40° (angle at L between LN going right and LM going down-left). Find ∠NOP (angle at O). [Hint: Draw lines parallel to LM and PQ through points N and O.]

SOLUTION

SOLUTION

Step 1: Draw a line through N parallel to LM, and a line through O parallel to PQ. Call the line through N parallel to LM as n_1, and the line through O parallel to PQ as n_2.

Step 2: Find the angle between n_1 and NO at N.
$n_1 \parallel$ LM, and LN is a transversal cutting both LM (at L) and n_1 (at N).
By alternate interior angles (since $n_1 \parallel$ LM, transversal LN):
$\angle(n_1, NL)$ at N = ∠NLM at L = 40°
So the angle between n_1 and NL (at N) = 40°.
Since ∠LNO = 96°, the angle between n_1 and NO at N = ∠LNO − 40° = 96° − 40° = 56°.

Step 3: Find the angle between n_2 and ON at O.
$n_2 \parallel$ PQ, and OP is a transversal cutting both PQ (at P) and n_2 (at O).
By alternate interior angles (since $n_2 \parallel$ PQ, transversal OP):
$\angle(n_2, OP)$ at O = ∠OPQ at P = 52°

Step 4: Since $n_1 \parallel n_2$ (both parallel to the same family of lines), NO acts as a transversal.
The alternate interior angles give: $\angle(n_1, NO) = \angle(n_2, ON) = 56°$.

Step 5: Find ∠NOP.
$\angle NOP = \angle(NO, n_2) + \angle(n_2, OP) = 56° + 52° = 108°$

Verification: 56 + 52 = 108 ✓

Answer: ∠NOP = 108°.