Chapter 7: A Tale of Three Intersecting Lines

CONTENTS 8 exercises · 20 questions

7.1	Equilateral and Isosceles Triangles (Page 150)	2 Q
7.2	Triangle Inequality — Are Triangles Possible for Any Lengths? (...	3 Q
7.3	Triangle Inequality — Which Lengths Form a Triangle? (Page 156)	1 Q
7.4	Triangle Inequality — Conclusion and Third Side Range (Page ...	3 Q
7.5	Two Sides and Included Angle (SAS) (Page 161)	1 Q
7.6	Two Angles and Included Side (ASA) (Page 162–163)	3 Q
7.7	Angle Sum Property of Triangles (Page 165)	3 Q
7.8	Types of Triangles and Altitudes (Page 170)	4 Q

Exercise 7.1 — Equilateral and Isosceles Triangles (Page 150)

Q1 Use the points on the circle and/or the centre to form isosceles triangles. A single circle is given with its centre marked as a dot.

SOLUTION

Recall the definition of an isosceles triangle: a triangle with at least two equal sides.

Key property of a circle: every point on the circle is the same distance (the radius) from the centre.

Strategy 1 — Use two points on the circle and the centre:
Choose any two points P and Q on the circle. Let O be the centre.
Then OP = OQ = radius (both are radii of the same circle).
Triangle OPQ has two equal sides OP and OQ, so it is isosceles.

Strategy 2 — Use three points on the circle:
Choose three points P, Q, R on the circle. Triangle PQR need not be isosceles in general.
However, if you choose P and Q symmetrically about a diameter, then PO = QO, making triangle POQ isosceles as in Strategy 1.

Example construction: Mark any two points A and B on the circle. Join A, B, and the centre O.

Triangle OAB is isosceles with OA = OB (both equal to the radius).
By choosing different pairs of points on the circle, you can form many different isosceles triangles.

Answer: Connect any two points on the circle to the centre. Since both connections are radii (equal lengths), the triangle formed is isosceles. Many such triangles are possible.

Q2 Two diagrams are given, each showing two or three circles of the same size. Diagram 1: Two circles with centres A and B. Diagram 2: Three circles with centres A, B, and C arranged so each circle passes through the other two centres. Use the points on the circles and/or their centres to form isosceles and equilateral triangles.

SOLUTION

Diagram 1 — Two circles of the same radius r, with centres A and B:
The two circles intersect at two points; call them P (top intersection) and Q (bottom intersection).
Since both circles have the same radius r: AP = AQ = BP = BQ = r.

Isosceles triangles:
Triangle APB: AP = BP = r, so it is isosceles (vertex P, equal sides PA = PB).
Triangle AQB: AQ = BQ = r, so it is isosceles (vertex Q, equal sides QA = QB).
Triangle APQ: AP = AQ = r, so it is isosceles (vertex A, equal sides AP = AQ).
Triangle BPQ: BP = BQ = r, so it is isosceles (vertex B, equal sides BP = BQ).

Equilateral triangles:
When two circles of the same radius r have centres A and B with AB = r (i.e., each circle passes through the other's centre), then AP = PB = AB = r.
Triangle APB is equilateral. Similarly triangle AQB is equilateral.

Diagram 2 — Three circles of the same radius r, with centres A, B, C, where each circle passes through the other two centres (AB = BC = CA = r):
The three centres A, B, C themselves form an equilateral triangle (all sides = r).
Each pair of circles intersects at two points. There are many isosceles and equilateral triangles:
Triangle ABC (the centres): equilateral.
Any triangle formed by one centre and two intersection points on its circle: isosceles (two sides are radii).
Specific equilateral triangle: the three intersection points of adjacent circle pairs

also form an equilateral triangle.

Answer: Diagram 1: Connecting any two points equidistant from a centre gives isosceles triangles. When AB equals the radius, triangles APB and AQB are equilateral. Diagram 2: Triangle ABC (centres) is equilateral. Multiple isosceles triangles can be formed using the centres and intersection points.

Exercise 7.2 — Triangle Inequality — Are Triangles Possible for Any Lengths? (Page 154

Q1 We checked by construction that there are no triangles having sidelengths 3 cm, 4 cm and 8 cm; and 2 cm, 3 cm and 6 cm. Check if you could have found this without trying to construct the triangle.

SOLUTION

For a triangle to exist, each side must be less than the sum of the other two sides (triangle inequality).
We only need to check the largest side against the sum of the other two.

Set 1: 3 cm, 4 cm, 8 cm
Largest side = 8 cm.
Sum of the other two sides = 3 + 4 = 7 cm.
Since 7 < 8, the sum of the two smaller sides is less than the largest side.
The triangle inequality is violated. So a triangle with these sides does not exist.

Set 2: 2 cm, 3 cm, 6 cm
Largest side = 6 cm.
Sum of the other two sides = 2 + 3 = 5 cm.
Since 5 < 6, the sum of the two smaller sides is less than the largest side.
The triangle inequality is violated. So a triangle with these sides does not exist.

Yes, we could have determined this without construction, simply by checking whether the sum of the two smaller sides exceeds the largest side.

Answer: Yes. For 3, 4, 8: sum of smaller two = 3+4 = 7 < 8, so no triangle. For 2, 3, 6: sum of smaller two = 2+3 = 5 < 6, so no triangle. No construction needed.

Q2 Can we say anything about the existence of a triangle for each of the following sets of lengths? (a) 10 km, 10 km and 25 km (b) 5 mm, 10 mm and 20 mm (c) 12 cm, 20 cm and 40 cm

SOLUTION

For each set, arrange the lengths in order and check: sum of the two smaller lengths vs the largest length.

(a) 10 km, 10 km, 25 km
Sum of the two smaller sides = 10 + 10 = 20 km.
Largest side = 25 km.
Since 20 < 25, the triangle inequality is not satisfied.

A triangle with these sidelengths does NOT exist.

(b) 5 mm, 10 mm, 20 mm
Sum of the two smaller sides = 5 + 10 = 15 mm.
Largest side = 20 mm.
Since 15 < 20, the triangle inequality is not satisfied.
A triangle with these sidelengths does NOT exist.

(c) 12 cm, 20 cm, 40 cm
Sum of the two smaller sides = 12 + 20 = 32 cm.
Largest side = 40 cm.
Since 32 < 40, the triangle inequality is not satisfied.
A triangle with these sidelengths does NOT exist.

Answer: (a) 10+10=20 < 25: triangle does NOT exist. (b) 5+10=15 < 20: triangle does NOT exist. (c) 12+20=32 < 40: triangle does NOT exist.

Q3 For each set of lengths seen so far, you might have noticed that in at least two of the comparisons, the direct length was less than the sum of the other two (if not, check again!). For example, for the set of lengths 10 cm, 15 cm and 30 cm, there are two comparisons where this happens. Identify them, and identify the comparison where it does not happen.

SOLUTION

For any three lengths, compare each length against the sum of the other two. There are three such comparisons.

Take the set 10, 15, 30:
Comparison 1: is 10 less than 15 + 30 = 45? Yes, 10 < 45. ✓
Comparison 2: is 15 less than 10 + 30 = 40? Yes, 15 < 40. ✓
Comparison 3: is 30 less than 10 + 15 = 25? No, 30 > 25. ✗

So two of the three comparisons hold (each of the two smaller lengths is less than the sum of the other two), but the comparison for the LARGEST length fails.

This is why we only need to check the largest length: for any set of lengths, the two smaller lengths are always less than the sum of the other two automatically. A triangle exists only when the largest length is ALSO less than the sum of the other two.

Answer: For 10, 15, 30 the two comparisons that hold are 10 < 15 + 30 and 15 < 10 + 30. The one that fails is 30 > 10 + 15. In general, only the comparison for the largest length can fail — which is why checking the largest side against the sum of the other two decides whether a triangle exists.

Exercise 7.3 — Triangle Inequality — Which Lengths Form a Triangle? (Page 156)

Q1 Which of the following lengths can be the sidelengths of a triangle? Explain your answers. Note that for each set, the three lengths have the same unit of measure. (a) 2, 2, 5 (b) 3, 4, 6 (c) 2, 4, 8 (d) 5, 5, 8 (e) 10, 20, 25 (f) 10, 20, 35 (g) 24, 26, 28

SOLUTION

Check each set using the triangle inequality: the sum of the two smaller sides must be greater than the largest side.

(a) 2, 2, 5:
Sum of the two smaller sides = 2 + 2 = 4.
Largest side = 5.
Since 4 < 5, the triangle inequality is NOT satisfied.
These lengths CANNOT be the sidelengths of a triangle.

(b) 3, 4, 6:
Sum of the two smaller sides = 3 + 4 = 7.
Largest side = 6.
Since 7 > 6, the triangle inequality is satisfied.
These lengths CAN be the sidelengths of a triangle.

(c) 2, 4, 8:
Sum of the two smaller sides = 2 + 4 = 6.
Largest side = 8.
Since 6 < 8, the triangle inequality is NOT satisfied.
These lengths CANNOT be the sidelengths of a triangle.

(d) 5, 5, 8:
Sum of the two smaller sides = 5 + 5 = 10.
Largest side = 8.
Since 10 > 8, the triangle inequality is satisfied.
These lengths CAN be the sidelengths of a triangle.

(e) 10, 20, 25:
Sum of the two smaller sides = 10 + 20 = 30.
Largest side = 25.
Since 30 > 25, the triangle inequality is satisfied.
These lengths CAN be the sidelengths of a triangle.

(f) 10, 20, 35:

Sum of the two smaller sides = 10 + 20 = 30.
Largest side = 35.
Since 30 < 35, the triangle inequality is NOT satisfied.
These lengths CANNOT be the sidelengths of a triangle.

(g) 24, 26, 28:
Sum of the two smaller sides = 24 + 26 = 50.
Largest side = 28.
Since 50 > 28, the triangle inequality is satisfied.
These lengths CAN be the sidelengths of a triangle.

Answer: Can form a triangle: (b) 3,4,6; (d) 5,5,8; (e) 10,20,25; (g) 24,26,28. Cannot form a triangle: (a) 2,2,5 (2+2=4<5); (c) 2,4,8 (2+4=6<8); (f) 10,20,35 (10+20=30<35).

Exercise 7.4 — Triangle Inequality — Conclusion and Third Side Range (Page 159)

Q1 Check if a triangle exists for each of the following set of lengths: (a) 1, 100, 100 (b) 3, 6, 9 (c) 1, 1, 5 (d) 5, 10, 12

SOLUTION

Check the triangle inequality: sum of the two smaller sides must be strictly greater than the largest side.

(a) 1, 100, 100:
Sum of the two smaller sides = 1 + 100 = 101.
Largest side = 100.
Since 101 > 100, the triangle inequality is satisfied.
A triangle EXISTS.

(b) 3, 6, 9:
Sum of the two smaller sides = 3 + 6 = 9.
Largest side = 9.
Since 9 = 9 (not strictly greater), the triangle inequality is NOT satisfied.
A triangle does NOT exist. (The three points would be collinear — a degenerate case.)

(c) 1, 1, 5:
Sum of the two smaller sides = 1 + 1 = 2.
Largest side = 5.
Since 2 < 5, the triangle inequality is NOT satisfied.
A triangle does NOT exist.

(d) 5, 10, 12:
Sum of the two smaller sides = 5 + 10 = 15.
Largest side = 12.
Since 15 > 12, the triangle inequality is satisfied.
A triangle EXISTS.

Answer: (a) Exists (1+100=101>100). (b) Does not exist (3+6=9, not > 9). (c) Does not exist (1+1=2<5). (d) Exists (5+10=15>12).

Q2 Does there exist an equilateral triangle with sides 50, 50, 50? In general, does there exist an equilateral triangle of any sidelength? Justify your answer.

SOLUTION

For an equilateral triangle with sides 50, 50, 50:
Check triangle inequality: sum of any two sides = 50 + 50 = 100 > 50 (the third side).
The triangle inequality is satisfied.
Yes, an equilateral triangle with sides 50, 50, 50 exists.

In general, for an equilateral triangle with any side length a (where a > 0):
Sum of any two sides = a + a = 2a.
Third side = a.
Since 2a > a for any positive a, the triangle inequality is always satisfied.
Therefore, an equilateral triangle exists for any positive sidelength.

Answer: Yes, an equilateral triangle with sides 50, 50, 50 exists (50+50=100>50). In general, for any positive sidelength a, an equilateral triangle exists because a+a=2a>a always holds.

Q3 For each of the following, give at least 5 possible values for the third length so there exists a triangle having these as sidelengths (decimal values could also be chosen): (a) 1, 100 (b) 5, 5 (c) 3, 7

SOLUTION

For two given sides p and q, the third side t must satisfy the triangle inequality.
The rule is: |p - q| < t < p + q (t must be strictly between the difference and the sum).

(a) Sides 1 and 100:
Difference = |100 - 1| = 99.
Sum = 1 + 100 = 101.
So the third side t must satisfy: 99 < t < 101.
Five possible values: 99.5, 99.8, 100, 100.2, 100.5.
Verification for t = 99.5: 1 + 99.5 = 100.5 > 100 ✓, 1 + 100 = 101 > 99.5 ✓, 99.5 + 100 = 199.5 > 1 ✓.

(b) Sides 5 and 5:
Difference = |5 - 5| = 0.
Sum = 5 + 5 = 10.

So the third side t must satisfy: 0 < t < 10.
Five possible values: 1, 3, 5, 7, 9.
Verification for t = 9: 5 + 5 = 10 > 9 ✓, 5 + 9 = 14 > 5 ✓.

(c) Sides 3 and 7:
Difference = |7 - 3| = 4.
Sum = 3 + 7 = 10.
So the third side t must satisfy: 4 < t < 10.
Five possible values: 5, 6, 7, 8, 9.
Verification for t = 5: 3 + 5 = 8 > 7 ✓, 3 + 7 = 10 > 5 ✓, 5 + 7 = 12 > 3 ✓.

Answer: (a) Third side t: 99 < t < 101. Five values: 99.5, 99.8, 100, 100.2, 100.5. (b) 0 < t < 10. Five values: 1, 3, 5, 7, 9. (c) 4 < t < 10. Five values: 5, 6, 7, 8, 9.

Exercise 7.5 — Two Sides and Included Angle (SAS) (Page 161)

Q1 Construct triangles for the following measurements where the angle is included between the sides: (a) 3 cm, 75°, 7 cm (b) 6 cm, 25°, 3 cm (c) 3 cm, 120°, 8 cm

SOLUTION

For each measurement, two sides and the included angle (the angle between those two sides) are given.

Construction steps for all three cases follow the same procedure:

Step 1: Draw the base — take one of the two given sides as the base AB.

Step 2: At vertex A, construct the given included angle.

Step 3: Along the other arm of the angle, mark vertex C at the second given side length from A.

Step 4: Join B and C to complete the triangle.

(a) 3 cm, 75°, 7 cm:

Draw AB = 3 cm (or 7 cm) as the base.

At A, construct angle of 75°.

Mark C on the other arm such that AC = 7 cm (or 3 cm).

Join BC. Triangle ABC is the required triangle with two sides 3 cm, 7 cm and included angle 75°.

(b) 6 cm, 25°, 3 cm:

Draw AB = 6 cm as the base.

At A, construct angle of 25°.

Mark C on the other arm such that AC = 3 cm.

Join BC. Triangle ABC is the required triangle.

(c) 3 cm, 120°, 8 cm:

Draw AB = 3 cm as the base.

At A, construct angle of 120° (an obtuse angle).

Mark C on the other arm such that AC = 8 cm.

Join BC. Triangle ABC is the required triangle.

Note: A triangle always exists when two sides and their included angle are given, provided the included angle is strictly between 0° and 180°. All three cases satisfy this.

Answer: All three triangles can be constructed. Draw one side as base, construct the included angle at one end, mark the second side along the other arm, and join the third vertex. A triangle always exists for any two sides with an included angle strictly between 0° and 180°.

Exercise 7.6 — Two Angles and Included Side (ASA) (Page 162–163)

Q1 Construct triangles for the following measurements (two angles and the included side between them): (a) 75°, 5 cm, 75° (b) 25°, 3 cm, 60° (c) 120°, 6 cm, 30°

SOLUTION

For each measurement, two angles at the ends of the included side are given.
Construction steps for all three cases:
Step 1: Draw the included side AB of the given length.
Step 2: At A, construct the first given angle.
Step 3: At B, construct the second given angle.
Step 4: The point where the two new line segments meet is the third vertex C.

(a) 75°, 5 cm, 75°:
Draw AB = 5 cm.
At A, construct angle of 75°. At B, construct angle of 75°.
Sum of given angles = 75° + 75° = 150° < 180°, so a triangle exists.
Third angle = 180° - 75° - 75° = 30°.
The two arms from A and B meet at C. Triangle ABC is isosceles (equal base angles).

(b) 25°, 3 cm, 60°:
Draw AB = 3 cm.
At A, construct angle of 25°. At B, construct angle of 60°.
Sum = 25° + 60° = 85° < 180°, so a triangle exists.
Third angle = 180° - 25° - 60° = 95°.
The arms from A and B meet at C to give triangle ABC.

(c) 120°, 6 cm, 30°:
Draw AB = 6 cm.
At A, construct angle of 120°. At B, construct angle of 30°.
Sum = 120° + 30° = 150° < 180°, so a triangle exists.
Third angle = 180° - 120° - 30° = 30°.
The arms from A and B meet at C to give triangle ABC.

Answer: All three triangles can be constructed. (a) Third angle = 30°, isosceles triangle. (b) Third angle = 95°. (c) Third angle = 30°. A triangle exists when the sum of the two given angles is less than 180°.

Q2 For each of the following angles, find another angle for which a triangle is (a) possible, (b) not possible. Find at least two different angles for each category: (a) 30° (b) 70° (c) 54° (d) 144°

SOLUTION

Rule: Given one angle A of a triangle, a second angle B forms a valid triangle if and only if A + B < 180°, i.e., B < 180° - A.
A triangle is NOT possible when A + B ≥ 180°, i.e., B ≥ 180° - A.

(a) Given angle = 30°. Limit = 180° - 30° = 150°.
Triangle POSSIBLE: second angle must be less than 150°.
Examples: 50° (30°+50°=80°<180°), 90° (30°+90°=120°<180°).
Triangle NOT POSSIBLE: second angle must be 150° or more.
Examples: 150° (30°+150°=180°, not < 180°), 170° (30°+170°=200°>180°).

(b) Given angle = 70°. Limit = 180° - 70° = 110°.
Triangle POSSIBLE: second angle must be less than 110°.
Examples: 60° (70°+60°=130°<180°), 80° (70°+80°=150°<180°).
Triangle NOT POSSIBLE: second angle must be 110° or more.
Examples: 110° (70°+110°=180°), 140° (70°+140°=210°>180°).

(c) Given angle = 54°. Limit = 180° - 54° = 126°.
Triangle POSSIBLE: second angle must be less than 126°.
Examples: 90° (54°+90°=144°<180°), 64° (54°+64°=118°<180°).
Triangle NOT POSSIBLE: second angle must be 126° or more.
Examples: 126° (54°+126°=180°), 134° (54°+134°=188°>180°).

(d) Given angle = 144°. Limit = 180° - 144° = 36°.
Triangle POSSIBLE: second angle must be less than 36°.
Examples: 20° (144°+20°=164°<180°), 35° (144°+35°=179°<180°).
Triangle NOT POSSIBLE: second angle must be 36° or more.
Examples: 36° (144°+36°=180°), 90° (144°+90°=234°>180°).

Answer: (a) Possible: 50°, 90°. Not possible: 150°, 170°. (b) Possible: 60°, 80°. Not possible: 110°, 140°. (c) Possible: 90°, 64°. Not possible: 126°, 134°. (d) Possible: 20°, 35°. Not possible: 36°, 90°.

Q3 Determine which of the following pairs can be the angles of a triangle and which cannot: (a) 35°, 150° (b) 70°, 30° (c) 90°, 85° (d) 50°, 150°

SOLUTION

A pair of angles can belong to a triangle if and only if their sum is strictly less than 180° (so that a positive third angle remains).

(a) 35° and 150°:
Sum = 35° + 150° = 185° > 180°.
This pair CANNOT be angles of a triangle.

(b) 70° and 30°:
Sum = 70° + 30° = 100° < 180°.
Third angle = 180° - 100° = 80° > 0°.
This pair CAN be angles of a triangle.

(c) 90° and 85°:
Sum = 90° + 85° = 175° < 180°.
Third angle = 180° - 175° = 5° > 0°.
This pair CAN be angles of a triangle.

(d) 50° and 150°:
Sum = 50° + 150° = 200° > 180°.
This pair CANNOT be angles of a triangle.

Answer: Can be angles of a triangle: (b) 70°, 30° (sum=100°<180°); (c) 90°, 85° (sum=175°<180°). Cannot be angles of a triangle: (a) 35°, 150° (sum=185°>180°); (d) 50°, 150° (sum=200°>180°).

Exercise 7.7 — Angle Sum Property of Triangles (Page 165)

Q1 Find the third angle of a triangle (using a parallel line) when two of the angles are: (a) 36°, 72° (b) 150°, 15° (c) 90°, 30° (d) 75°, 45°

SOLUTION

By the angle sum property of triangles: the sum of all three angles of a triangle is 180°.
Third angle = 180° - (sum of the other two angles).

Reasoning using a parallel line (as in the textbook's approach for part a):
In triangle ABC, draw line XY through vertex A parallel to BC.
Then angle XAB = angle B (alternate angles, since AB is a transversal).
And angle YAC = angle C (alternate angles, since AC is a transversal).
Since XAY is a straight line: angle XAB + angle BAC + angle YAC = 180°.
Therefore angle BAC = 180° - angle B - angle C.

(a) Angles 36° and 72°:
Third angle = 180° - 36° - 72° = 180° - 108° = 72°.

(b) Angles 150° and 15°:
Third angle = 180° - 150° - 15° = 180° - 165° = 15°.

(c) Angles 90° and 30°:
Third angle = 180° - 90° - 30° = 180° - 120° = 60°.

(d) Angles 75° and 45°:
Third angle = 180° - 75° - 45° = 180° - 120° = 60°.

Answer: (a) 72°. (b) 15°. (c) 60°. (d) 60°.

Q2 Can you construct a triangle all of whose angles are equal to 70°? If two of the angles are 70°, what would the third angle be? If all the angles in a triangle have to be equal, then what must its measure be? Explore and find out.

SOLUTION

Part 1: Can all three angles of a triangle equal 70°?
Sum of three angles = 70° + 70° + 70° = 210°.
But the angle sum property requires the three angles to add up to exactly 180°.
Since 210° ≠ 180°, it is NOT possible to construct a triangle with all three angles equal to 70°.

Part 2: If two angles are 70°, what is the third angle?
Third angle = 180° - 70° - 70° = 180° - 140° = 40°.

Part 3: If all three angles must be equal, what is the measure of each?
Let each angle = x°.
Then x + x + x = 180°, so 3x = 180°, giving x = 60°.
Each angle must be 60°. This is an equilateral triangle.

Answer: No, a triangle with all angles 70° is not possible (70°×3=210°≠180°). If two angles are 70°, the third = 40°. For all equal angles: each must be 60° (equilateral triangle).

Q3 Here is a triangle in which we know angle_B = angle_C and angle_A = 50°. Can you find angle_B and angle_C? [Figure shows triangle ABC with angle A = 50° at the top and equal angles at B and C.]

SOLUTION

Given: angle_A = 50°, and angle_B = angle_C (isosceles triangle with equal base angles).

Using the angle sum property:
angle_A + angle_B + angle_C = 180°.
50° + angle_B + angle_C = 180°.
angle_B + angle_C = 180° - 50° = 130°.

Since angle_B = angle_C:
2 × angle_B = 130°.
angle_B = 65°.
Therefore angle_C = 65°.

Verification: 50° + 65° + 65° = 180°. ✓

Answer: angle_B = angle_C = 65°.

Exercise 7.8 — Types of Triangles and Altitudes (Page 170)

Q1 Construct a triangle ABC with BC = 5 cm, AB = 6 cm, CA = 5 cm. Construct an altitude from A to BC.

SOLUTION

Step 1: Check that a triangle exists.
Sides: BC = 5, AB = 6, CA = 5. Sorted: 5, 5, 6.
Triangle inequality: 5 + 5 = 10 > 6. ✓ Triangle exists.

Step 2: Construct triangle ABC.
Draw BC = 5 cm as the base.
From B, draw an arc of radius 6 cm (for AB).
From C, draw an arc of radius 5 cm (for CA).
Let A be the intersection of the two arcs above BC.
Join AB and AC. Triangle ABC is complete.

Step 3: Observe the triangle.
BC = CA = 5 cm, so triangle ABC is isosceles with the equal sides being BC and CA.
The vertex angle is at C, and the base is AB.

Step 4: Construct the altitude from A to BC.
The altitude from A is the perpendicular from A to the line BC.
Using a set square: align one edge of the set square along BC; slide until the perpendicular edge passes through A. Draw the perpendicular line. Let D be the foot on BC.

Note on the altitude length:
Using coordinates: place B at (0,0), C at (5,0). Since AB=6 and CA=5:
A = (3.6, 4.8). The foot of the altitude D = (3.6, 0).
Altitude length AD = 4.8 cm.
BD = 3.6 cm, DC = 1.4 cm. The foot D lies inside segment BC.

Answer: Triangle ABC is isosceles (BC=CA=5 cm, AB=6 cm). Draw BC=5 cm, construct arcs of radius 6 cm from B and 5 cm from C, mark intersection as A. The altitude from A meets BC at D (inside BC). Altitude length AD = 4.8 cm.

Q2 Construct a triangle TRY with RY = 4 cm, TR = 7 cm, angle_R = 140°. Construct an altitude from T to RY.

SOLUTION

Step 1: Construct triangle TRY using SAS (two sides and included angle).
Draw RY = 4 cm as the base.
At vertex R, construct angle_R = 140° (an obtuse angle).
Along the other arm of the angle at R, mark T such that TR = 7 cm.
Join T and Y to complete triangle TRY.

Step 2: Understand where the altitude from T lands.
Angle R = 140° is obtuse. This means the altitude from T to line RY falls OUTSIDE the triangle.
Specifically, the foot of the altitude (let us call it D) lies on the extension of RY beyond R (not between R and Y).

Step 3: Construct the altitude from T to RY.
Extend the line RY beyond R.
Using a set square or compass, draw a perpendicular from T to the extended line RY.
The foot D is on the extension beyond R.
The altitude TD is the distance from T to the line RY.

Note: For a triangle with an obtuse angle at R, all altitudes to the sides adjacent to R will have feet outside the triangle on the extended sides.

Answer: Draw RY=4 cm, construct 140° at R, mark T at 7 cm along the other arm, join TY. Since angle R=140° is obtuse, the altitude from T to RY has its foot D on the extension of YR beyond R. Extend RY beyond R and drop a perpendicular from T to meet the extended line at D.

Q3 Construct a right-angled triangle ABC with angle_B = 90°, AC = 5 cm. How many different triangles exist with these measurements? [Hint: Note that the other measurements can take any values. Take AC as the base. What values can angle_A and angle_C take so that the other angle is 90°?]

SOLUTION

Given: angle_B = 90°, AC = 5 cm (AC is the hypotenuse, since it is opposite the right angle at B).

The remaining measurements are not fixed: angle_A and angle_C must satisfy:

angle_A + angle_C = 90° (since angle_A + angle_B + angle_C = 180° and angle_B = 90°).
Both angle_A and angle_C must be strictly between 0° and 90°.
angle_A can be any value in the range (0°, 90°), and then angle_C = 90° - angle_A.

Since angle_A can take infinitely many values (e.g., 1°, 10°, 30°, 45°, 60°, 89°, etc.), there are infinitely many different right-angled triangles with hypotenuse AC = 5 cm.

Examples:
angle_A = 30°, angle_C = 60°: AB = 5cos(30°) ≈ 4.33 cm, BC = 5sin(30°) = 2.5 cm.
angle_A = 45°, angle_C = 45°: AB = BC = 5/√2 ≈ 3.54 cm (isosceles right triangle).
angle_A = 60°, angle_C = 30°: AB = 5cos(60°) = 2.5 cm, BC = 5sin(60°) ≈ 4.33 cm.

Therefore, infinitely many different right-angled triangles exist with angle_B = 90° and AC = 5 cm.

Answer: Infinitely many different triangles exist. AC=5 cm is the hypotenuse. angle_A and angle_C must sum to 90°, and angle_A can be any value strictly between 0° and 90°. Each different choice of angle_A gives a different triangle.

Q4 Through construction, explore if it is possible to construct an equilateral triangle that is (i) right-angled (ii) obtuse-angled. Also construct an isosceles triangle that is (i) right-angled (ii) obtuse-angled.

SOLUTION

Equilateral triangle:
An equilateral triangle has all three sides equal and therefore all three angles equal.
By the angle sum property: each angle = 180° ÷ 3 = 60°.

(i) Equilateral and right-angled:
A right-angled triangle must have one angle equal to 90°.
But in an equilateral triangle, every angle is 60° — none can be 90°.
It is NOT possible to construct an equilateral triangle that is right-angled.

(ii) Equilateral and obtuse-angled:

An obtuse-angled triangle must have one angle greater than 90°.
But in an equilateral triangle, every angle is 60° — none can exceed 90°.
It is NOT possible to construct an equilateral triangle that is obtuse-angled.

Isosceles triangle:
An isosceles triangle has two equal sides (and two equal base angles). The apex angle can vary.

(i) Isosceles and right-angled:
Choose the right angle to be at the apex (vertex between the equal sides).
Angles: 90°, 45°, 45°. Sum = 180°. ✓
The two equal legs are the sides adjacent to the 90° angle.
Example: an isosceles right triangle with legs of 3 cm each. Hypotenuse = $3\sqrt{2} \approx 4.24$ cm.
This is possible to construct.

(ii) Isosceles and obtuse-angled:
Choose the obtuse angle to be at the apex.
Example: angles 120°, 30°, 30°. Sum = 180°. ✓
The two base angles are equal (30° each), making it isosceles.
Another example: 100°, 40°, 40°.
These are possible to construct.

Answer: Equilateral triangle: cannot be right-angled (all angles are 60°, none is 90°) or obtuse-angled (60°<90°). Isosceles right-angled: possible — angles 90°, 45°, 45°. Isosceles obtuse-angled: possible — e.g., angles 120°, 30°, 30°.