Chapter 6: Number Play

CONTENTS 8 exercises · 33 questions

6.1	Numbers Tell us Things (Page 128)	2 Q
6.2	Picking Parity (Page 131)	3 Q
6.3	Small Squares in Grids and Parity of Expressions (Page 131–132)	3 Q
6.4	Fill the Grids (Page 133)	2 Q
6.5	Magic Squares (Page 136)	5 Q
6.6	Generalising a 3 × 3 Magic Square (Page 137)	5 Q
6.7	The Virahāṅka–Fibonacci Sequence (Page 142)	2 Q
6.8	Digits in Disguise (Page 143–144)	11 Q

Exercise 6.1 — Numbers Tell us Things (Page 128)

Q1 Arrange the stick figure cutouts (or draw a height arrangement) such that when each child calls out the number of people in front of them who are taller, the sequence reads:
(a) 0, 1, 1, 2, 4, 1, 5
(b) 0, 0, 0, 0, 0, 0, 0
(c) 0, 1, 2, 3, 4, 5, 6
(d) 0, 1, 0, 1, 0, 1, 0
(e) 0, 1, 1, 1, 1, 1, 1
(f) 0, 0, 0, 3, 3, 3, 3

SOLUTION

The rule: each child counts the number of children in front of them (to their left) who are taller than them.

(a) 0, 1, 1, 2, 4, 1, 5
Using the stick-figure cutouts at the end of the book (labelled by height), one valid arrangement is: F, C, B, G, A, D, E.
The first person always says 0 (no one in front). Each later number counts how many people to that person's left are taller, so choose the heights so that each position produces its required count. Many valid arrangements exist.

(b) 0, 0, 0, 0, 0, 0, 0
Every child says 0, meaning no one to their left is taller. This requires heights to be strictly decreasing from left to right (each child is shorter than all children to their left).

(c) 0, 1, 2, 3, 4, 5, 6
Each child has exactly 1 more taller person in front than the previous child. This requires heights to be strictly increasing from left to right (each child is taller than all children to their left).

(d) 0, 1, 0, 1, 0, 1, 0
Positions 1, 3, 5, 7 say 0 (no one taller in front of them); positions 2, 4, 6 say 1 (exactly one taller person in front). Arrange by alternating taller-shorter-taller-shorter… heights.

(e) 0, 1, 1, 1, 1, 1, 1
Person 1 says 0. Every other person says 1 (exactly one taller person in front of them). Person 1 must be the tallest; all others see only person 1 as taller.

(f) 0, 0, 0, 3, 3, 3, 3
Persons 1, 2, 3 say 0 (no taller person in front of each of them within positions 1–3). Persons 4, 5, 6, 7 each say 3 (exactly 3 taller people in front of them). The first 3 children must be the tallest 3 (in some decreasing order), and the last 4 children must each be shorter than all three of the first three.

Answer: Arrangements exist for all parts. Key: the first person always says 0; subsequent numbers depend on relative heights of those to the left.

Q2 For each statement, identify if it is Always True, Only Sometimes True, or Never True. Share your reasoning.
(a) If a person says '0', then they are the tallest in the group.
(b) If a person is the tallest, then their number is '0'.
(c) The first person's number is '0'.
(d) If a person is not first or last in line, they cannot say '0'.
(e) The person who calls out the largest number is the shortest.
(f) What is the largest number possible in a group of 8 people?

SOLUTION

(a) If a person says '0', then they are the tallest in the group. → Only Sometimes True

A person says 0 when no one in front of them is taller. The first person always says 0 (no one in front), but is not necessarily the tallest. Also in a group of decreasing heights, everyone says 0, but only the first is tallest.

(b) If a person is the tallest, then their number is '0'. → Always True
The tallest person has nobody taller than them anywhere. So nobody in front of them can be taller. They always say 0. This is always true regardless of their position.

(c) The first person's number is '0'. → Always True
The first person has nobody in front of them, so the count of taller people in front is 0. This is always true.

(d) If a person is not first or last in line, they cannot say '0'. → Only Sometimes True
A middle person CAN say 0 if no one to their left is taller than them (for example, they could be the tallest person seen so far). In a decreasing sequence, every person says 0.

(e) The person who calls out the largest number is the shortest. → Only Sometimes True
A high number means many people in front are taller, but there could be someone else who is even shorter but positioned differently (with fewer tall people in front).

(f) Largest number in a group of 8 people:
The last person (8th position) can have at most 7 people in front of them. If all 7 are taller than the 8th person, they say 7.
Answer: 7

Answer: (a) Only Sometimes True (b) Always True (c) Always True (d) Only Sometimes True (e) Only Sometimes True (f) 7

Exercise 6.2 — Picking Parity (Page 131)

Q1 Using your understanding of the pictorial representation of odd and even numbers, find out the parity of the following sums:
(a) Sum of 2 even numbers and 2 odd numbers (e.g., even + even + odd + odd)
(b) Sum of 2 odd numbers and 3 even numbers
(c) Sum of 5 even numbers
(d) Sum of 8 odd numbers

SOLUTION

Key rules: even + even = even; odd + odd = even; even + odd = odd.

(a) even + even + odd + odd
= (even + even) + (odd + odd)
= even + even
= even

(b) odd + odd + even + even + even
= (odd + odd) + (even + even + even)
= even + even
= even

(c) Sum of 5 even numbers
Any sum of even numbers is always even (even = 2k, so sums of 2k values are always 2×(sum of k values) = even).
= even

(d) Sum of 8 odd numbers
8 is even, so we have an even count of odd numbers.
Pair them: (odd+odd) + (odd+odd) + (odd+odd) + (odd+odd) = even + even + even + even = even.
= even

Answer: (a) Even (b) Even (c) Even (d) Even

Q2 Lakpa has an odd number of 1 coins, an odd number of 5 coins and an even number of 10 coins in his piggy bank. He calculated the total and got 205. Did he make a mistake? If he did, explain why. If he didn't, how many coins of each type could he have?

SOLUTION

Find the parity of each coin type's contribution:

1 coins: (odd number) × 1 = odd × 1 = odd total value
5 coins: (odd number) × 5 = odd × 5 = odd total value (odd × odd = odd)
10 coins: (even number) × 10 = even × 10 = even total value

Total = odd + odd + even = (odd + odd) + even = even + even = even

But 205 is an odd number.
An even total can never equal 205 (odd).
Therefore, Lakpa made a mistake.

Answer: Yes, Lakpa made a mistake. The total value must be even, but 205 is odd — this is impossible with the given coin counts.

Q3 We know that:
(a) even + even = even
(b) odd + odd = even
(c) even + odd = odd
Similarly, find the parity for:
(d) even − even = ?
(e) odd − odd = ?
(f) even − odd = ?
(g) odd − even = ?

SOLUTION

Write even = 2a and odd = 2b + 1 for whole numbers a and b.

(d) even − even = 2a − 2b = 2(a − b) = even
Example: 8 − 4 = 4 (even) ✓

(e) odd − odd = (2a + 1) − (2b + 1) = 2(a − b) = even
Example: 9 − 5 = 4 (even) ✓

(f) even − odd = 2a − (2b + 1) = 2(a − b) − 1 = odd
Example: 8 − 3 = 5 (odd) ✓

(g) odd − even = (2a + 1) − 2b = 2(a − b) + 1 = odd
Example: 9 − 4 = 5 (odd) ✓

Answer: (d) even (e) even (f) odd (g) odd

Exercise 6.3 — Small Squares in Grids and Parity of Expressions (Page 131–132)

Q1 Find the parity of the number of small squares in these grids:
(a) 27 × 13
(b) 42 × 78
(c) 135 × 654

SOLUTION

Rule: The number of small squares in an m × n grid equals m × n.
Parity of a product: odd × odd = odd; even × anything = even.

(a) 27 × 13: both 27 and 13 are odd → product = odd
27 × 13 = 351 (odd) ✓

(b) 42 × 78: 42 is even → product = even
42 × 78 = 3276 (even) ✓

(c) 135 × 654: 654 is even → product = even
135 × 654 = 88290 (even) ✓

Answer: (a) Odd (b) Even (c) Even

Q2 What is the nth even number? Write a formula. What is the 100th even number?

SOLUTION

The even numbers in order are: 2, 4, 6, 8, 10, …
The nth even number = 2n.

100th even number = 2 × 100 = 200.

Answer: The nth even number is 2n. The 100th even number is 200.

Q3 Write a formula to find the nth odd number. What is the 100th odd number?

SOLUTION

Compare the even and odd sequences term by term:
Even: 2, 4, 6, 8, 10, … (nth term = 2n)
Odd: 1, 3, 5, 7, 9, … (each is one less than the corresponding even)

The nth odd number = 2n − 1.

100th odd number = 2 × 100 − 1 = 200 − 1 = 199.

Answer: The nth odd number is 2n − 1. The 100th odd number is 199.

Exercise 6.4 — Fill the Grids (Page 133)

Q1 Fill the 3 × 3 grid with numbers from 1 to 9 (no repeats) so that the row and column sums match the circled numbers shown.

Grid 1 — known cell: top-left = 9, bottom-right = 5
Row sums (right circles): 13, 14, 18
Column sums (bottom circles): 24, 9, 12

(Pipe-delimited layout: each row is | c1 | c2 | c3 |)
Partial grid:
| 9 | _ | _ | → 13
| _ | _ | _ | → 14
| _ | _ | 5 | → 18
↓ ↓ ↓
24 9 12

SOLUTION

Known: top-left = 9, bottom-right = 5. Numbers 1–9, no repeats.

Row 1: 9 + a + b = 13 → a + b = 4
Row 3: f + g + 5 = 18 → f + g = 13
Col 1: 9 + c + f = 24 → c + f = 15
Col 2: a + d + g = 9
Col 3: b + e + 5 = 12 → b + e = 7

Remaining digits: {1, 2, 3, 4, 6, 7, 8} for the 7 empty cells.

Try a = 1, b = 3 (since a + b = 4): then e = 7 − 3 = 4.
c + f = 15 from remaining {2, 6, 7, 8}: try c = 8, f = 7.
f + g = 13: g = 13 − 7 = 6.
a + d + g = 9: 1 + d + 6 = 9 → d = 2.

Check all digits used: 9, 1, 3, 8, 2, 4, 7, 6, 5 = {1,2,3,4,5,6,7,8,9} ✓

Verify row sums: 9+1+3 = 13 ✓, 8+2+4 = 14 ✓, 7+6+5 = 18 ✓
Verify col sums: 9+8+7 = 24 ✓, 1+2+6 = 9 ✓, 3+4+5 = 12 ✓

Answer: | 9 | 1 | 3 |
| 8 | 2 | 4 |
| 7 | 6 | 5 |
(Row sums 13, 14, 18; Col sums 24, 9, 12 ✓)

Q2 Fill the 3 × 3 grid with numbers from 1 to 9 (no repeats) so that the row and column sums match the circled numbers shown.

Grid 2 — known cells: middle-left = 4, bottom-right = 3
Row sums (right circles): 24, 15, 6
Column sums (bottom circles): 12, 16, 17

Partial grid:
| _ | _ | _ | → 24
| 4 | _ | _ | → 15
| _ | _ | 3 | → 6
↓ ↓ ↓
12 16 17

SOLUTION

Known: middle-left = 4, bottom-right = 3. Numbers 1–9, no repeats.

Row 2: 4 + d + e = 15 → d + e = 11
Row 3: f + g + 3 = 6 → f + g = 3
Col 1: a + 4 + f = 12 → a + f = 8
Col 2: b + d + g = 16
Col 3: c + e + 3 = 17 → c + e = 14

Total check: 24 + 15 + 6 = 45 ✓ (sum of 1 to 9 = 45)

Remaining digits: {1, 2, 5, 6, 7, 8, 9} for the 7 empty cells.

f + g = 3 and f, g from remaining: only 1 + 2 = 3, so {f, g} = {1, 2}.
a + f = 8: if f = 1, a = 7; if f = 2, a = 6.

Case f = 1, g = 2, a = 7:
b + d + 2 = 16 → b + d = 14
c + e = 14
Remaining: {5, 6, 8, 9} for b, c, d, e.
b + d = 14: try b = 9, d = 5 → row1: 7 + 9 + c = 24 → c = 8.
Then e = 14 − 8 = 6. Check d + e = 5 + 6 = 11 ✓.
Digits used: 7, 9, 8, 4, 5, 6, 1, 2, 3 = {1,2,3,4,5,6,7,8,9} ✓

Verify row sums: 7+9+8 = 24 ✓, 4+5+6 = 15 ✓, 1+2+3 = 6 ✓
Verify col sums: 7+4+1 = 12 ✓, 9+5+2 = 16 ✓, 8+6+3 = 17 ✓

Answer: | 7 | 9 | 8 |
| 4 | 5 | 6 |
| 1 | 2 | 3 |
(Row sums 24, 15, 6; Col sums 12, 16, 17 ✓)

Exercise 6.5 — Magic Squares (Page 136)

Q1 How many different magic squares can be made using the numbers 1 to 9?

SOLUTION

We established that in any 3 × 3 magic square using numbers 1–9:
• The magic sum must be 15.
• The centre must be 5.
• 1 and 9 must be in edge-middle positions (not corners).

Working through the constraints systematically, there is essentially ONE magic square structure. However, it can be rotated (4 ways) and reflected (2 ways), giving 4 × 2 = 8 distinct arrangements.

The fundamental (Lo Shu) magic square is:

	8	1	6	
	3	5	7	
	4	9	2	

Magic sum: 8+1+6 = 3+5+7 = 4+9+2 = 15 ✓; diagonals: 8+5+2 = 4+5+6 = 15 ✓

If we consider rotations and reflections as the same, there is exactly 1 unique magic square.

Answer: There is exactly 1 essentially unique magic square using 1–9 (ignoring rotations and reflections). Counting all rotations and reflections gives 8 arrangements.

Q2 Create a magic square using the numbers 2 to 10. What strategy would you use? Compare it with magic squares made using 1 to 9.

SOLUTION

Strategy: Add 1 to every number in the 1–9 magic square.
When you add the same constant to every cell, each row, column and diagonal sum increases by 3 × (constant) = 3 × 1 = 3.
New magic sum = 15 + 3 = 18.

Starting from the Lo Shu (1–9 magic square):

	8	1	6	→	9	2	7	
	3	5	7	→	4	6	8	
	4	9	2	→	5	10	3	

Verify: 9+2+7 = 4+6+8 = 5+10+3 = 18 ✓; diagonals: 9+6+3 = 5+6+7 = 18 ✓

Comparison: The structure is identical; the numbers are each 1 more. The magic sum changed from 15 to 18.

Answer: Add 1 to every entry of the 1–9 magic square. Result:
9	2	7
4	6	8
5	10	3
Magic sum = 18.

Q3 Take the magic square:
8	1	6
3	5	7
4	9	2
(a) Increase each number by 1. Is the resulting grid also a magic square? How does the magic sum change?
(b) Double each number. Is the resulting grid also a magic square? How does the magic sum change?

SOLUTION

(a) Increase each number by 1:

	9	2	7	
	4	6	8	
	5	10	3	

Each row, column and diagonal sum increases by 3 (one added per cell, 3 cells per line).
New magic sum = 15 + 3 = 18. The result IS a magic square.

(b) Double each number:

	16	2	12	
	6	10	14	
	8	18	4	

Each row, column and diagonal sum is doubled.
New magic sum = 15 × 2 = 30. The result IS a magic square.

Verify (b): 16+2+12 = 30 ✓, 6+10+14 = 30 ✓, 8+18+4 = 30 ✓;
diagonals: 16+10+4 = 30 ✓, 12+10+8 = 30 ✓

Answer: (a) Yes, magic square with sum 18. (b) Yes, magic square with sum 30.

Q4 What other operations can be performed on a magic square to yield another magic square?

SOLUTION

Any operation applied uniformly to every cell preserves the magic square property, because rows, columns, and diagonals all have the same number of cells (3 each), so the operation affects all line-sums equally.

Operations that produce another magic square:
• Add the same constant to every cell (magic sum increases by 3 × constant).
• Subtract the same constant from every cell.
• Multiply every cell by the same non-zero constant (magic sum scales by that constant).
• Divide every cell by the same non-zero constant (provided results are integers, or we allow fractions).

Also, any rotation (90°, 180°, 270°) or reflection of a magic square is again a magic square.

Answer: Adding, subtracting, multiplying, or dividing every cell by the same constant all produce another magic square. Rotations and reflections also preserve the property.

Q5 Discuss ways of creating a magic square using any set of 9 consecutive numbers (like 2–10, 3–11, 9–17, etc.).

SOLUTION

Strategy: Start with the 1–9 magic square and add a constant to every cell.

For the set starting at k (i.e., k, k+1, …, k+8):
Add (k − 1) to every cell of the 1–9 magic square.
New magic sum = 15 + 3(k − 1) = 3(k + 4).

Examples:

• 2–10: add 1 to each cell → magic sum = 18

	9	2	7	sum = 18
	4	6	8	
	5	10	3	

• 3–11: add 2 to each cell → magic sum = 21

	10	3	8	
	5	7	9	
	6	11	4	

• 9–17: add 8 to each cell → magic sum = 39

	16	9	14	
	11	13	15	
	12	17	10	

Answer: Add (k − 1) to every cell of the 1–9 magic square to get a magic square with k through k+8. The magic sum becomes 3(k + 4).

Exercise 6.6 — Generalising a 3 × 3 Magic Square (Page 137)

Q1 Using the generalised form of a magic square (where m is the centre number), find a magic square if the centre number is 25.

Generalised form:
m+3	m−4	m+1
m−2	m	m+2
m−1	m+4	m−3

SOLUTION

Substitute m = 25 into the generalised form:

m+3 = 28, m−4 = 21, m+1 = 26
m−2 = 23, m = 25, m+2 = 27
m−1 = 24, m+4 = 29, m−3 = 22

Magic square:

	28	21	26	
	23	25	27	
	24	29	22	

Verify row sums: 28+21+26 = 75, 23+25+27 = 75, 24+29+22 = 75 ✓
Verify col sums: 28+23+24 = 75, 21+25+29 = 75, 26+27+22 = 75 ✓
Diagonals: 28+25+22 = 75, 26+25+24 = 75 ✓
Magic sum = 3 × 25 = 75 ✓

Answer: | 28 | 21 | 26 |
| 23 | 25 | 27 |
| 24 | 29 | 22 |
Magic sum = 75.

Q2 What is the expression obtained by adding the 3 terms of any row, column or diagonal in the generalised form?

SOLUTION

Take any row of the generalised magic square. For example, Row 1: (m+3) + (m−4) + (m+1).
(m+3) + (m−4) + (m+1) = 3m + (3 − 4 + 1) = 3m + 0 = 3m

Row 2: (m−2) + m + (m+2) = 3m + (−2+0+2) = 3m ✓
Row 3: (m−1) + (m+4) + (m−3) = 3m + (−1+4−3) = 3m ✓

All rows, columns and diagonals sum to 3m.

Answer: The sum of any row, column or diagonal equals 3m.

Q3 Write the result obtained by:
(a) Adding 1 to every term in the generalised form.
(b) Doubling every term in the generalised form.

SOLUTION

(a) Add 1 to every cell. The new entries are:

	(m+3)+1	(m−4)+1	(m+1)+1		m+4	m−3	m+2	
	(m−2)+1	m+1	(m+2)+1	=	m−1	m+1	m+3	
	(m−1)+1	(m+4)+1	(m−3)+1		m	m+5	m−2	

New magic sum = 3m + 3 = 3(m + 1). The new centre is m+1.

(b) Double every cell. The new entries are:

	2(m+3)	2(m−4)	2(m+1)		2m+6	2m−8	2m+2	
	2(m−2)	2m	2(m+2)	=	2m−4	2m	2m+4	
	2(m−1)	2(m+4)	2(m−3)		2m−2	2m+8	2m−6	

New magic sum = 6m. The new centre is 2m.

Answer: (a) Add 1: new generalised form has centre m+1, magic sum 3(m+1).
(b) Double: new generalised form has centre 2m, magic sum 6m.

Q4 Create a magic square whose magic sum is 60.

SOLUTION

We need 3m = 60, so m = 20.

Substitute m = 20 into the generalised form:
m+3 = 23, m−4 = 16, m+1 = 21
m−2 = 18, m = 20, m+2 = 22
m−1 = 19, m+4 = 24, m−3 = 17

Magic square:

	23	16	21	
	18	20	22	
	19	24	17	

Verify: 23+16+21 = 60 ✓, 18+20+22 = 60 ✓, 19+24+17 = 60 ✓
Col sums: 23+18+19 = 60 ✓, 16+20+24 = 60 ✓, 21+22+17 = 60 ✓
Diagonals: 23+20+17 = 60 ✓, 21+20+19 = 60 ✓

Answer: | 23 | 16 | 21 |
| 18 | 20 | 22 |
| 19 | 24 | 17 |
Magic sum = 60.

Q5 Is it possible to get a magic square by filling nine non-consecutive numbers?

SOLUTION

Yes, it is possible. For example, use multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27. These are non-consecutive integers with centre 15.

Apply the generalised form with m = 15 and common difference 3:
(Equivalently, multiply the 1–9 magic square by 3.)

	24	3	18	
	9	15	21	
	12	27	6	

Verify: 24+3+18 = 45 ✓, 9+15+21 = 45 ✓, 12+27+6 = 45 ✓
Col sums: 24+9+12 = 45 ✓, 3+15+27 = 45 ✓, 18+21+6 = 45 ✓
Diagonals: 24+15+6 = 45 ✓, 18+15+12 = 45 ✓

Answer: Yes. Example using multiples of 3:
24	3	18
9	15	21
12	27	6
Magic sum = 45.

Exercise 6.7 — The Virahāṅka–Fibonacci Sequence (Page 142)

Q1 Write the next 3 numbers in the Virahāṅka sequence:
1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ___, ___, ___, …
Also, can you tell whether the number after 377 is odd or even (without computing the sum)?

SOLUTION

The Virahāṅka rule: each term = sum of the two previous terms.

Term 11: 55 + 89 = 144
Term 12: 89 + 144 = 233
Term 13: 144 + 233 = 377

Next 3 numbers: 144, 233, 377.

Parity of the number after 377:
The parity pattern of the Virahāṅka sequence is:
O, E, O, O, E, O, O, E, O, O, E, O, O, E, …
(Starting from term 2, the pattern E, O, O repeats with period 3.)
Term 13 = 377 is Odd (position 13: (13−1)/3 = 4 complete cycles, remainder 0 → O).
Term 14 = 233 + 377 = 610 is Even.
So the number after 377 is EVEN — confirmed by the parity pattern (O, O is followed by E in the repeating block).

Answer: Next 3 terms: 144, 233, 377. The number after 377 is even (610).

Q2 What is the parity of each number in the Virahāṅka sequence? Do you notice any pattern?

SOLUTION

List the first several terms with their parity:
1(O), 2(E), 3(O), 5(O), 8(E), 13(O), 21(O), 34(E), 55(O), 89(O), 144(E), …

Pattern: starting from term 1, the parities are O, E, O, O, E, O, O, E, O, O, E, …
After the first term (O), the pattern (E, O, O) repeats forever.

Why this pattern holds:
- O + E = O (terms 1+2 → term 3: O)
- E + O = O (terms 2+3 → term 4: O)
- O + O = E (terms 3+4 → term 5: E)

- O + E = O (terms 4+5 → term 6: O)

… and then the same three-step cycle repeats.

Answer: Parity pattern: O, E, O, O, E, O, O, E, O, O, … After the first term (O), the block (E, O, O) repeats with period 3.

Exercise 6.8 — Digits in Disguise (Page 143–144)

Q1 A light bulb is ON. Dorjee toggles its switch 77 times. Will the bulb be on or off? Why?

SOLUTION

Each toggle changes the state: ON → OFF → ON → OFF → …
Starting state: ON.
After 1 toggle: OFF (odd number of toggles → OFF).
After 2 toggles: ON.
After an odd number of toggles: OFF.
After an even number of toggles: ON.

77 is odd.
Therefore, after 77 toggles, the bulb is OFF.

Answer: The bulb will be OFF. (77 is odd; an odd number of toggles starting from ON ends in OFF.)

Q2 Liswini has a large old encyclopaedia. When she opened it, several loose pages fell out of it. She counted 50 sheets in total, each printed on both sides. Can the sum of the page numbers of the loose sheets be 6000? Why or why not?

SOLUTION

Each sheet of a book has two consecutive page numbers, e.g., page $2k-1$ and page $2k$ (one odd, one even).

Sum of page numbers from one sheet = $(2k-1) + 2k = 4k - 1$ (odd).

With 50 sheets, the total sum is the sum of 50 such values, each of which is odd.
Sum of 50 odd numbers: 50 is even, so this is the sum of an even count of odd numbers = even.

6000 is even.
Therefore, it IS possible for the sum to be 6000.

(Alternatively: the 50 sheets give 50 odd-numbered pages and 50 even-numbered pages.
Sum of 50 odd numbers = even; sum of 50 even numbers = even; total = even + even = even.
6000 is even, so this is possible.)

Answer: Yes, the sum can be 6000. The sum of page numbers from 50 sheets is always even, and 6000 is even.

Q3 Here is a 2 × 3 grid. For each row and column, the parity of the sum is written in the circle: 'e' for even and 'o' for odd.
Row parities (right circles): top row = o, bottom row = e
Column parities (bottom circles): col 1 = e, col 2 = e, col 3 = o
Fill the 6 boxes with 3 odd numbers and 3 even numbers to satisfy the row and column sum parities.

SOLUTION

Label the grid cells:	a	b	c	(row 1)
	d	e	f	(row 2)

Constraints:
Row 1: a + b + c is odd
Row 2: d + e + f is even
Col 1: a + d is even → a, d have the same parity
Col 2: b + e is even → b, e have the same parity
Col 3: c + f is odd → c, f have different parities

We need exactly 3 odd (o) and 3 even (e) across all 6 cells.

Since col 1 pair is same parity and col 2 pair is same parity, and col 3 pair is different:
Col 3 contributes one odd and one even.
Remaining 4 cells (cols 1 and 2) must have 2 odd and 2 even.
With same-parity pairs per column: col 1 = (o,o) or (e,e); col 2 = (o,o) or (e,e).
Need 2 odd + 2 even: so one column gets (o,o) and the other gets (e,e).

Try col 1 = (o,o), col 2 = (e,e), col 3 = one o and one e:
Row 1: a+b+c = o+e+c → for row 1 to be odd, c must be even.
Row 2: d+e+f = o+e+f → for row 2 to be even, f must be odd.
Col 3 check: c(even)+f(odd) = odd ✓

One valid assignment: a=1(o), b=2(e), c=4(e), d=3(o), e=6(e), f=5(o)
Row 1: 1+2+4 = 7 (odd) ✓
Row 2: 3+6+5 = 14 (even) ✓
Col 1: 1+3 = 4 (even) ✓

Col 2: 2+6 = 8 (even) ✓
Col 3: 4+5 = 9 (odd) ✓
Odd count: 1,3,5 → 3 ✓; Even count: 2,4,6 → 3 ✓

Answer: One valid solution:
| 1 | 2 | 4 | → sum 7 (odd)
| 3 | 6 | 5 | → sum 14 (even)
Col sums: 4(e), 8(e), 9(o). Many other solutions exist.

Q4 Make a 3 × 3 magic square with 0 as the magic sum. All numbers cannot be zero. Use negative numbers as needed.

SOLUTION

We need 3m = 0, so the centre number m = 0.

Using the generalised form with m = 0:

	m+3	m−4	m+1		3	−4	1	
	m−2	m	m+2	=	−2	0	2	
	m−1	m+4	m−3		−1	4	−3	

Verify row sums: 3+(−4)+1 = 0 ✓, (−2)+0+2 = 0 ✓, (−1)+4+(−3) = 0 ✓
Col sums: 3+(−2)+(−1) = 0 ✓, (−4)+0+4 = 0 ✓, 1+2+(−3) = 0 ✓
Diagonals: 3+0+(−3) = 0 ✓, 1+0+(−1) = 0 ✓
Not all zero ✓ (uses 3, −4, 1, −2, 0, 2, −1, 4, −3)

Answer: | 3 | −4 | 1 |
| −2 | 0 | 2 |
| −1 | 4 | −3 |
Magic sum = 0.

Q5 Fill in the following blanks with 'odd' or 'even':
(a) Sum of an odd number of even numbers is _____
(b) Sum of an even number of odd numbers is _____
(c) Sum of an even number of even numbers is _____
(d) Sum of an odd number of odd numbers is _____

SOLUTION

(a) Sum of an odd number of even numbers:
Each even number contributes an even value. The sum of any collection of even numbers is always even (even + even + … = even).
Answer: even

(b) Sum of an even number of odd numbers:
Pair up the odd numbers: each pair sums to even. With an even count, all pair up completely.
Sum of evens = even.
Answer: even

(c) Sum of an even number of even numbers:
Sum of any even numbers is always even.
Answer: even

(d) Sum of an odd number of odd numbers:
With an odd count, pair up all but one: (odd−1) odds pair to give even sum, plus 1 odd remaining.
even + odd = odd.
Verify: 1+3+5 = 9 (odd) ✓; 1+3+5+7+9 = 25 (odd) ✓
Answer: odd

Answer: (a) even (b) even (c) even (d) odd

Q6 What is the parity of the sum of the numbers from 1 to 100?

SOLUTION

Pair the numbers: (1+100), (2+99), (3+98), …, (50+51).
Each pair sums to 101 (odd).
There are 50 such pairs.

Sum = 50 × 101.
Parity of 50 × 101: 50 is even, so even × odd = even.

Alternatively: sum of 50 odd numbers (from pairing) with 50 being even gives even sum.

The sum 1 + 2 + … + 100 = (100 × 101)/2 = 5050.
5050 is even ✓

Answer: Even. The sum 1+2+…+100 = 5050, which is even.

Q7 Two consecutive numbers in the Virahāṅka sequence are 987 and 1597. What are the next 2 numbers in the sequence? What are the previous 2 numbers in the sequence?

SOLUTION

The Virahāṅka rule: each term = sum of the two previous terms.
So: next term = current + next-current, and previous term = next − current.

Next 2 numbers (going forward):
987 + 1597 = 2584
1597 + 2584 = 4181

Previous 2 numbers (going backward):
The term before 987: since (previous) + 987 = 1597 → previous = 1597 − 987 = 610
The term before 610: since (previous) + 610 = 987 → previous = 987 − 610 = 377

The sequence: …, 377, 610, 987, 1597, 2584, 4181, …

Answer: Next 2: 2584, 4181. Previous 2: 610, 377.

Q8 Angaan wants to climb an 8-step staircase. His rule is that he can take either 1 step or 2 steps at a time (e.g., one path is 1, 2, 2, 1, 2). In how many different ways can he reach the top?

SOLUTION

This is exactly the Virahāṅka problem: count ways to write n as an ordered sum of 1s and 2s.

Let W(n) = number of ways to climb n steps.
W(1) = 1 (only: 1)
W(2) = 2 (1+1, or 2)
W(n) = W(n−1) + W(n−2) [first step is 1 → W(n−1) ways; first step is 2 → W(n−2)

ways]

W(1) = 1
W(2) = 2
W(3) = 3
W(4) = 5
W(5) = 8
W(6) = 13
W(7) = 21
W(8) = 34

W(8) = W(7) + W(6) = 21 + 13 = 34

Answer: 34 different ways. (This is the 8th Virahāṅka number.)

Q9 What is the parity of the 20th term of the Virahāṅka sequence?

SOLUTION

The parity pattern of the Virahāṅka sequence:
Term: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 …
Par: O E O O E O O E O O E O O E O O E O O E …

From term 2 onward, the pattern (E, O, O) repeats with period 3.

Position of term 20 within the repeating part:
Terms from position 2: position within cycle = (20 − 2) mod 3 + 1 = 18 mod 3 + 1 = 0 + 1 = 1st position of (E, O, O) = E.

The 20th term is 10946, which is even ✓

Answer: Even. The 20th term of the Virahāṅka sequence is 10946.

Q10 Identify the statements that are true:
(a) The expression 4m − 1 always gives odd numbers.
(b) All even numbers can be expressed as 6j − 4.
(c) Both expressions 2p + 1 and 2q − 1 describe all odd numbers.
(d) The expression 2f + 3 gives both even and odd numbers.

SOLUTION

(a) 4m − 1:
4m is always even (4m = 2 × 2m). Subtracting 1 from an even number gives an

odd number.
4(1)−1=3, 4(2)−1=7, 4(3)−1=11, 4(4)−1=15 — all odd. TRUE.

(b) 6j − 4 for j = 1, 2, 3, …: gives 2, 8, 14, 20, 26, …
These are only even numbers of the form 6j − 4. Even numbers like 4, 6, 10, 12, 16, 18, … are missing.
Not all even numbers can be expressed this way. FALSE.

(c) 2p + 1 and 2q − 1 for natural numbers p, q (starting at 1):
2p + 1 with p = 1, 2, 3, … gives 3, 5, 7, … (misses the odd number 1).
2q − 1 with q = 1, 2, 3, … gives 1, 3, 5, 7, … (covers all positive odd numbers).
Since 2p + 1 does not cover 1, both expressions together do NOT each describe ALL odd numbers. FALSE.

(d) 2f + 3:
2f is always even. 2f + 3 = even + odd = always odd. Never even. FALSE.

Only statement (a) is true.

Answer: Only (a) is true.

Q11 Solve this cryptarithm:

```
  UT
+ TA
----
 TAT
```

(Each letter stands for a unique digit 0–9. UT is a 2-digit number, TA is a 2-digit number, TAT is a 3-digit number.)

SOLUTION

UT = 10U + T; TA = 10T + A; TAT = 100T + 10A + T = 101T + 10A

Equation: (10U + T) + (10T + A) = 101T + 10A
10U + 11T + A = 101T + 10A
10U = 90T + 9A
10U = 9(10T + A)

Since the left side is divisible by 10, the right side 9(10T + A) must be divisible by 10.
As gcd(9, 10) = 1, we need (10T + A) divisible by 10, so A = 0.

Then: 10U = 90T → U = 9T.

U must be a single digit (0–9) and T ≥ 1 (since TA is a 2-digit number with leading digit T):
T = 1 → U = 9 ✓
T = 2 → U = 18 (not a single digit) ✗

So T = 1, A = 0, U = 9.

Verify: UT = 91, TA = 10, TAT = 101
91 + 10 = 101 ✓
All digits T=1, A=0, U=9 are distinct ✓

Answer: U = 9, T = 1, A = 0. Check: 91 + 10 = 101 ✓