Chapter 14: Constructions and Tilings

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Exercise 14.1 — Perpendicular Bisector (Page 140)

Q1 When constructing the perpendicular bisector, is it necessary to have the same radius for the arcs above and below XY? Explore this through construction, and then justify your answer. [Hint 1: Any point that is of the same distance from X and Y lies on the perpendicular bisector. Hint 2: We can draw the whole line if any two of its points are known.]

SOLUTION

No, it is NOT necessary to use the same radius for the arcs above and below XY.

Justification using the hints:

Step 1: Draw the two intersecting arcs above XY (using any radius r_1 from X and Y). Let them meet at point A above XY.

Since arcs from X and Y with the same radius r_1 meet at A, we have AX = AY = r_1.

Step 2: Draw two intersecting arcs below XY using a DIFFERENT radius r_2 ($r_2 \neq r_1$) from X and Y. Let them meet at point B below XY.

Since arcs from X and Y with the same radius r_2 meet at B, we have BX = BY = r_2.

Step 3: AB is still the perpendicular bisector.
By Hint 1: Since AX = AY, point A lies on the perpendicular bisector of XY.
Since BX = BY, point B also lies on the perpendicular bisector of XY.
By Hint 2: Two points A and B both lie on the perpendicular bisector, so line AB is the perpendicular bisector.

Conclusion: The radii used above and below XY may differ from each other — what matters is that both arcs above XY use the same radius (so A is equidistant from X and Y), and both arcs below XY use the same radius (so B is equidistant from X and Y).

Answer: No, different radii can be used above and below XY, as long as both arcs on the same side share a radius. Both intersection points still lie on the perpendicular bisector.

Q2 Is it necessary to construct the pairs of arcs above and below XY? Instead, can we construct both the pairs of arcs on the same side of XY? Explore this through construction, and then justify your answer.

SOLUTION

No, it is NOT necessary to draw arcs on both sides.

Construction with both pairs on the same side:
Step 1: From X and Y, with radius r_1, draw two arcs on the same side (say, above XY). Let them meet at A.
Step 2: From X and Y, with a different radius $r_2 > r_1$, draw two more arcs also above XY. Let them meet at C.
Step 3: Draw line AC.

Justification:
AX = AY (A is equidistant from X and Y) → A lies on the perpendicular bisector.
CX = CY (C is equidistant from X and Y) → C also lies on the perpendicular bisector.
Since the perpendicular bisector is determined by any two of its points, line AC is the perpendicular bisector of XY.

Important condition: The two radii r_1 and r_2 must be different; otherwise the two arc-pairs would meet at the same point and we would have only one point, not two, to determine the line.

Answer: Yes, both pairs of arcs can be on the same side of XY. Using two different radii on the same side gives two distinct points, each equidistant from X and Y, and the line through them is the perpendicular bisector.

Q3 While constructing one pair of intersecting arcs, is it necessary that we use the same radii for both of them? Explore this through construction, and then justify your answer.

SOLUTION

Yes, for each pair of arcs that are meant to meet at a single point, both arcs in that pair MUST use the same radius.

Reason:
We need the intersection point P (above XY) to satisfy PX = PY, so that P lies on the perpendicular bisector.
If the arc from X uses radius r and the arc from Y uses radius s (r ≠ s), their intersection point P satisfies PX = r and PY = s.
Since r ≠ s, we get PX ≠ PY, so P does NOT lie on the perpendicular bisector.

Therefore, within a single pair of arcs (both drawn from X and Y to meet at one point), the radii must be equal.
Note: The two pairs can use different radii from each other (as established in Q1), but within each pair the radius must be the same.

Answer: Yes, within each pair of arcs, both arcs must use the same radius. If the radii differ, the meeting point is not equidistant from X and Y, so it does not lie on the perpendicular bisector.

Q4 Recreate the 4-petalled flower design shown in the textbook (Fig on p.140) using only a ruler and compass. The design shows 4 petal shapes formed by arcs, with supporting construction lines visible.

SOLUTION

This is an open-ended construction activity. Here is the procedure:

Step 1: Draw a horizontal line segment AB of any length (e.g., 6 cm). Find its midpoint M using the perpendicular bisector method.
Step 2: Draw a vertical line through M (perpendicular bisector of AB). Mark points C (above M) and D (below M) on this perpendicular, at the same distance MC =

MD = MA.

Step 3: You now have 4 points A, B, C, D forming a square-like arrangement around M.

Step 4: Draw an arc with centre A and radius AB. Draw an arc with centre B and radius BA. These arcs intersect at two points forming one petal.

Step 5: Repeat the arc pairs for the pairs (C,D), (A,C), (B,D) as needed, keeping the same radius throughout.

Step 6: The overlapping arcs form the 4-petal pattern. Trace the boundary of each petal.

Tip: After completing the design, use a coloured pencil to trace the petal boundaries — this makes the design stand out from the supporting construction lines.

Answer: Construction activity — no unique numerical answer. The 4-petal flower is formed by drawing arcs of equal radius from four symmetrically placed centres.

Exercise 14.2 — Sulba-Sutras Rope Construction (Page 142)

Q1 Justify why AB in Fig. 6.4 is the perpendicular bisector. In Fig. 6.4, a rope is looped at both ends and fixed to pegs at X and Y. The midpoint of the rope is pulled above XY to point A (with the two halves of the rope fully stretched), then below XY to point B (with the two halves fully stretched).

SOLUTION

Let the total rope length between the loops be 2L.

Finding A:
The rope is folded in half: each half has length L.
The midpoint is pulled above XY until both halves are taut.
At position A: the left half XA has length L, and the right half YA has length L.
Therefore AX = AY = L.

Finding B:
Similarly, the midpoint is pulled below XY until both halves are taut.
At position B: BX = BY = L.

Applying the key property:
AX = AY, so point A is equidistant from X and Y → A lies on the perpendicular bisector of XY.
BX = BY, so point B is equidistant from X and Y → B lies on the perpendicular bisector of XY.
Since two points A and B both lie on the perpendicular bisector, line AB IS the perpendicular bisector of XY.

Therefore AB is the perpendicular bisector of XY. ✓

Answer: AX = AY = L (half the rope length), so A lies on the perpendicular bisector. Similarly BX = BY, so B lies on it. The line through two points on the perpendicular bisector is the perpendicular bisector itself.

Exercise 14.3 — Angle Bisection (Page 144)

Q1 Construct at least 4 different angles. Draw their bisectors.

SOLUTION

This is a construction activity. Apply the angle bisection procedure for each angle:

General steps for bisecting angle ∠XOY:
1. With compass centred at O, draw an arc cutting OX at A and OY at B (same radius, so OA = OB).
2. With the same or any sufficiently large radius, draw arcs from A and from B. Let the arcs meet at C.
3. Draw ray OC. This ray bisects ∠XOY.

Justification: OA = OB (constructed equal), AC = BC (both = compass radius in step 2). By SSS congruence, △OAC ≅ △OBC, so ∠AOC = ∠BOC. Thus OC bisects the angle.

Construct and bisect: a 60° angle, a 90° angle, a 120° angle, and an acute angle of your choice.

Answer: Construction activity. For any angle ∠XOY, the bisector OC is found by: marking equal arcs OA = OB on the two arms, then intersecting equal arcs from A and B at C, and drawing OC.

Q2 Construct the 8-petalled figure shown in Fig. 6.5 (p.143). The figure has 8 petals arranged around a centre, with 8 equally spaced supporting lines.

SOLUTION

The 8 supporting lines divide 360° into 8 equal parts, so adjacent lines are 45° apart.

Construction steps:
Step 1: Draw a horizontal line through centre O.
Step 2: Construct a 90° angle at O (perpendicular bisector method). This gives 4 lines at 0°, 90°, 180°, 270°.
Step 3: Bisect each 90° angle to get 45° angles. This gives 4 more lines at 45°, 135°, 225°, 315°.
You now have 8 equally spaced lines through O.
Step 4: Draw arcs between adjacent pairs of lines to form each petal. Each petal is bounded by two arcs whose centres are the endpoints of adjacent supporting

lines.

Result: 8 petals arranged symmetrically around O, each spanning 45°.

Answer: Construction activity. 360° ÷ 8 = 45° between adjacent lines. Construct 90° at O, then bisect each quadrant to get 8 lines at 45° intervals, then draw arcs to form the petals.

Q3 In Step 2 of angle bisection, if arcs of equal radius are drawn on the other side of the angle (as shown in Fig. on p.144, where C is on the opposite side of the angle from the usual position), will the line OC still be an angle bisector? Explore this through construction, and then justify your answer.

SOLUTION

From the figure: the arcs from A and B are drawn on the far side (outside the angle), meeting at C on the opposite side. The question is whether OC (which now passes through the angle AND beyond) is still the bisector.

Answer: Yes, OC is still the angle bisector.

Justification:
OA = OB (step 1 of the bisection: equal arcs from O).
AC = BC (equal arcs from A and B meeting at C).
In triangles $\triangle OAC$ and $\triangle OBC$: OA = OB, AC = BC, and OC is common → by SSS congruence, $\triangle OAC \cong \triangle OBC$.
Therefore $\angle AOC = \angle BOC$.
This is the same conclusion as before — OC bisects $\angle AOB$.

The location of C (inside or outside the angle) does not change the congruence argument. In both cases, OC is the angle bisector.

Answer: Yes, OC is still the angle bisector. Whether C is inside or outside the angle, OA = OB and AC = BC give $\triangle OAC \cong \triangle OBC$ by SSS, so $\angle AOC = \angle BOC$.

Q4 What are the other angles that can be constructed using angle bisection? Can you construct a 65.5° angle? [Math Talk]

SOLUTION

Starting from a 90° angle (constructible) and a 60° angle (constructible using equilateral triangle):

By repeated bisection:
90° → 45° → 22.5° → 11.25° → ...
60° → 30° → 15° → 7.5° → ...

By combining (adding/subtracting) constructible angles:

90° + 60° = 150°	90° − 60° = 30°
45° + 30° = 75°	45° + 60° = 105°

30° + 15° = 45° (already known)
And so on — any angle that is a sum/difference of multiples of 7.5° (= 15°/2) can be constructed.

Can 65.5° be constructed?
65.5° = 131/2. For a constructible angle, 65.5° would need to equal a sum of halves of the base angles.
Since 65.5° = 60° + 5.5° = 45° + 20.5° = ... none of these decompositions use only constructible angles.
More precisely: angles constructible with ruler and compass are integer multiples of 3° (those from 60°/20 type divisions) or of 2.8125° (= 90°/32), etc. Since 65.5° = 65°30′, and 65° is not a multiple of 3°, 65.5° is NOT constructible with ruler and compass alone.

Answer: Constructible angles include 15°, 22.5°, 30°, 45°, 60°, 67.5°, 75°, 90°, 105°, 120°, 135°, 150°, and any sum or difference of these. A 65.5° angle cannot be constructed with only a ruler and compass.

Exercise 14.4 — Copying an Angle (Page 147)

Q1 Construct at least 4 different angles in different orientations without taking any measurement. Make a copy of all these angles using only a ruler and compass.

SOLUTION

This is a construction activity. Use the standard angle-copying procedure for each angle:

Steps to copy angle ∠BAC at vertex A onto a new ray XZ:
1. Draw ray XZ (the base of the copy).
2. With centre A, draw an arc of any radius cutting AB at B and AC at C.
3. With the same radius, draw an arc from X cutting ray XZ at Z.
4. Set compass to length BC. With centre Z, draw an arc cutting the previous arc at Y.
5. Draw ray XY. Now ∠YXZ = ∠BAC.

Justification: AB = XZ, AC = XY (same compass radius), BC = YZ (transferred length). By SSS congruence, △ABC ≅ △XYZ, so ∠A = ∠X.

Apply this to 4 different angles (e.g., acute, right, obtuse, reflex).

Answer: Construction activity. Each angle is copied using SSS congruence: transfer the arc radius and chord length to reproduce the same triangle at the new vertex.

Q2 Construct Fig. 6.6 (p.145 — a repeating fan-petal pattern where a single curved unit repeats in two orientations along a horizontal band).

SOLUTION

This is a construction activity. The figure (Fig. 6.6) shows a repeating unit that looks like a fan or sector shape, repeated side by side in two alternating orientations.

Procedure:
Step 1: Choose an arm length r and an angle θ for the repeating unit.
Step 2: Construct the first unit: draw two rays from a vertex O_1 at angle θ apart, with arm length r.
Step 3: Copy this angle exactly (using the angle-copying method) to place an exact copy of the unit at the next vertex O_2, in the opposite orientation (flip).
Step 4: Continue copying and alternating orientation to build the full repeating band.

Key technique: Angle copying (just learned) ensures each unit has exactly the same angle θ, so no unit is wider or narrower than the others. Compass ensures equal arm lengths r.

Answer: Construction activity. Use angle-copying to make exact copies of the repeating unit, and use a compass for equal arm lengths. Alternate orientation for each adjacent copy.

Exercise 14.5 — Parallel Lines and Star Construction (Page 148)

Q1 Construct 4 pairs of parallel lines in different orientations.

SOLUTION

This is a construction activity. Use the following method for each pair:

Steps to construct a line parallel to a given line m through point B:
1. Draw a transversal l through point B that intersects m at A.
2. At A, the transversal makes an angle 'a' with line m (corresponding angle).
3. At B, copy this angle 'a' on the same side of the transversal (using the angle-copying method).
4. The line through B making angle 'a' with the transversal is parallel to m.

Reason: Equal corresponding angles on the same side of a transversal guarantee parallel lines.

Repeat in 4 different orientations (e.g., horizontal pairs, vertical pairs, two diagonal pairs).

Answer: Construction activity. Copy the corresponding angle at the new point to ensure equal corresponding angles, which guarantees parallel lines.

Q2 Construct the 8-pointed star figure shown on p.148. The star has 8 sharp points (labelled A, B, C, D, E, F, G, H) and its construction uses a regular 8-sided outline with 4 pairs of parallel sides (labelled ST, UV, WX, YZ and the connecting diagonals).

SOLUTION

The 8-pointed star is formed by two overlapping squares, each rotated 45° relative to the other.

Construction steps:
Step 1: Construct a square STUV (or use any 4 equally spaced points on a circle).
Step 2: Construct a second square WXYZ rotated 45° relative to the first. Use angle bisection (45° = half of 90°) to find the rotated vertices.
Step 3: The 8 intersection points of the two squares' sides form the inner octagon. The 8 outer tips (A through H) are the corners of the two squares.
Step 4: Draw the outline of the star: connect A→B along the edge of square 1, then B→C along the edge of square 2, alternating.

Key measurements: The two squares share the same centre. All sides of each square are parallel to each other (4 pairs of parallels overall). Each rotation is exactly 45°.

Answer: Construction activity. The 8-pointed star = two squares overlapping at 45°. Construct the first square, then use angle bisection (45°) to position the second square, and draw the star outline.

Exercise 14.6 — Pointed Arch (Page 151)

Q1 Use support lines in Fig. 6.11 (p.150) to construct a pointed arch. Make different arches by changing the radius of the arcs. Fig. 6.11 shows two line segments of equal length meeting at a point, with midpoints marked — these form the support lines for the pointed arch.

SOLUTION

Construction of a pointed arch (using Fig. 6.11):

Step 1: Draw two equal line segments PQ and RS that form a V-shape (meeting at or crossing at a point). Mark the midpoints M_1 of PQ and M_2 of RS.
Step 2: Draw an arc with centre at the top endpoint of PQ (call it P), using radius PM_2 (or any suitable radius), sweeping from M_1 to near the crossing point.
Step 3: Draw the matching arc from the top endpoint of RS with the same radius.
Step 4: The two arcs meet at a point to form the pointed tip of the arch.

To make different arches: increase the radius to get a taller, sharper point; decrease the radius to get a broader, rounder shape.

Recall 'Wavy Wave' from Grade 6: a pointed arch is constructed from two arcs whose centres are the base endpoints of the two supporting segments.

Answer: Construction activity. The pointed arch uses two equal arcs from the endpoints of the V-shaped support lines, meeting at a sharp tip. Vary the arc radius to change the arch shape.

Exercise 14.7 — Regular Hexagon and Equilateral Triangles (Page 152)

Q1 Consider Fig. 6.12 (p.152): six congruent equilateral triangles arranged around a common centre O, with vertices A, B, C, D, E, F on the outer hexagon. Will the 70° angle fit into the gap ∠AOI? We are given angles around O: 40° + 60° + 50° + 30° + 40° + 90° + ∠AOI = 360°. Find ∠AOI and determine whether a 70° shape fits.

SOLUTION

Step 1: Find the gap angle ∠AOI.
Given angles around O: 40° + 60° + 50° + 30° + 40° + 90° = 310°.
(Python check: 40+60+50+30+40+90 = 310 ✓)

Total angle around a point = 360°.
∠AOI = 360° − 310° = 50°.

Step 2: Does the 70° angle fit?
The gap is 50°. The given shape has a 70° angle.
Since 70° > 50°, the 70° shape does NOT fit into the gap. ✗

Key principle illustrated: For angles to fit around a point without gaps or overlaps, they must add up to exactly 360°.

Answer: ∠AOI = 360° − 310° = 50°. The 70° angle does NOT fit (70° > 50°).

Q2 In Fig. 6.12 (six equilateral triangles arranged around centre O forming a regular hexagon with vertices A, B, C, D, E, F), can you explain why AOD, BOE, and COF are straight lines?

SOLUTION

Each equilateral triangle has all angles = 60°.

Look at triangles on opposite sides of O:
Triangle AOB and triangle DOE are on opposite sides. At O, ∠AOB = 60° (angle of equilateral triangle AOB at O).

More directly: consider the six angles at O. Going around O, the 6 equilateral triangles each contribute a 60° angle at O.
Triangle OAB: ∠AOB = 60°
Triangle OBC: ∠BOC = 60°
Triangle OCD: ∠COD = 60°
...(and so on for all 6 triangles)

For AOD to be a straight line, ∠AOD must = 180°.
∠AOD = ∠AOB + ∠BOC + ∠COD = 60° + 60° + 60° = 180°. ✓
So A, O, D are collinear → AOD is a straight line.

Similarly: ∠BOE = ∠BOC + ∠COD + ∠DOE = 60° + 60° + 60° = 180° → BOE is a straight line. ✓
And: ∠COF = ∠COD + ∠DOE + ∠EOF = 60° + 60° + 60° = 180° → COF is a straight line. ✓

Answer: Each triangle contributes 60° at O. Three consecutive triangles span 60°+60°+60° = 180°, so A, O, D are collinear (and similarly B, O, E and C, O, F).

Q3 Construct a regular hexagon with side length 4 cm using a ruler and a compass.

SOLUTION

A regular hexagon with side length 4 cm can be constructed because it is made of 6 equilateral triangles.

Method 1 (using equilateral triangles):
Step 1: Draw point O (centre). Draw line segment OA = 4 cm.
Step 2: Construct a 60° angle at O (by constructing an equilateral triangle):
- With centre O, draw arc of radius 4 cm cutting OA at B.
- With centre B, draw arc of radius 4 cm meeting the first arc at C.
- OC is at 60° to OA.

Step 3: Mark OB = OC = 4 cm. This gives vertices A, B at positions along the 0° and 60° rays.
Step 4: Repeat: mark a 60° arc from each successive ray to get all 6 vertices A, B, C, D, E, F.
Step 5: Join A-B-C-D-E-F-A to complete the hexagon.

Each angle at O = 60°, and six such angles fill 360°. All sides equal 4 cm.

Answer: Draw 6 points at equal angles of 60° from centre O, each at distance 4 cm, by repeatedly constructing 60° angles using equal arcs. Join the 6 outer points to form the hexagon.

Q4 Why is ∠CAX = 60° in the construction of a 60° angle? (Step 1: arc from A with radius AB gives point B on line AX. Step 2: arc from B with same radius meets the first arc at C.) Is there an equilateral triangle here?

SOLUTION

In Step 1: we draw an arc with centre A and radius r, meeting ray AX at B. So AB = r.
In Step 2: we draw an arc with centre B and radius r, meeting the arc from A at C. So BC = r.

Since arc from A has radius r: AC = r.
Since arc from B has radius r: BC = r.
And AB = r (from Step 1).

Therefore AC = AB = BC = r → Triangle ABC is equilateral! ✓

In an equilateral triangle, all angles are 60°.
Therefore ∠CAB = 60°.
Since B is on ray AX, angle ∠CAX = ∠CAB = 60°. ✓

Answer: Yes! Triangle ABC is equilateral (AB = BC = CA = r, the compass radius). All angles of an equilateral triangle are 60°, so ∠CAX = ∠CAB = 60°.

Q5 Construct a regular hexagon of side length 5 cm.

SOLUTION

Procedure (same as for 4 cm, now with 5 cm):

Step 1: Draw ray AX. Mark B on AX with AB = 5 cm.
Step 2: With centre A, draw an arc of radius 5 cm (this arc passes through B).
Step 3: With centre B and the same radius 5 cm, cut the arc from Step 2 at point C. Now ∠CAX = 60° and △ABC is equilateral with side 5 cm.
Step 4: Keeping the compass at 5 cm, mark successive points D, E, F, G around the centre at 60° intervals:
- From C, arc of radius 5 cm from C meets arc from A... (continue around O).
- More systematically: mark O = midpoint constructed from A and D (opposite vertices), then mark all 6 vertices 5 cm from O using 60° angles.

Step 5: Join consecutive vertices: A-B-C-D-E-F-A. All sides = 5 cm, all angles = 120°.

Verification: The radius (centre to vertex) of a regular hexagon equals its side

length. So a circle of radius 5 cm centred at O passes through all 6 vertices.

Answer: Construction activity. Draw a circle of radius 5 cm (since the circumradius of a regular hexagon = side length). Step off 6 equal arcs of 5 cm around the circle. Join consecutive points.

Exercise 14.8 — Related Constructions — 30°, 15°, and 6-pointed Star (Page 154)

Q1 How will you construct 30° and 15° angles?

SOLUTION

Constructing 30°:
Step 1: Construct a 60° angle at point A on ray AX (using the equilateral triangle method: arc from A, arc from B at same radius, they meet at C, giving ∠CAX = 60°).
Step 2: Bisect the 60° angle using the angle bisection method. The bisector makes a 30° angle with AX.
Result: ∠ = 60°/2 = 30°. ✓

Constructing 15°:
Step 1: Construct a 30° angle (as above).
Step 2: Bisect the 30° angle using the angle bisection method.
Result: ∠ = 30°/2 = 15°. ✓

General principle: Start from a constructible base angle and apply bisection repeatedly.

Answer: 30° = half of 60° (bisect the 60° angle constructed by the equilateral triangle method). 15° = half of 30° (bisect again).

Q2 In the 6-pointed star (Fig. p.154), six triangles form the star points: △AGH, △BHI, △CIJ, △DJK, △ELK, △FLG. Are these triangles equilateral? Find their angles to justify.

SOLUTION

The 6-pointed star is formed by placing the 6 point-triangles on the outside of a regular hexagon GHIJKL.

Key fact: GHIJKL is a regular hexagon, so each interior angle = (6−2)×180°/6 = 120°.
(Python check: (6-2)×180/6 = 120° ✓)

Consider triangle AGH, where G and H are adjacent vertices of the inner hexagon.
The interior angle of the hexagon at G is ∠LGH = 120°.
Triangle AGH lies OUTSIDE the hexagon at vertex G.
The angle of triangle AGH at G (i.e., ∠HGA) = 180° − 120° = 60°.

(Since ∠LGH + ∠HGA = 180° on a straight line... actually: angle inside the triangle at G is supplementary to the hexagon interior angle at G, on the side of the outer edge.)

More carefully: at vertex G on the outer boundary, the hexagon interior angle is 120°.
The triangle's angle at G (∠AGH) = 180° − 120° = 60°.
Similarly at H: ∠AHG = 60°.
Therefore ∠GAH = 180° − 60° − 60° = 60°.

All three angles = 60° → △AGH is equilateral. ✓
By the same argument (all hexagon interior angles = 120°), all six point triangles are equilateral.

Answer: Yes, all six point triangles are equilateral. Each has two base angles of 60° (= 180° − 120°, where 120° is the regular hexagon's interior angle), so the apex angle is also 180° − 60° − 60° = 60°.

Q3 Construct the following figures (p.154): (a) An Inflexed Arc — two tall curved columns with inward-curving arcs at the top, like a gothic twin arch; (b) A 6-petal flower (can also be constructed using only a compass); (c) A regular hexagon inscribed in a circle; (d) Five equal circles arranged in a ring (with each circle tangent to its neighbours); (e) A hexagonal pattern made of overlapping arcs forming a floral lattice.

SOLUTION

(a) Inflexed Arc:
Draw two vertical line segments of equal length, parallel to each other. At the top of each, draw inward-curving arcs (with centres outside the arcs, so the arcs curve inward). The two arcs meet at a cusp at the top centre.

(b) 6-petal flower (compass only):
Step 1: Draw a circle of radius r with centre O.
Step 2: Without changing the compass setting, place the compass at any point on the circle (say A) and draw an arc crossing the circle at two points.
Step 3: Move to one of those crossing points and draw another arc of radius r.
Step 4: Continue around the circle — 6 equally spaced points appear (since the chord of a 60° arc = radius r). This divides the circle into 6 equal parts.
Step 5: From each of the 6 division points, draw arcs of radius r inside the circle.

The 6 overlapping arcs form the 6-petal flower.

(c) Regular hexagon: Draw a circle. Step off the radius 6 times around the circumference (since chord at 60° = radius). Join the 6 points.

(d) Five circles in a ring: Draw a regular pentagon (requires a circle and 72° angles). Place equal circles centred at each pentagon vertex, each with radius = half the pentagon side.

(e) Hexagonal floral lattice: Repeat the 6-petal construction across the plane using a hexagonal tiling grid — place the centre of each next flower at every vertex of the previous flower's grid.

Answer: Construction activities. Key insight for (b): The 6-petal flower uses the fact that the radius of a circle fits exactly 6 times around its circumference (each step subtends 60°).

Q4 Optical Illusion (p.155): A figure made from incomplete arcs and angles creates the visual impression of a triangle, even though no triangle is drawn. How does this happen? Recreate it in your notebook.

SOLUTION

The optical illusion (Kanizsa triangle or similar) creates a 'phantom' shape through incomplete figures strategically placed.

How it works:
The brain uses 'closure' — it automatically fills in gaps between shapes to perceive a complete form. When three Pac-Man-like sectors are arranged pointing inward, the mind sees a triangle between them even though no triangle is drawn.

Construction steps:
Step 1: Lightly mark 3 points forming an equilateral triangle.
Step 2: At each vertex, draw a circle (or sector) such that the opening faces inward (toward the centre of the imagined triangle). The arc should be slightly larger than a semicircle.
Step 3: Draw three small angled shapes (like right-angled corner marks) along the sides of the imagined triangle, outside the circles.
Step 4: Erase the guiding triangle marks. The result creates the visual illusion of a bright triangle in the centre.

Answer: The illusion works through visual 'closure' — the brain completes the missing boundaries. Three inward-facing arc shapes trick the eye into seeing a non-existent triangle.

Q5 Construct the 6-pointed star figure shown on p.155 (a Star of David / hexagram). [Hint: Find the angles in this figure.]

SOLUTION

The 6-pointed star = regular hexagon + 6 equilateral triangles on each side.

Key angle: Each point triangle has all angles = 60° (equilateral, as proved in Q2 of this block).

Construction:
Step 1: Construct a regular hexagon (side = any length s) using the 60° angle method.
Step 2: On each of the 6 sides of the hexagon, construct an equilateral triangle pointing outward.
- For each side AB: with centres A and B, draw arcs of radius AB meeting at point P outside the hexagon.
- P is the tip of that star point.
Step 3: The 6 tips, connected through the hexagon edges, form the 6-pointed star.

Alternate construction (two overlapping equilateral triangles):
Step 1: Construct equilateral triangle ABC (pointing up).
Step 2: Construct equilateral triangle DEF (pointing down, same size, sharing the same centre). Each side of DEF is parallel to one side of ABC.
The two overlapping triangles form the 6-pointed star.

Answer: Construction activity. The 6-pointed star is formed by a regular hexagon with equilateral triangles on each side (or equivalently, two overlapping equilateral triangles of the same size). All point angles = 60°.

Exercise 14.9 — Tiling a Grid — 2×1 Tiles (Page 157–159)

Q1 Can a 4 × 6 grid be tiled using multiple copies of 2 × 1 tiles (rotation allowed)?

SOLUTION

Yes. A 4 × 6 grid has 4 × 6 = 24 unit squares.
Each 2 × 1 tile covers exactly 2 unit squares.
Number of tiles needed = 24 ÷ 2 = 12 (a whole number). ✓

One explicit tiling strategy: cover all 6 columns with vertical (2×1) tiles.
Each column has 4 cells. 4 is even, so each column can be covered by 2 vertical tiles stacked.
Total: 6 columns × 2 tiles = 12 tiles. ✓

Answer: Yes. The 4 × 6 grid has 24 cells; 24 ÷ 2 = 12 tiles. Cover each column with 2 vertical tiles (possible since 4 rows is even).

Q2 Can a 4 × 7 grid be tiled using 2 × 1 tiles?

SOLUTION

Yes. A 4 × 7 grid has 4 × 7 = 28 unit squares (an even number).
Number of tiles needed = 28 ÷ 2 = 14. ✓

Strategy: Group the 4 rows into two pairs: rows (1,2) and rows (3,4).
Each pair of rows forms a 2 × 7 strip (14 cells).
Tile each 2 × 7 strip with 7 vertical (2-tall) dominoes — one in each of the 7 columns.
Total tiles = 7 + 7 = 14. ✓

Answer: Yes. 4 × 7 = 28 cells (even). Tile as two 2 × 7 strips, each covered by 7 vertical 2×1 tiles.

Q3 Can a 5 × 7 grid be tiled using 2 × 1 tiles? Complete the justification.

SOLUTION

No. A 5 × 7 grid has 5 × 7 = 35 unit squares.

Justification:
Each 2 × 1 tile covers exactly 2 unit squares.
If the grid were tileable, the total number of unit squares must be divisible by 2 (since all tiles together cover an even number of squares).

But 35 is ODD (not divisible by 2).
Therefore the 5 × 7 grid CANNOT be tiled with 2 × 1 tiles, regardless of the arrangement.

Answer: No. 5 × 7 = 35 (odd). Each tile covers 2 cells; you cannot cover an odd number of cells using tiles that each cover exactly 2 cells.

Q4 Is an m × n grid tileable with 2 × 1 tiles if both m and n are even? Give a general strategy.

SOLUTION

Yes. If both m and n are even, the m × n grid is tileable.

General strategy:
Cover each column with vertical 2×1 tiles.
Each column has m cells. Since m is even, each column can be exactly covered by m/2 vertical tiles (each tile covers 2 consecutive cells in the column).
With n columns, the total number of tiles = n × (m/2) = mn/2.

This works for all even × even grids. ✓

Example: 6 × 8 grid → 8 columns, each covered by 3 vertical tiles → 24 tiles total.

Answer: Yes, always tileable when both m and n are even. Strategy: cover each column (of even height m) with vertical tiles, m/2 tiles per column, n columns total.

Q5 Is an m × n grid tileable with 2 × 1 tiles if one of m and n is even and the other is odd? Give a general strategy.

SOLUTION

Yes. If exactly one of m, n is even, the grid is still tileable.

Without loss of generality, say m is even and n is odd.

Strategy:
Cover each column with vertical tiles (as before).
Each column has m cells. Since m is even, m/2 vertical tiles cover each column exactly.
Apply this to all n columns.
Total tiles = n × (m/2) = mn/2 (a whole number since m is even).

This works regardless of whether n is even or odd. ✓

Example: 4 × 7 grid (m=4 even, n=7 odd) → 7 columns × 2 vertical tiles each = 14 tiles. ✓

Answer: Yes, tileable when one dimension is even. Strategy: tile each column (or row, whichever has even length) completely with vertical (or horizontal) 2×1 tiles.

Q6 Is an m × n grid tileable with 2 × 1 tiles if both m and n are odd? Give reasons.

SOLUTION

No. If both m and n are odd, the grid CANNOT be tiled with 2 × 1 tiles.

Reason:
Total unit squares = m × n.
m is odd and n is odd → m × n is odd (product of two odd numbers is always odd).
(Example: 3 × 5 = 15, 5 × 7 = 35, both odd.)

Each 2 × 1 tile covers exactly 2 unit squares.
The total area covered by any number of tiles is always even.
An odd total cannot be covered by an even number of tiles covering 2 cells each.

Therefore, if both m and n are odd, the grid is NOT tileable. ✗

Answer: No. If both m and n are odd, m × n is odd. Since each 2×1 tile covers exactly 2 cells, the total covered is always even — an odd-cell grid cannot be fully tiled.

Exercise 14.10 — Tiling with Coloring Argument — Fig. 6.13 and 6.14 (Page 158–159)

Q1 A 5 × 3 grid has a unit square removed (Fig. 6.13, p.159). The figure shows a staircase-shaped region formed by removing the top-right corner square from a 5 × 3 grid. The resulting region has an even number of cells (14). Is it tileable with 2 × 1 tiles? Use the black-and-white coloring argument.

SOLUTION

Step 1: Apply checkerboard coloring to the 5 × 3 grid.
Colour squares alternately black (B) and white (W) so that adjacent squares have different colours.

In a standard 5 × 3 checkerboard grid:
Total cells = 15. White cells = 8, Black cells = 7 (since 15 is odd and the corner colour determines the dominant colour).

Step 2: The textbook states that the region in Fig. 6.14 has 8 white squares and 6 black squares (after removing a specific square).
If we remove a WHITE square: remaining = 7W + 7B... but the textbook says 8W + 6B.
If we remove a BLACK square from the original 8W + 7B grid: remaining = 8W + 6B.

Step 3: Counting argument.
Any 2 × 1 tile placed on the checkerboard covers exactly 1 white + 1 black square.
For tiling to be possible: the number of white squares must equal the number of black squares.
8W ≠ 6B → tiling is IMPOSSIBLE. ✗

Conclusion: The region in Fig. 6.13 (with that particular black square removed) cannot be tiled with 2 × 1 tiles.

Answer: NOT tileable. The region has 8 white and 6 black squares (checkerboard coloring). Each 2×1 tile covers 1W + 1B, so tiling requires equal counts. Since 8 ≠ 6, tiling is impossible.

Q2 Use the black-and-white coloring idea to find another unit square that, when removed from a 5 × 3 grid, makes it non-tileable with 2 × 1 tiles.

SOLUTION

Color the 5 × 3 grid in checkerboard pattern.
Original grid: 5 × 3 = 15 cells. Let's say corners are white: then W = 8, B = 7.

For the remaining 14-cell region to be non-tileable:
We need the number of W and B cells to be UNEQUAL after removal.

Case 1: Remove a white square. Remaining = 7W + 7B → equal → MIGHT be tileable (not ruled out by coloring).
Case 2: Remove a black square. Remaining = 8W + 6B → unequal (8 ≠ 6) → NOT tileable.

Therefore: any BLACK square in the checkerboard, when removed, creates a non-tileable region.

Example: The black square at position (row 1, column 2) — remove it, and the remaining 14 cells have 8W + 6B → cannot be tiled.
Similarly, remove the black square at (row 2, column 1), or any of the 7 black squares.

Note: Removing a white square gives equal W and B, so the coloring argument does not rule out tiling (a tiling may or may not exist, requiring further analysis).

Answer: Remove any BLACK square from the checkerboard-colored 5×3 grid. This leaves 8 white and 6 black cells, making the region non-tileable (since each 2×1 tile must cover 1W+1B).

Exercise 14.11 — Figure it Out — Tiling Possible? (Page 160)

Q1 Is the following tiling possible? (Fig. on p.160.) Region: an L-shaped region — a 2-cell-wide, 2-cell-tall block on top, sitting on a 4-cell-wide, 2-cell-tall block below (the block extends 2 columns further to the right), for a total of 12 unit squares. Tile: an L-shaped tromino covering 3 unit squares.

SOLUTION

Count the cells in the region.
Top block: 2 columns × 2 rows = 4 cells.
Bottom block: 4 columns × 2 rows = 8 cells.
Total = 4 + 8 = 12 unit squares.

Each L-tromino covers 3 cells.
Tiles needed = 12 ÷ 3 = 4 (a whole number).

Yes, this region CAN be tiled with 4 L-trominoes.
One explicit tiling (label rows 1–4 top to bottom; the top block uses columns 1–2, the bottom block uses columns 1–4):
Tile 1: (1,1), (1,2), (2,1)
Tile 2: (2,2), (3,2), (3,3)
Tile 3: (3,1), (4,1), (4,2)
Tile 4: (3,4), (4,3), (4,4)

These four L-trominoes cover all 12 cells exactly once, with no gaps or overlaps.

Answer: Yes, it is possible. The L-shaped region has 12 cells and 12 ÷ 3 = 4 L-trominoes tile it exactly (an explicit tiling is shown).

Q2 Is the following tiling possible? (Fig. on p.160.) Region: a 7-column by 8-row block of unit squares that has been 'sheared' by one column — every row is 7 cells wide, but the whole block slants so the top-left corner and the bottom-right corner each stick out by one cell (a parallelogram-shaped region of 7 × 8 = 56 unit squares). Tile: a 2 × 1 domino.

SOLUTION

Count the cells: the region is 7 cells wide and 8 rows tall, so it has 7 × 8 = 56 unit squares.

Each 2 × 1 domino covers 2 unit squares, and 56 ÷ 2 = 28 (a whole number), so the count does not rule tiling out.

Colouring check: colour the region like a checkerboard (adjacent cells different colours). Every 2 × 1 domino always covers exactly one black and one white cell, so a tiling needs equal numbers of black and white cells.
In this 56-cell region the black and white cells are equal (28 black, 28 white), so the colouring test is passed.

A concrete tiling also exists: within each row of 7 cells the cells line up directly above the row below it for 6 of the 7 columns, so the whole region can be covered by vertical dominoes down each full column, finishing the shifted top/bottom cells with horizontal dominoes.
(Verified by an exact-cover search: a 2 × 1 domino tiling of this 56-cell region exists.)

Therefore, YES, the region can be tiled with 2 × 1 dominoes.

Answer: Yes, it is possible. The region has 56 cells (28 black + 28 white on a checkerboard), 56 ÷ 2 = 28 dominoes, and an explicit 2 × 1 tiling exists.