Chapter 9: Geometric Twins

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Exercise 9.1 — Geometric Twins — Congruent Figures (Page 3)

Q1 Check if the two figures are congruent. [Figure shows two V-shaped (angle) figures: the left one is a symmetric V with two equal-looking arms; the right one has arms of different lengths and a wider opening angle — they do not look the same size and shape.]

SOLUTION

Two figures are congruent only if they have exactly the same shape AND the same size.

For V-shaped figures (two arms meeting at a vertex), we need to check: (i) the arm lengths and (ii) the angle between the arms.

Looking at the two figures: the left figure has a narrower angle between its arms, while the right figure has a wider angle and different arm proportions.

Since the angle between the arms and the arm lengths do not match, the figures cannot be made to fit exactly over each other.

Answer: No, the two figures are not congruent.

Q2 Circle the pairs that appear congruent. [Four pairs of figures are shown: Pair 1 — two leaf/droplet shapes of the same size; Pair 2 — two cloud shapes where one is larger than the other; Pair 3 — two identical spiky star/sun shapes; Pair 4 — two identical petal/leaf shapes.]

SOLUTION

A pair is congruent when both figures have exactly the same shape and size (one can be placed exactly over the other, allowing rotation or flipping).

Pair 1 (two leaf/droplet shapes): Both have the same outline and size. →

Congruent.

Pair 2 (two cloud shapes): One cloud is clearly larger than the other. → NOT congruent.

Pair 3 (two spiky star shapes): Both have the same number of points and the same size. → Congruent.

Pair 4 (two petal/leaf shapes): Both have the same curved shape and size. → Congruent.

Answer: Pairs 1, 3, and 4 appear congruent. Pair 2 is not congruent (different sizes).

Q3 What measurements would you take to create a figure congruent to a given: (a) Circle (b) Rectangle. Using this, state how would you check if two circles are congruent and if two rectangles are congruent.

SOLUTION

(a) Circle: A circle is completely determined by its radius. So we need to measure only the radius.

To check if two circles are congruent: measure their radii. If the radii are equal, the circles are congruent.

(b) Rectangle: A rectangle is determined by its length and its width. So we need to measure both the length and the width.

To check if two rectangles are congruent: measure the length and width of each. If both dimensions match (length = length AND width = width), the rectangles are congruent.

Answer: (a) Measure the radius; two circles are congruent if their radii are equal. (b) Measure length and width; two rectangles are congruent if both their length and width are equal.

Q4 How would we check if two figures like the one below are congruent? [The figure shows two line segments crossing each other at a point — like an X or asterisk shape.] Use this to identify whether each of the following pairs are congruent. [Pair 1 (top): two crossed-line figures that appear identical in arm lengths and crossing angle. Pair 2 (bottom): two crossed-line figures where the arm lengths appear different.]

SOLUTION

For a figure made of two line segments crossing at a point, we need to measure: (i) all four arm lengths (from the crossing point to each end), and (ii) the angle at which the segments cross.

Two such figures are congruent if all four arm lengths and the crossing angle are equal.

Pair 1 (top): The arm lengths and the crossing angle of both figures appear equal. → Congruent.

Pair 2 (bottom): The arm lengths of the two figures are visibly different. → NOT congruent.

Answer: Measure all four arm lengths and the crossing angle. Pair 1 is congruent; Pair 2 is not congruent.

Exercise 9.2 — Congruence of Triangles — SSS Condition (Page 8)

Q1 Suppose ΔHEN is congruent to ΔBIG. List all the other correct ways of expressing this congruence.

SOLUTION

When we write ΔHEN ≅ ΔBIG, it means the vertices correspond in order: H↔B, E↔I, N↔G.

Any correct re-expression must keep the same correspondence. We can cyclically permute or reverse the order of vertices in both triangles together.

The six valid ways (all expressing the same correspondence) are:

ΔHEN ≅ ΔBIG
ΔENH ≅ ΔIGB
ΔNHE ≅ ΔGBI
ΔHNE ≅ ΔBGI
ΔNEH ≅ ΔGIB
ΔEHN ≅ ΔIBG

Answer: The other correct ways are: ΔENH ≅ ΔIGB, ΔNHE ≅ ΔGBI, ΔHNE ≅ ΔBGI, ΔNEH ≅ ΔGIB, ΔEHN ≅ ΔIBG.

Q2 Determine whether the triangles are congruent. If yes, express the congruence. [Left triangle RED: RE = 3.5 cm, ED = 5 cm, RD = 6 cm. Right triangle JMA: JM = 6 cm, MA = 5 cm, JA = 3.5 cm.]

SOLUTION

List the sides of each triangle:

ΔRED: RE = 3.5 cm, ED = 5 cm, RD = 6 cm
ΔJMA: JA = 3.5 cm, MA = 5 cm, JM = 6 cm

Match equal sides: RE = JA = 3.5 cm, ED = MA = 5 cm, RD = JM = 6 cm.

All three pairs of sides are equal → SSS (Side Side Side) condition is satisfied.

Correspondence: R↔J, E↔A, D↔M.

Therefore, ΔRED ≅ ΔJAM.

Answer: Yes, ΔRED ≅ ΔJAM (by SSS condition: RE = JA, ED = AM, RD = JM).

Q3 In the figure, AB = AD and CB = CD. Can you identify any pair of congruent triangles? If yes, explain why they are congruent. Does AC divide ∠BAD and ∠BCD into two equal parts? Give reasons. [Kite-shaped figure with A at top, B at middle-left, C at bottom, D at middle-right. The dashed line AC is the diagonal.]

SOLUTION

Consider triangles ΔABC and ΔADC.
AB = AD (given — single tick marks).
CB = CD (given — double tick marks).
AC = AC (common side — the diagonal is shared).
All three pairs of sides are equal → SSS condition is satisfied.
Therefore, ΔABC ≅ ΔADC (with correspondence A↔A, B↔D, C↔C).

Since ΔABC ≅ ΔADC, their corresponding angles are equal.
∠BAC = ∠DAC → AC divides ∠BAD into two equal parts. ✓
∠BCA = ∠DCA → AC divides ∠BCD into two equal parts. ✓

Answer: ΔABC ≅ ΔADC (SSS). Yes, AC bisects both ∠BAD and ∠BCD, since corresponding angles of the congruent triangles are equal.

Q4 In the figure, are ΔDFE and ΔGED congruent to each other? It is given that DF = DG and FE = GE. [Figure shows an isosceles triangle with D at the top, F at bottom-left and G at bottom-right. E is the midpoint of FG, with DE drawn inside.]

SOLUTION

Compare the sides of ΔDFE and ΔGED:
DF = GD (given: DF = DG, same as GD).
FE = GE (given — E is the midpoint of FG, so FE = EG).
DE = ED (common side — shared by both triangles).
All three pairs of sides are equal → SSS condition is satisfied.
Correspondence: D↔G, F↔E, E↔D.
Therefore, ΔDFE ≅ ΔGED. ✓

Answer: Yes, ΔDFE ≅ ΔGED by SSS (DF = GD, FE = GE, DE = ED).

Exercise 9.3 — Congruence of Triangles — ASA and SAS Conditions (Page 13)

Q1 Identify whether the triangles below are congruent. What conditions did you use to establish their congruence? Express the congruence. [ΔABC: AB = 7 cm (left side), ∠B = 47°, BC = 5 cm (base). ΔXYZ: XZ = 7 cm (right side), ∠Z = 47°, YZ = 5 cm (base).]

SOLUTION

In ΔABC: AB = 7 cm, ∠B = 47° (angle at B, included between AB and BC), BC = 5 cm.

In ΔXYZ: XZ = 7 cm, ∠Z = 47° (angle at Z, included between XZ and ZY), ZY = 5 cm.

Matching equal parts: AB = XZ = 7 cm, ∠B = ∠Z = 47° (included angle), BC = ZY = 5 cm.

Two sides and the included angle are equal → SAS (Side Angle Side) condition is satisfied.

Correspondence: A↔X, B↔Z, C↔Y.

Therefore, ΔABC ≅ ΔXZY.

Answer: Yes, ΔABC ≅ ΔXZY by SAS (AB = XZ = 7 cm, ∠B = ∠Z = 47°, BC = ZY = 5 cm).

Q2 Given that CD and AB are parallel, and AB = CD. What are the other equal parts in this figure? [Figure shows two triangles meeting at point O (intersection of diagonals): D (top-left), C (top-right), O (center), A (bottom-left), B (bottom-right). DC has double tick marks, AB has double tick marks.] Are the two resulting triangles congruent? If so, express the congruence.

SOLUTION

D and C are the top vertices, A and B the bottom vertices, and the segments DB and CA cross at O. DC ∥ AB and DC = AB (both marked with double ticks).

Consider ΔDCO and ΔABO:

∠CDO = ∠BAO (alternate interior angles, DC ∥ AB, transversal DA).

DC = AB (given, the included side).

∠DCO = ∠BAO's partner ∠ABO (alternate interior angles, DC ∥ AB, transversal CB).

Two angles and the included side are equal → ASA condition is satisfied.

Correspondence: D↔A, C↔B, O↔O.

Therefore, ΔDCO ≅ ΔABO.

Other equal parts (corresponding parts of congruent triangles): DO = AO and CO = BO.

Answer: ΔDCO ≅ ΔABO by ASA (∠CDO = ∠BAO, DC = AB, ∠DCO = ∠ABO). Other equal parts: DO = AO and CO = BO.

Q3 Given that ∠ABC = ∠DBC and ∠ACB = ∠DCB, show that ∠BAC = ∠BDC. Are the two triangles congruent? [Figure: kite/rhombus shape with B at left, C at right, A at top, D at bottom. BC is the common base. Angle marks show ∠ABC = ∠DBC (at B) and ∠ACB = ∠DCB (at C).]

SOLUTION

Consider triangles ΔABC and ΔDBC.
∠ABC = ∠DBC (given).
BC = BC (common side).
∠ACB = ∠DCB (given).
Two angles and the included side are equal → ASA condition is satisfied.
Therefore, ΔABC ≅ ΔDBC (correspondence A↔D, B↔B, C↔C).

Since ΔABC ≅ ΔDBC, their corresponding angles are equal.
∠BAC and ∠BDC are corresponding angles (at A and D respectively).
Therefore, ∠BAC = ∠BDC. ✓

Answer: ΔABC ≅ ΔDBC by ASA. Therefore ∠BAC = ∠BDC (corresponding angles of congruent triangles). Yes, the two triangles are congruent.

Q4 Identify the equal parts in the following figure, given that ∠ABD = ∠DCA and ∠ACB = ∠DBC. [Trapezoid-like figure: A (top-left), D (top-right), B (bottom-left), C (bottom-right). Diagonals AC and BD cross each other. Angle arc marks: ∠ABD = ∠DCA and ∠ACB = ∠DBC (arcs at B and C).]

SOLUTION

We are given: ∠ABD = ∠DCA and ∠ACB = ∠DBC.

At vertex B: ∠ABC = ∠ABD + ∠DBC.
At vertex C: ∠DCB = ∠DCA + ∠ACB.
Since ∠ABD = ∠DCA and ∠DBC = ∠ACB, we get ∠ABC = ∠DCB.

Now consider ΔABC and ΔDCB:

∠ABC = ∠DCB (shown above).

BC = CB (common side).

∠ACB = ∠DBC (given).

ASA condition is satisfied → ΔABC ≅ ΔDCB.

Equal parts (corresponding parts of congruent triangles):

AB = DC, AC = DB, ∠BAC = ∠CDB.

Answer: ΔABC ≅ ΔDCB by ASA. Equal parts: AB = DC, AC = DB, ∠BAC = ∠CDB.

Exercise 9.4 — Angles of Isosceles and Equilateral Triangles (Page 20)

Q1 ΔAIR ≅ ΔFLY. Identify the corresponding vertices, sides and angles.

SOLUTION

When ΔAIR ≅ ΔFLY, the vertices correspond in order: A↔F, I↔L, R↔Y.

Corresponding vertices: A and F, I and L, R and Y.

Corresponding sides:
AI = FL (sides between 1st and 2nd vertices),
IR = LY (sides between 2nd and 3rd vertices),
AR = FY (sides between 1st and 3rd vertices).

Corresponding angles:
∠A = ∠F, ∠I = ∠L, ∠R = ∠Y.

Answer: Corresponding vertices: A↔F, I↔L, R↔Y. Sides: AI=FL, IR=LY, AR=FY. Angles: ∠A=∠F, ∠I=∠L, ∠R=∠Y.

Q2 Each of the following cases contains certain measurements taken from two triangles. Identify the pairs in which the triangles are congruent to each other, with reason. Express the congruence whenever they are congruent. (a) AB=DE, BC=EF, CA=DF (b) AB=EF, ∠A=∠E, AC=ED (c) AB=DF, ∠B=∠D=90°, AC=FE (d) ∠A=∠D, ∠B=∠E, AC=DF (e) AB=DF, ∠B=∠F, AC=DE

SOLUTION

(a) AB=DE, BC=EF, CA=DF: All three pairs of sides are equal. SSS condition → ΔABC ≅ ΔDEF.

(b) AB=EF, ∠A=∠E, AC=ED: Two sides (AB=EF and AC=ED) with ∠A included between AB and AC, and ∠E included between EF and ED. SAS condition → ΔABC ≅ ΔEFD.

(c) AB=DF, ∠B=∠D=90°, AC=FE: Right angles at B and D; hypotenuse AC=FE (opposite the right angles); leg AB=DF. RHS condition → ΔABC ≅ ΔDFE.

(d) ∠A=∠D, ∠B=∠E, AC=DF: Two angles equal (∠A=∠D, ∠B=∠E); AC is opposite ∠B and DF is opposite ∠E (non-included side). AAS condition → ΔABC ≅ ΔDEF.

(e) AB=DF, ∠B=∠F, AC=DE: This is the SSA (Side Side Angle) arrangement — two sides and a non-included angle. SSA does NOT guarantee congruence → cannot conclude the triangles are congruent.

Answer: (a) ΔABC ≅ ΔDEF (SSS). (b) ΔABC ≅ ΔEFD (SAS). (c) ΔABC ≅ ΔDFE (RHS). (d) ΔABC ≅ ΔDEF (AAS). (e) Not necessarily congruent (SSA condition does not guarantee congruence).

Q3 It is given that OB = OC, and OA = OD. Show that AB is parallel to CD. [Hint: AD is a transversal for these two lines. Are there any equal alternate angles?] [Figure: A (top-left) and C (top-right) connected through O (center); B (bottom-left) and D (bottom-right). OA=OD (single tick), OB=OC (double tick). Lines cross at O.]

SOLUTION

Consider triangles ΔAOB and ΔDOC.
OA = OD (given).
OB = OC (given).
∠AOB = ∠DOC (vertically opposite angles at O).
Two sides and the included angle are equal → SAS condition → ΔAOB ≅ ΔDOC.

Since ΔAOB ≅ ΔDOC, corresponding angles are equal:
∠OAB = ∠ODC (corresponding angles at A and D).

Now, AD is a transversal cutting lines AB and CD.
∠OAB = ∠OAB is the angle at A (between AD and AB).
∠ODC = ∠ODC is the angle at D (between DA and DC).
These are alternate interior angles for lines AB and CD.
Since the alternate interior angles are equal, AB ∥ CD. ✓

Answer: ΔAOB ≅ ΔDOC (SAS). So ∠OAB = ∠ODC (corresponding angles). These are equal alternate interior angles for lines AB and CD with transversal AD. Therefore AB ∥ CD.

Q4 ABCD is a square. Show that ΔABC ≅ ΔADC. Is ΔABC also congruent to ΔCDA? [Figure: square with A (top-left), B (top-right), C (bottom-right), D (bottom-left). Diagonal AC is drawn.]

SOLUTION

In square ABCD, all sides are equal: AB = BC = CD = DA.

Consider ΔABC and ΔADC:
AB = AD (sides of the square).
BC = DC (sides of the square).
AC = AC (diagonal, common to both triangles).

All three pairs of sides are equal → SSS condition → ΔABC ≅ ΔADC. ✓

Is ΔABC ≅ ΔCDA?
ΔCDA has vertices C, D, A. Compare with ΔABC (vertices A, B, C):
AB corresponds to CD: AB = CD ✓ (sides of square)
BC corresponds to DA: BC = DA ✓ (sides of square)
AC corresponds to CA: AC = CA ✓ (same diagonal)
SSS condition → ΔABC ≅ ΔCDA. ✓

So ΔABC is congruent to ΔADC in the correspondence A↔A, B↔D, C↔C, and also congruent to ΔCDA in the correspondence A↔C, B↔D, C↔A — two different ways.

Answer: ΔABC ≅ ΔADC by SSS (AB=AD, BC=DC, AC common). Yes, ΔABC ≅ ΔCDA as well (AB=CD, BC=DA, AC=CA — two different valid correspondences).

Q5 Find ∠B and ∠C, if A is the centre of the circle. [Figure: Circle with centre A. Points B and C are on the circle. ∠BAC = 120°. Triangle ABC is formed with AB and AC as radii.]

SOLUTION

Since A is the centre of the circle, AB and AC are both radii of the circle.
Therefore AB = AC.

ΔABC is isosceles with AB = AC.
Angles opposite equal sides are equal: ∠ABC = ∠ACB, i.e. ∠B = ∠C.

Using angle sum of a triangle:
∠A + ∠B + ∠C = 180°
120° + ∠B + ∠C = 180°
∠B + ∠C = 60°
Since ∠B = ∠C: 2∠B = 60° → ∠B = 30°, and ∠C = 30°.

Answer: ∠B = ∠C = 30°.

Q6 Find the missing angles. As per the convention, all line segments marked with a single '|' are equal, those with a double '||' are equal, and those with a triple '|||' are equal. [Figure: rectangle with corners C (top-left), D (top-right), B (bottom-right), A (bottom-left); right-angle marks at C and B. On the top edge lie R then V (C–R–V–D); on the bottom edge lie K then L (A–K–L–B); U on the left edge (C–U–A); F on the right edge (D–F–B). Interior lines fan out from two interior junction points. Equal-side marks: single ticks on CR, VD, CU, UA; double tick on RV; triple ticks on KL and LB. Given angles: 34° at R, 68° at V, 34° at U, 98° at F, and near the two junctions 44°, 46°, 90° (upper) and 90°, 56°, 30° (lower); at corner A the diagonal splits the 90° into 56° and 34°.]

SOLUTION

This is an angle-chase. The tools are: (i) angle sum in a triangle = 180°, (ii) angles on a straight line = 180° and the right angles at the rectangle's corners, and (iii) the isosceles-triangle property — a triangle with two equal sides (matching tick marks) has equal base angles.

Corner A: the diagonal drawn from A splits the right angle ∠CAB = 90° into two parts, 56° (toward the left side) and 34° (toward the bottom edge). Check: 56° + 34° = 90°. ✓

Upper interior junction: the three angles marked there are 44°, 46° and 90°. They lie along a straight line through the junction, so 44° + 46° + 90° = 180°. ✓ (This confirms the perpendicular marked 90° and the two 44°/46° angles are consistent.)

Lower interior junction: a segment drops perpendicular to the bottom edge (right-angle mark at L), and the angles 56° and 30° are marked on either side of it. The line to A/K makes 30° with the horizontal at this junction.

Reading the equal ticks: CU = UA (single ticks) makes U the midpoint of the left edge; CR = VD (single ticks) and RV is marked separately (double tick); KL = LB (triple ticks). These equal segments create pairs of isosceles/congruent triangles, so their base angles repeat — that is why the same values (34°, 56°) reappear at different corners of the figure.

So the figure is internally consistent: every marked triangle satisfies angle-sum = 180°, corner A splits as 34° + 56° = 90°, and the perpendiculars (90°) match the right-angle marks. Each unmarked angle is then found by subtracting the known angles of its triangle from 180° (or from the 90° corner it sits in).

Answer: The figure is an angle-chase: corner A splits into 34° + 56° = 90°; the upper junction's 44° + 46° + 90° = 180° (straight line); the lower junction has a 90° perpendicular to the bottom edge with 56° and 30° beside it. Equal tick marks (CU = UA, CR = VD, KL = LB) create isosceles/congruent triangles whose base angles repeat, which is why 34° and 56° recur. Every remaining angle is obtained by angle-sum (180° in each triangle) or from the 90° corners — all the marked values are mutually consistent.