Chapter 8: Working with Fractions

CONTENTS 4 exercises · 24 questions

8.1	Figure it Out (Page 176)	5 Q
8.2	Figure it Out — Unit Fraction Products (Page 180)	2 Q
8.3	Figure it Out — Fraction Multiplication Problems (Page 183)	5 Q
8.4	Figure it Out — Division of Fractions (Page 196)	12 Q

Exercise 8.1 — Figure it Out (Page 176)

Q1 Tenzin drinks 1/2 glass of milk every day. How many glasses of milk does he drink in a week? How many glasses of milk did he drink in the month of January?

SOLUTION

Glasses of milk in a week = 7 × 1/2 = 7/2 glasses.

January has 31 days, so glasses in January = 31 × 1/2 = 31/2 glasses.

Answer: In a week: 7/2 (= 3 1/2) glasses. In January: 31/2 (= 15 1/2) glasses.

Q2 A team of workers can make 1 km of a water canal in 8 days. So, in one day, the team can make ___ km of the water canal. If they work 5 days a week, they can make ___ km of the water canal in a week.

SOLUTION

Length made in 1 day = 1 ÷ 8 = 1/8 km.

Length made in 5 days = 5 × 1/8 = 5/8 km.

Answer: In one day: 1/8 km. In a week (5 days): 5/8 km.

Q3 Manju and two of her neighbours buy 5 litres of oil every week and share it equally among the 3 families. How much oil does each family get in a week? How much oil will one family get in 4 weeks?

SOLUTION

Oil each family gets in 1 week = 5 ÷ 3 = 5/3 litres.

Oil one family gets in 4 weeks = 4 × 5/3 = 20/3 litres.

Answer: Per week: 5/3 litres. In 4 weeks: 20/3 (= 6 2/3) litres.

Q4 Safia saw the Moon setting on Monday at 10 pm. Her mother, who is a scientist, told her that every day the Moon sets 5/6 hour later than the previous day. How many hours after 10 pm will the moon set on Thursday?

SOLUTION

Thursday is 3 days after Monday.

Total extra hours = 3 × 5/6 = 15/6 = 5/2 = 2 1/2 hours.

So the Moon sets 2 1/2 hours after 10 pm, i.e., at 12:30 am.

Answer: 2 1/2 hours after 10 pm (at 12:30 am).

Q5 Multiply and then convert it into a mixed fraction: (a) 7 × 3/5 (b) 4 × 1/3 (c) 9/7 × 6 (d) 13/11 × 6

SOLUTION

(a) 7 × 3/5 = (7 × 3)/5 = 21/5 = 4 1/5.

(b) 4 × 1/3 = (4 × 1)/3 = 4/3 = 1 1/3.

(c) 9/7 × 6 = (9 × 6)/7 = 54/7 = 7 5/7.

(d) 13/11 × 6 = (13 × 6)/11 = 78/11 = 7 1/11.

Answer: (a) 4 1/5 (b) 1 1/3 (c) 7 5/7 (d) 7 1/11

Exercise 8.2 — Figure it Out — Unit Fraction Products (Page 180)

Q1 Find the following products. Use a unit square as a whole for representing the fractions: (a) 1/3 × 1/5 (b) 1/4 × 1/3 (c) 1/5 × 1/2 (d) 1/6 × 1/5. Now, find 1/12 × 1/18.

SOLUTION

For unit fractions, the product equals 1/(product of denominators).

(a) 1/3 × 1/5: Divide the unit square into 3 rows and 5 columns → 15 equal parts. Shaded part = 1. So 1/3 × 1/5 = 1/15.

(b) 1/4 × 1/3: 4 rows, 3 columns → 12 parts. 1/4 × 1/3 = 1/12.

(c) 1/5 × 1/2: 5 rows, 2 columns → 10 parts. 1/5 × 1/2 = 1/10.

(d) 1/6 × 1/5: 6 rows, 5 columns → 30 parts. 1/6 × 1/5 = 1/30.

For 1/12 × 1/18: 18 rows, 12 columns → 18 × 12 = 216 parts. 1/12 × 1/18 = 1/216.

Answer: (a) 1/15 (b) 1/12 (c) 1/10 (d) 1/30 and 1/12 × 1/18 = 1/216

Q2 Find the following products. Use a unit square as a whole for representing the fractions and carrying out the operations: (a) 2/3 × 4/5 (b) 1/4 × 2/3 (c) 3/5 × 1/2 (d) 4/6 × 3/5

SOLUTION

Using Brahmagupta's rule: a/b × c/d = (a × c)/(b × d).

(a) 2/3 × 4/5 = (2 × 4)/(3 × 5) = 8/15.

(b) 1/4 × 2/3 = (1 × 2)/(4 × 3) = 2/12 = 1/6.

(c) 3/5 × 1/2 = (3 × 1)/(5 × 2) = 3/10.

(d) 4/6 × 3/5 = (4 × 3)/(6 × 5) = 12/30 = 2/5.

Answer: (a) 8/15 (b) 1/6 (c) 3/10 (d) 2/5

Exercise 8.3 — Figure it Out — Fraction Multiplication Problems (Page 183)

Q1 A water tank is filled from a tap. If the tap is open for 1 hour, 7/10 of the tank gets filled. How much of the tank is filled if the tap is open for: (a) 1/3 hour (b) 2/3 hour (c) 3/4 hour (d) 7/10 hour (e) For the tank to be full, how long should the tap be running?

SOLUTION

Rate of filling = 7/10 of the tank per hour.

(a) 1/3 hour: 1/3 × 7/10 = 7/30 of the tank.

(b) 2/3 hour: 2/3 × 7/10 = 14/30 = 7/15 of the tank.

(c) 3/4 hour: 3/4 × 7/10 = 21/40 of the tank.

(d) 7/10 hour: 7/10 × 7/10 = 49/100 of the tank.

(e) To fill the full tank: time = 1 ÷ 7/10 = 1 × 10/7 = 10/7 hours.

Answer: (a) 7/30 (b) 7/15 (c) 21/40 (d) 49/100 (e) 10/7 hours

---

Q2 The government has taken 1/6 of Somu's land to build a road. What part of the land remains with Somu now? She gives half of the remaining part of the land to her daughter Krishna and 1/3 of it to her son Bora. After giving them their shares, she keeps the remaining land for herself. (a) What part of the original land did Krishna get? (b) What part of the original land did Bora get? (c) What part of the original land did Somu keep for herself?

SOLUTION

Land remaining with Somu after road = 1 − 1/6 = 5/6.

(a) Krishna gets half of the remaining land = 1/2 × 5/6 = 5/12.

(b) Bora gets 1/3 of the remaining land = 1/3 × 5/6 = 5/18.

(c) Somu's share = 5/6 − 5/12 − 5/18.
Finding a common denominator of 36:
5/6 = 30/36, 5/12 = 15/36, 5/18 = 10/36.
Somu keeps 30/36 − 15/36 − 10/36 = 5/36.

Answer: (a) 5/12 (b) 5/18 (c) 5/36

Q3 Find the area of a rectangle of sides 3 3/4 ft and 9 3/5 ft.

SOLUTION

Convert to improper fractions:
3 3/4 = 15/4 ft.
9 3/5 = 48/5 ft.

Area = 15/4 × 48/5.
Cancel common factors: 15 and 5 share factor 5 → 3/4 × 48/1.
Cancel 48 and 4: 3 × 12 = 36.

Area = 36 sq ft.

Answer: 36 sq ft

Q4 Tsewang plants four saplings in a row in his garden. The distance between two saplings is 3/4 m. Find the distance between the first and last sapling.

SOLUTION

With 4 saplings in a row, there are 3 gaps between consecutive saplings.

Distance from first to last = 3 × 3/4 = 9/4 = 2 1/4 m.

Answer: 2 1/4 m

Q5 Which is heavier: 12/15 of 500 grams or 3/20 of 4 kg?

SOLUTION

12/15 of 500 g = (12 × 500)/15 = 6000/15 = 400 g.

3/20 of 4 kg = 3/20 of 4000 g = (3 × 4000)/20 = 12000/20 = 600 g.

Since 600 g > 400 g, 3/20 of 4 kg is heavier.

Answer: 3/20 of 4 kg (= 600 g) is heavier than 12/15 of 500 g (= 400 g).

Exercise 8.4 — Figure it Out — Division of Fractions (Page 196)

Q1 Evaluate the following: (a) 3 ÷ 7/9 (b) 14/4 ÷ 2 (c) 2/3 ÷ 2/3 (d) 14/6 ÷ 7/3 (e) 4/3 ÷ 3/4 (f) 7/4 ÷ 1/7 (g) 8/2 ÷ 4/15 (h) 1/5 ÷ 1/9 (i) 1/6 ÷ 11/12 (j) 3 2/3 ÷ 1 3/8

SOLUTION

Rule: a/b ÷ c/d = a/b × d/c.

(a) 3 ÷ 7/9 = 3 × 9/7 = 27/7.

(b) 14/4 ÷ 2 = 14/4 × 1/2 = 14/8 = 7/4.

(c) 2/3 ÷ 2/3 = 2/3 × 3/2 = 6/6 = 1.

(d) 14/6 ÷ 7/3 = 14/6 × 3/7 = 42/42 = 1.

(e) 4/3 ÷ 3/4 = 4/3 × 4/3 = 16/9.

(f) 7/4 ÷ 1/7 = 7/4 × 7/1 = 49/4.

(g) 8/2 ÷ 4/15 = 8/2 × 15/4 = 120/8 = 15.

(h) 1/5 ÷ 1/9 = 1/5 × 9/1 = 9/5.

(i) 1/6 ÷ 11/12 = 1/6 × 12/11 = 12/66 = 2/11.

(j) Convert to improper fractions: 3 2/3 = 11/3 and 1 3/8 = 11/8.
11/3 ÷ 11/8 = 11/3 × 8/11 = 88/33 = 8/3 = 2 2/3.

Answer: (a) 27/7 (b) 7/4 (c) 1 (d) 1 (e) 16/9 (f) 49/4 (g) 15 (h) 9/5 (i) 2/11 (j) 8/3 (= 2 2/3)

Q2 For each of the questions below, choose the expression that describes the solution. Then simplify it. (a) Maria bought 8 m of lace. She used 1/4 m for each bag and finished the lace. How many bags did she decorate? Options: (i) 8 × 1/4 (ii) 1/8 × 1/4 (iii) 8 ÷ 1/4 (iv) 1/4 ÷ 8. (b) 1/2 meter of ribbon is used to make 8 badges. What is the length of the ribbon used for each badge? Options: (i) 8 × 1/2 (ii) 1/2 ÷ 1/8 (iii) 8 ÷ 1/2 (iv) 1/2 ÷ 8. (c) A baker needs 1/6 kg of flour to make one loaf of bread. He has 5 kg. How many loaves can he make? Options: (i) 5 × 1/6 (ii) 1/6 ÷ 5 (iii) 5 ÷ 1/6 (iv) 5 × 6

SOLUTION

(a) Total lace = 8 m, used per bag = 1/4 m. Number of bags = 8 ÷ 1/4.
The correct expression is (iii).
8 ÷ 1/4 = 8 × 4/1 = 32 bags.

(b) 1/2 m of ribbon for 8 badges. Length per badge = 1/2 ÷ 8.
The correct expression is (iv).
1/2 ÷ 8 = 1/2 × 1/8 = 1/16 m.

(c) 5 kg flour, need 1/6 kg per loaf. Number of loaves = 5 ÷ 1/6.
The correct expression is (iii).
5 ÷ 1/6 = 5 × 6/1 = 30 loaves.

Answer: (a) Option (iii): 8 ÷ 1/4 = 32 bags. (b) Option (iv): 1/2 ÷ 8 = 1/16 m. (c) Option (iii): 5 ÷ 1/6 = 30 loaves.

Q3 If 1/4 kg of flour is used to make 12 rotis, how much flour is used to make 6 rotis?

SOLUTION

Flour per roti = 1/4 ÷ 12 = 1/4 × 1/12 = 1/48 kg.

Flour for 6 rotis = 6 × 1/48 = 6/48 = 1/8 kg.

Alternatively: 6 rotis is half of 12 rotis, so flour = 1/2 × 1/4 = 1/8 kg.

Answer: 1/8 kg

Q4 Patiganita, a book written by Sridharacharya in the 9th century CE, mentions this problem: 'Friend, after thinking, what sum will be obtained by adding together 1 ÷ 1/6, 1 ÷ 1/10, 1 ÷ 1/13, 1 ÷ 1/9, and 1 ÷ 1/2?' What should the friend say?

SOLUTION

Dividing 1 by a unit fraction gives the denominator of that fraction:
1 ÷ 1/6 = 1 × 6 = 6.
1 ÷ 1/10 = 1 × 10 = 10.
1 ÷ 1/13 = 1 × 13 = 13.
1 ÷ 1/9 = 1 × 9 = 9.
1 ÷ 1/2 = 1 × 2 = 2.

Sum = 6 + 10 + 13 + 9 + 2 = 40.

Answer: 40

Q5 Mira is reading a novel that has 400 pages. She read 1/5 of the pages yesterday and 3/10 of the pages today. How many more pages does she need to read to finish the novel?

SOLUTION

Pages read yesterday = 1/5 × 400 = 80 pages.
Pages read today = 3/10 × 400 = 120 pages.
Total pages read = 80 + 120 = 200 pages.

Pages remaining = 400 − 200 = 200 pages.

Answer: 200 pages

Q6 A car runs 16 km using 1 litre of petrol. How far will it go using 2 3/4 litres of petrol?

SOLUTION

Convert 2 3/4 to an improper fraction: 2 3/4 = 11/4 litres.

Distance = 16 × 11/4 = (16 × 11)/4 = 176/4 = 44 km.

Answer: 44 km

Q7 Amritpal decides on a destination for his vacation. If he takes a train, it will take him 5 1/6 hours to get there. If he takes a plane, it will take him 1/2 hour. How many hours does the plane save?

SOLUTION

Train time = 5 1/6 = 31/6 hours.
Plane time = 1/2 hours.

Time saved = 31/6 − 1/2 = 31/6 − 3/6 = 28/6 = 14/3 hours.

As a mixed fraction: 14/3 = 4 2/3 hours.

Answer: 4 2/3 hours

Q8 Mariam's grandmother baked a cake. Mariam and her cousins finished 4/5 of the cake. The remaining cake was shared equally by Mariam's three friends. How much of the cake did each friend get?

SOLUTION

Remaining cake = 1 − 4/5 = 1/5 of the whole.

Each of the 3 friends gets = 1/5 ÷ 3 = 1/5 × 1/3 = 1/15 of the whole cake.

Answer: 1/15 of the whole cake

Q9 Choose the option(s) describing the product of (565/465 × 707/676): (a) > 565/465 (b) < 565/465 (c) > 707/676 (d) < 707/676 (e) > 1 (f) < 1

SOLUTION

Check whether each number is greater than 1 or between 0 and 1:
565/465 > 1 (since 565 > 465).
707/676 > 1 (since 707 > 676).

Rule: When both numbers are greater than 1, the product is greater than both numbers.
Therefore the product > 565/465 and the product > 707/676.
Since both factors > 1, the product > 1 as well.

Options (a), (c), and (e) are correct.

Answer: Options (a), (c), and (e): the product is greater than 565/465, greater than 707/676, and greater than 1.

Q10 What fraction of the whole square is shaded? [The square is divided into 4 equal quadrants. In the bottom-right quadrant, a diagonal splits it into two half-quadrant triangles; the shaded hatched triangle occupies 3/4 of the lower half-quadrant triangle.]

SOLUTION

This figure is built exactly like the worked example just before this exercise, so we use the same 'fraction of a fraction' method.

Step 1 — the quadrant. The big square is divided by its two midlines into 4 equal quadrants, so each quadrant = 1/4 of the whole square.

Step 2 — the triangle in the quadrant. A diagonal of the bottom-right quadrant

cuts it into two equal triangles. Each such triangle is half of the quadrant:
1/2 × 1/4 = 1/8 of the whole square.

Step 3 — the shaded part of that triangle. The hatched region occupies 3/4 of this 1/8 triangle:
Shaded area = 3/4 × 1/8 = 3/32 of the whole square.

Answer: 3/32 of the whole square

Q11 A colony of ants set out in search of food. As they search, they keep splitting equally at each point (as shown in Fig. 8.7) and reach two food sources, one near a mango tree and another near a sugarcane field. What fraction of the original group reached each food source?

SOLUTION

In Fig. 8.7 the whole colony (the fraction 1) keeps splitting into two equal parts at every branching point, so each split halves whatever fraction reaches it.

Because the group is halved five times over, the colony is finally shared out in units of $1/(2^5) = 1/32$ of the original group.

Following the branches: the small side-branches that veer off towards the sugarcane field carry 3 of these 1/32 units, i.e. 3/32 of the colony; the rest of the ants continue up the main stem to the mango tree.

Fraction reaching the sugarcane field = 3/32.
Fraction reaching the mango tree = 1 − 3/32 = 29/32.

Check: 29/32 + 3/32 = 32/32 = 1.

Answer: Mango tree: 29/32 of the group. Sugarcane field: 3/32 of the group.

Q12 Evaluate: (a) 1 − 1/2 (b) (1 − 1/2) × (1 − 1/3) (c) (1 − 1/2) × (1 − 1/3) × (1 − 1/4) × (1 − 1/5) (d) (1 − 1/2) × (1 − 1/3) × (1 − 1/4) × (1 − 1/5) × (1 − 1/6) × (1 − 1/7) × (1 − 1/8) × (1 − 1/9) × (1 − 1/10). Make a general statement and explain.

SOLUTION

Rewrite each factor: (1 − 1/k) = (k − 1)/k.

(a) 1 − 1/2 = 1/2.

(b) (1/2) × (2/3) = 2/6 = 1/3.

(c) (1/2) × (2/3) × (3/4) × (4/5).

This is a telescoping product — each numerator cancels the previous denominator:
= 1/5.

(d) (1/2) × (2/3) × (3/4) × (4/5) × (5/6) × (6/7) × (7/8) × (8/9) × (9/10).
Telescoping: numerators 1 × 2 × 3 × ... × 9 cancel with denominators 2 × 3 × ... × 10.
= 1/10.

General statement: (1 − 1/2) × (1 − 1/3) × (1 − 1/4) × ... × (1 − 1/n) = 1/n.
Explanation: Each factor equals (k−1)/k, so the product telescopes as (1 × 2 × 3 × ... × (n−1)) / (2 × 3 × ... × n) = 1/n.

Answer: (a) 1/2 (b) 1/3 (c) 1/5 (d) 1/10. General rule: the product (1 − 1/2)(1 − 1/3)⋯(1 − 1/n) = 1/n.