Chapter 15: Finding the Unknown

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Exercise 15.1 — Unknown Weights — Balance Scales (Page 164)

Q1 A sack is placed on one side of a balance scale together with a 2 kg weight. The other side holds a 10 kg weight and a 2 kg weight, and the scale is balanced. Find the weight of the sack. (Fig. 7.9)

SOLUTION

Let the weight of the sack = s kg.

Left side: s + 2
Right side: 10 + 2 = 12

Since the scale is balanced:
s + 2 = 12
s = 12 - 2
s = 10

Answer: The sack weighs 10 kg.

Q2 Three sacks of equal weight are on one side of a balance scale. The other side has two sacks (same weight) and a 10 kg weight and a 4 kg weight. The scale is balanced. Find the weight of one sack. [Hint: Remove one sack from each side.] (Fig. 7.10)

SOLUTION

Let the weight of one sack = s kg.

Left side: 3s
Right side: 2s + 10 + 4 = 2s + 14

Since the scale is balanced: 3s = 2s + 14

Remove one sack from each side (subtract s from both sides):
2s = s + 14
s = 14

Answer: The weight of one sack is 14 kg.

Q3 Four sacks are on the left side of a balance scale. The right side has one sack, a 1 kg weight, a 10 kg weight, and another 10 kg weight. The scale is balanced. Find the weight of one sack. [Hint: Remove objects so that sacks are only on one plate.] (Fig. 7.11)

SOLUTION

Let the weight of one sack = s kg.

Left side: 4s
Right side: s + 1 + 10 + 10 = s + 21

Since the scale is balanced: 4s = s + 21

Subtract s from both sides:
3s = 21
s = 21 / 3 = 7

Answer: The weight of one sack is 7 kg.

Q4 A balance scale has 90 sacks and a 50 kg weight on the left, and 60 sacks and a 500 kg weight on the right. All sacks have the same weight. Find the weight of one sack. (Fig. 7.12)

SOLUTION

Let the weight of one sack = s kg.

Left side: 90s + 50
Right side: 60s + 500

Since the scale is balanced: 90s + 50 = 60s + 500

Subtract 60s from both sides: 30s + 50 = 500
Subtract 50 from both sides: 30s = 450
Divide both sides by 30: s = 15

Answer: The weight of one sack is 15 kg.

Exercise 15.2 — Solving Equations — Figure it Out (Page 172)

Q1 Solve the equation 3x - 10 = 35 and check the solution.

SOLUTION

3x - 10 = 35
Add 10 to both sides: 3x = 35 + 10 = 45
Divide both sides by 3: x = 45 / 3 = 15

Check: LHS = 3(15) - 10 = 45 - 10 = 35 = RHS ✓

Answer: x = 15

Q2 Solve the equation 5s = 3s and check the solution.

SOLUTION

5s = 3s
Subtract 3s from both sides: 5s - 3s = 0
2s = 0
Divide both sides by 2: s = 0

Check: LHS = 5(0) = 0, RHS = 3(0) = 0 ✓

Answer: s = 0

Q3 Solve the equation 3u - 7 = 2u + 3 and check the solution.

SOLUTION

3u - 7 = 2u + 3
Subtract 2u from both sides: u - 7 = 3
Add 7 to both sides: u = 3 + 7 = 10

Check: LHS = 3(10) - 7 = 23, RHS = 2(10) + 3 = 23 ✓

Answer: u = 10

Q4 Solve the equation 4(m + 6) - 8 = 2m - 4 and check the solution.

SOLUTION

4(m + 6) - 8 = 2m - 4
Expand: 4m + 24 - 8 = 2m - 4
Simplify LHS: 4m + 16 = 2m - 4

Subtract 2m from both sides: 2m + 16 = -4
Subtract 16 from both sides: 2m = -4 - 16 = -20
Divide both sides by 2: m = -10

Check: LHS = 4(-10+6) - 8 = 4(-4) - 8 = -16 - 8 = -24
RHS = 2(-10) - 4 = -20 - 4 = -24 ✓

Answer: m = -10

Q5 Solve the equation u/15 = 6 and check the solution.

SOLUTION

u/15 = 6
Multiply both sides by 15: u = 6 x 15 = 90

Check: LHS = 90/15 = 6 = RHS ✓

Answer: u = 90

Exercise 15.3 — Solving Equations — Figure it Out (Page 181)

Q1 Find the value of each unknown: (a) 2y = 60 (b) -8 = 5x - 3 (c) -53w = -15 (d) 13 - z = 8 (e) k + 8 = 12 - k (f) 7m = m - 3 (g) 3n = 10 + n

SOLUTION

(a) 2y = 60 => y = 60 / 2 = 30
Check: 2(30) = 60 ✓

(b) -8 = 5x - 3 => 5x = -8 + 3 = -5 => x = -1
Check: 5(-1) - 3 = -5 - 3 = -8 ✓

(c) -53w = -15 => w = (-15) / (-53) = 15/53
Check: -53 x (15/53) = -15 ✓

(d) 13 - z = 8 => z = 13 - 8 = 5
Check: 13 - 5 = 8 ✓

(e) k + 8 = 12 - k => k + k = 12 - 8 => 2k = 4 => k = 2
Check: 2 + 8 = 10, 12 - 2 = 10 ✓

(f) 7m = m - 3 => 7m - m = -3 => 6m = -3 => m = -1/2
Check: 7(-1/2) = -7/2, (-1/2) - 3 = -7/2 ✓

(g) 3n = 10 + n => 3n - n = 10 => 2n = 10 => n = 5
Check: 3(5) = 15, 10 + 5 = 15 ✓

Answer: (a) y = 30 (b) x = -1 (c) w = 15/53 (d) z = 5 (e) k = 2 (f) m = -1/2 (g) n = 5

Q2 I am a 3-digit number. My hundred's digit is 3 less than my ten's digit. My ten's digit is 3 less than my unit's digit. The sum of all three digits is 15. Who am I?

SOLUTION

Let the unit's digit = u.
Then ten's digit = u - 3, and hundred's digit = (u - 3) - 3 = u - 6.

Sum of digits: (u - 6) + (u - 3) + u = 15
3u - 9 = 15
3u = 24
u = 8

Unit's digit = 8, Ten's digit = 8 - 3 = 5, Hundred's digit = 5 - 3 = 2.

The number is 258.
Check: 2 + 5 + 8 = 15 ✓, hundred (2) is 3 less than ten (5) ✓, ten (5) is 3 less than unit (8) ✓

Answer: The 3-digit number is 258.

Q3 The weight of a brick is 1 kg more than half its weight. What is the weight of the brick?

SOLUTION

Let the weight of the brick = w kg.

w = (w/2) + 1

Subtract w/2 from both sides: w - w/2 = 1
w/2 = 1
Multiply both sides by 2: w = 2

Check: Half of 2 = 1, and 1 + 1 = 2 ✓

Answer: The brick weighs 2 kg.

Q4 One quarter of a number increased by 9 gives the same number. What is the number?

SOLUTION

Let the number = n.

n/4 + 9 = n

Subtract n/4 from both sides: 9 = n - n/4 = 3n/4
Multiply both sides by 4/3: n = 9 x (4/3) = 12

Check: 12/4 + 9 = 3 + 9 = 12 ✓

Answer: The number is 12.

Q5 Given 4k + 1 = 13, find the values of: (a) 8k + 2 (b) 4k (c) k (d) 4k - 1 (e) -k - 2

SOLUTION

First solve 4k + 1 = 13:
4k = 13 - 1 = 12

k = 12 / 4 = 3

(a) 8k + 2 = 2(4k + 1) = 2 x 13 = 26

(b) 4k = 12

(c) k = 3

(d) 4k - 1 = 12 - 1 = 11

(e) -k - 2 = -3 - 2 = -5

Answer: (a) 26 (b) 12 (c) 3 (d) 11 (e) -5

Exercise 15.4 — Mind the Mistake, Mend the Mistake (Page 181)

Q1 Find and correct the mistake in the following solution:

4x + 6 = 10
4x = 10 + 6
4x = 16
x = 4

SOLUTION

Mistake: The term +6 was added to the RHS instead of subtracted.
When moving +6 from the LHS, its additive inverse (-6) must appear on the RHS.

Correct solution:
4x + 6 = 10
4x = 10 - 6 = 4
x = 4 / 4 = 1

Check: 4(1) + 6 = 10 ✓

Answer: Mistake in the sign when moving 6. Correct answer: x = 1.

Q2 Find and correct the mistake in the following solution:

7 - 8z = 5
8z = 7 - 5
8z = 2
z = 4

SOLUTION

Mistake: After getting 8z = 2, they wrote z = 4 instead of z = 2/8.

Correct solution:
7 - 8z = 5
Subtract 7 from both sides: -8z = 5 - 7 = -2
Divide both sides by -8: z = (-2) / (-8) = 1/4

Check: 7 - 8(1/4) = 7 - 2 = 5 ✓

Answer: Mistake in dividing 2 by 8. Correct answer: z = 1/4.

Q3 Find and correct the mistake in the following solution:

2v - 4 = 6

v - 4 = 6 - 2

v - 4 = 4

v = 8

SOLUTION

Mistake: The step v - 4 = 6 - 2 is wrong. Dividing the LHS (2v-4) by 2 to get v-4 must also divide the full RHS by 2, giving v - 4 = 6/2 = 3, not 6 - 2.

Correct solution:

2v - 4 = 6

Add 4 to both sides: 2v = 10

Divide both sides by 2: v = 5

Check: 2(5) - 4 = 6 ✓

Answer: Mistake in applying the operation to the RHS. Correct answer: v = 5.

Q4 Find and correct the mistake in the following solution:

5z + 2 = 3z - 4

5z + 3z = -4 + 2

8z = -2

z = -2/8

SOLUTION

Mistake: Instead of subtracting 3z from both sides, they added 3z to the LHS, making 5z + 3z instead of 5z - 3z.

Also the constant 2 moved to the wrong side (should be -2 not +2).

Correct solution:

5z + 2 = 3z - 4

Subtract 3z from both sides: 2z + 2 = -4

Subtract 2 from both sides: 2z = -6

Divide both sides by 2: z = -3

Check: 5(-3) + 2 = -13, 3(-3) - 4 = -13 ✓

Answer: Mistake in moving 3z (should subtract, not add). Correct answer: z = -3.

Q5 Find and correct the mistake in the following solution:

15w - 4w = 26

15w = 26 + 4w

15w = 30

w = 2

SOLUTION

Mistake: 15w - 4w = 26 simplifies directly to 11w = 26.

Moving -4w to the RHS to get 15w = 26 + 4w was a backward step, and then claiming 15w = 30 is incorrect (it substitutes w = 2 without justification).

Correct solution:

15w - 4w = 26

11w = 26

w = 26/11

Check: 15(26/11) - 4(26/11) = 11(26/11) = 26 ✓

Answer: Mistake in rearranging after simplification. Correct answer: w = 26/11.

Q6 Find and correct the mistake in the following solution:

3x + 1 = -12

x + 1 = -12/3

x + 1 = -4

x = -5

SOLUTION

Mistake: Only 3x was divided by 3 to get x, but the +1 was not also divided. When dividing both sides by 3, ALL terms on the LHS must be divided.

Correct solution:

3x + 1 = -12

Subtract 1 from both sides: 3x = -12 - 1 = -13

Divide both sides by 3: x = -13/3

Check: 3(-13/3) + 1 = -13 + 1 = -12 ✓

Answer: Mistake in dividing only part of LHS. Correct answer: x = -13/3.

Q7 Find and correct the mistake in the following solution:

4(4q + 2) = 50
4(4q) = 50 - 2
16q = 48
q = 3

SOLUTION

Mistake: The step '4(4q) = 50 - 2' is wrong. The 4 must be distributed over the whole bracket: 4(4q + 2) = 16q + 8. You cannot subtract 2 from the RHS without first distributing the 4.

Correct solution:
4(4q + 2) = 50
Expand: 16q + 8 = 50
Subtract 8 from both sides: 16q = 42
Divide both sides by 16: q = 42/16 = 21/8

Check: 4(4 x 21/8 + 2) = 4(21/2 + 2) = 4(25/2) = 50 ✓

Answer: Mistake in distributing before moving terms. Correct answer: q = 21/8.

Q8 Find and correct the mistake in the following solution:

-2(3 - 4x) = 14
-6v - 8x = 14
-8x = 14 + 6
-8x = 20
x = -20/8

SOLUTION

Mistake 1: A new variable v appeared (it should still be x throughout).
Mistake 2: -2 x (-4x) = +8x, not -8x. The sign is wrong.

Correct solution:
-2(3 - 4x) = 14
Expand: -6 + 8x = 14
Add 6 to both sides: 8x = 20
Divide both sides by 8: x = 20/8 = 5/2

Check: -2(3 - 4 x 5/2) = -2(3 - 10) = -2(-7) = 14 ✓

Answer: Two mistakes: wrong variable and sign error in expansion. Correct answer: x = 5/2.

Q9 Find and correct the mistake in the following solution:

3(7y + 4) = 9 + 5y
7y + 4 = 9/3 + 5y
7y + 4 = 3 + 5y
7y - 5y + 4 = 3
2y = 1/2

SOLUTION

Mistake: Only the 9 on the RHS was divided by 3, but the 5y term was not divided. When dividing both sides by 3, every term on both sides must be divided.

Correct solution:
3(7y + 4) = 9 + 5y
Expand LHS: 21y + 12 = 9 + 5y
Subtract 5y from both sides: 16y + 12 = 9
Subtract 12 from both sides: 16y = -3
y = -3/16

Check: 3(7 x (-3/16) + 4) = 3(-21/16 + 64/16) = 3(43/16) = 129/16
RHS: 9 + 5(-3/16) = 9 - 15/16 = 144/16 - 15/16 = 129/16 ✓

Answer: Mistake in dividing only part of the RHS by 3. Correct answer: y = -3/16.

Exercise 15.5 — Applications and Problem Solving — Figure it Out (Page 184)

Q1 Fill in the blanks with integers.

(a) 5 x ___ - 8 = 37

(b) 37 - (33 - __) = 35

(c) -3 x (-11 + __) = 45

SOLUTION

(a) 5 x n - 8 = 37 => 5n = 45 => n = 9

Check: 5(9) - 8 = 45 - 8 = 37 ✓

(b) 37 - (33 - n) = 35 => 37 - 33 + n = 35 => 4 + n = 35 => n = 31

Check: 37 - (33 - 31) = 37 - 2 = 35 ✓

(c) -3 x (-11 + n) = 45 => -11 + n = 45 / (-3) = -15 => n = -15 + 11 = -4

Check: -3 x (-11 + (-4)) = -3 x (-15) = 45 ✓

Answer: (a) 9 (b) 31 (c) -4

Q2 Ranju is a daily wage labourer. She earns Rs 750 a day. Her employer pays her in 50 and 100 rupee notes. If Ranju gets an equal number of 50 and 100 rupee notes, how many notes of each does she have?

SOLUTION

Let the number of 50-rupee notes = n (and also the number of 100-rupee notes = n).

Total value: 50n + 100n = 750

150n = 750

n = 750 / 150 = 5

Check: 5 x 50 + 5 x 100 = 250 + 500 = 750 ✓

Answer: Ranju has 5 notes of Rs 50 and 5 notes of Rs 100.

Q3 In the given picture, each black blob hides an equal number of blue dots. There are 3 blobs and 4 visible blue dots. If there are 25 dots in total, how many dots are covered by one blob? Write an equation to describe this problem.

SOLUTION

Let each blob hide b dots.

Equation: 3b + 4 = 25

Subtract 4 from both sides: 3b = 21
Divide both sides by 3: b = 7

Check: 3(7) + 4 = 21 + 4 = 25 ✓

Answer: Each blob hides 7 dots. Equation: 3b + 4 = 25.

Q4 Machine (a) takes an input, adds 3, multiplies by 4, then subtracts 5. The example shows input 12 gives output 55. Find the inputs when the outputs are: (i) 43 (ii) 75

SOLUTION

The machine performs: (x + 3) x 4 - 5 = output.
Check with example: (12 + 3) x 4 - 5 = 60 - 5 = 55 ✓

(i) Output = 43:
(x + 3) x 4 - 5 = 43
(x + 3) x 4 = 48
x + 3 = 12
x = 9
Check: (9+3)x4-5 = 48-5 = 43 ✓

(ii) Output = 75:
(x + 3) x 4 - 5 = 75
(x + 3) x 4 = 80
x + 3 = 20
x = 17
Check: (17+3)x4-5 = 80-5 = 75 ✓

Answer: (i) Input = 9 (ii) Input = 17

Q5 Machine (b) takes an input, splits into two paths: the top path multiplies by 3; the bottom path adds 3. The two results are then subtracted (top minus bottom). The example shows input 12 gives output 21. Find the inputs when the outputs are: (i) 63 (ii) 227

SOLUTION

The machine performs: 3x - (x + 3) = 2x - 3 = output.
Check with example: 2(12) - 3 = 21 ✓

(i) Output = 63:
2x - 3 = 63
2x = 66
x = 33
Check: 2(33) - 3 = 63 ✓

(ii) Output = 227:
2x - 3 = 227
2x = 230
x = 115
Check: 2(115) - 3 = 227 ✓

Answer: (i) Input = 33 (ii) Input = 115

Q6 What are the inputs to these machines?
(i) Machine: input -> add 3 -> add 3 -> output = 5
(ii) Machine: input -> subtract 4 -> subtract 4 -> output = -11

SOLUTION

(i) Machine performs: (x + 3) + 3 = x + 6 = 5
Subtract 6 from both sides: x = 5 - 6 = -1
Check: (-1 + 3) + 3 = 2 + 3 = 5 ✓

(ii) Machine performs: (x - 4) - 4 = x - 8 = -11
Add 8 to both sides: x = -11 + 8 = -3
Check: (-3 - 4) - 4 = -7 - 4 = -11 ✓

Answer: (i) Input = -1 (ii) Input = -3

Q7 A taxi driver charges a fixed fee of Rs 800 per day plus Rs 20 for each kilometre travelled. If the total cost for a taxi ride is Rs 2200, determine the number of kilometres travelled.

SOLUTION

Let the number of kilometres = k.

Total cost: 800 + 20k = 2200
Subtract 800 from both sides: 20k = 1400
Divide both sides by 20: k = 70

Check: 800 + 20(70) = 800 + 1400 = 2200 ✓

Answer: 70 kilometres were travelled.

Q8 The sum of two numbers is 76. One number is three times the other number. What are the numbers?

SOLUTION

Let the smaller number = x. Then the larger number = 3x.

x + 3x = 76
4x = 76
x = 19

Larger number = 3 x 19 = 57

Check: 19 + 57 = 76 ✓, 57 = 3 x 19 ✓

Answer: The two numbers are 19 and 57.

Q9 The figure shows a window grill with a total inner height of 34 cm. The top and bottom frame borders are each 3 cm. There are 4 horizontal rods each of thickness 2 cm, with equal gaps between and around them. What is the gap between two rods?

SOLUTION

Let the gap between two rods (and from the inner frame to the nearest rod) = g cm.

There are 4 rods and 5 equal gaps (one above the top rod, three between rods, one below the bottom rod).

Equation: top frame (3) + 5 gaps (5g) + 4 rods (4 x 2) + bottom frame (3) = 34
3 + 5g + 8 + 3 = 34
14 + 5g = 34
5g = 20
g = 4

Check: 3 + 5(4) + 4(2) + 3 = 3 + 20 + 8 + 3 = 34 ✓

Answer: The gap between two rods is 4 cm.

Q10 In a restaurant, a fruit juice costs Rs 15 less than a chocolate milkshake. If 4 fruit juices and 7 chocolate milkshakes cost Rs 600, find the cost of the fruit juice and milkshake.

SOLUTION

Let the cost of a milkshake = m rupees.
Then the cost of a fruit juice = m - 15 rupees.

4(m - 15) + 7m = 600
4m - 60 + 7m = 600
11m - 60 = 600
11m = 660
m = 60

Cost of juice = 60 - 15 = 45

Check: 4(45) + 7(60) = 180 + 420 = 600 ✓

Answer: Milkshake costs Rs 60 and fruit juice costs Rs 45.

Q11 Given 28p - 36 = 98, find the value of 14p - 19 and 28p - 38.

SOLUTION

28p - 36 = 98
Note: 14p - 19 = (28p - 38) / 2 and 28p - 38 = (28p - 36) - 2 = 98 - 2 = 96.

28p - 38 = 28p - 36 - 2 = 98 - 2 = 96

14p - 19 = (28p - 38) / 2 = 96 / 2 = 48

Verification: Solve for p: 28p = 134, p = 67/14.
14(67/14) - 19 = 67 - 19 = 48 ✓
28(67/14) - 38 = 134 - 38 = 96 ✓

Answer: 14p - 19 = 48 and 28p - 38 = 96.

Q12 The steps to solve three equations are shown below. Identify and correct any mistakes.
(a) 6x + 9 = 66: x + 9 = 11, x = 2
(b) 14y + 24 = 36: 7y + 12 = 18, 7y = 6, y = 6/7
(c) 4x - 5 = 9x + 8: 4x = 9x + 8 - 5, 4x = 9x + 3, -5x = 3, x = -5/3

SOLUTION

(a) Mistake: The student divided 6x by 6 to get x but did not divide 9 by 6.
Correct: 6x + 9 = 66 => 6x = 57 => x = 57/6 = 19/2
Check: 6(19/2) + 9 = 57 + 9 = 66 ✓

(b) No mistake. This solution is correct.
14y + 24 = 36 => (divide both sides by 2) 7y + 12 = 18 => 7y = 6 => y = 6/7
Check: 14(6/7) + 24 = 12 + 24 = 36 ✓

(c) Mistake: In step '4x = 9x + 8 - 5', the -5 was moved to the RHS as -5, but it should be moved as +5 (additive inverse of -5). i.e., 4x - 5 = 9x + 8 => 4x - 9x = 8 + 5.
Correct: 4x - 5 = 9x + 8 => 4x - 9x = 8 + 5 => -5x = 13 => x = -13/5
Check: 4(-13/5) - 5 = -52/5 - 25/5 = -77/5
9(-13/5) + 8 = -117/5 + 40/5 = -77/5 ✓

Answer: (a) Mistake — correct answer: x = 19/2. (b) No mistake — y = 6/7. (c) Mistake — correct answer: x = -13/5.

Q13 Find the measures of the angles of these triangles.
Triangle 1: angles are y, y, and (y + 15).
Triangle 2: angles are x, (x - 10), and (x + 10).

SOLUTION

Triangle 1: The angle sum of a triangle is 180 degrees.
y + y + (y + 15) = 180
3y + 15 = 180
3y = 165
y = 55
Angles: 55 degrees, 55 degrees, 70 degrees.
Check: 55 + 55 + 70 = 180 ✓

Triangle 2:
x + (x - 10) + (x + 10) = 180
3x = 180
x = 60
Angles: 60 degrees, 50 degrees, 70 degrees.
Check: 60 + 50 + 70 = 180 ✓

Answer: Triangle 1: 55°, 55°, 70°. Triangle 2: 60°, 50°, 70°.

Q14 The Bakhshali Manuscript (300 CE) mentions the following problem. The amount given to the first person is not known. The second person is given twice as much as the first. The third person is given thrice as much as the second; and the fourth person four times as much as the third. The total amount distributed is 132. What is the amount given to the first person?

SOLUTION

Let the amount given to the first person = a.
Second person = 2a
Third person = 3 x 2a = 6a
Fourth person = 4 x 6a = 24a

Total: a + 2a + 6a + 24a = 132
33a = 132
a = 4

Check: 4 + 8 + 24 + 96 = 132 ✓

Answer: The first person receives 4 units.

Q15 The height of a giraffe is two and a half metres more than half its height. How tall is the giraffe?

SOLUTION

Let the height of the giraffe = h metres.

h = h/2 + 5/2

Subtract h/2 from both sides: h/2 = 5/2
Multiply both sides by 2: h = 5

Check: half of 5 = 2.5, and 2.5 + 2.5 = 5 ✓

Answer: The giraffe is 5 metres tall.

Q16 Pattern 1 — Arrow shapes: Position 1 has 5 sticks, each new position adds 4 sticks; sticks(n) = 4n + 1, squares(n) = n.
(a) How many squares at position 11?
(b) How many sticks at position 11?
(c) Can exactly 85 sticks make a position? If yes, which?
(d) Can exactly 150 sticks make a position? If yes, which?

Pattern 2 — Staircase: Position n has n(n+1)/2 squares and $n^2 + 3n$ sticks.
(a) How many squares at position 11?
(b) How many sticks at position 11?
(c) Can exactly 85 sticks make a position? If yes, which?
(d) Can exactly 150 sticks make a position? If yes, which?

SOLUTION

Pattern 1 (arrow shapes): sticks(n) = 4n + 1, squares(n) = n.

(a) Squares at position 11 = 11

(b) Sticks at position 11 = 4(11) + 1 = 45

(c) 4n + 1 = 85 => 4n = 84 => n = 21. Yes, position 21.

(d) 4n + 1 = 150 => 4n = 149 => n = 149/4 = 37.25. Not a whole number, so NO.

Pattern 2 (staircase): squares(n) = n(n+1)/2, sticks(n) = $n^2 + 3n$ = n(n+3).

(a) Squares at position 11 = 11 x 12 / 2 = 66

(b) Sticks at position 11 = $11^2 + 3(11) = 121 + 33 = 154$

(c) $n^2 + 3n = 85 \Rightarrow n^2 + 3n - 85 = 0$.
Discriminant = 9 + 340 = 349, sqrt(349) ≈ 18.68. Not a perfect square, so n is not a whole number. NO.

(d) $n^2 + 3n = 150 \Rightarrow n^2 + 3n - 150 = 0$.
Discriminant = 9 + 600 = 609, sqrt(609) ≈ 24.68. Not a perfect square. NO.

Answer: Pattern 1: (a) 11 squares (b) 45 sticks (c) Yes, position 21 (d) No.
Pattern 2: (a) 66 squares (b) 154 sticks (c) No (d) No.

Q17 A number increased by 36 is equal to ten times itself. What is the number?

SOLUTION

Let the number = n.

n + 36 = 10n
36 = 10n - n = 9n
n = 36 / 9 = 4

Check: 4 + 36 = 40 = 10 x 4 ✓

Answer: The number is 4.

Q18 Solve these equations:

(a) 5(r + 2) = 10 (b) -3(u + 2) = 2(u - 1) (c) 2(7 - 2n) = -6 (d) 2(x - 4) = -16 (e) 6(x - 1) = 2(x - 1) - 4 (f) 3 - 7s = 7 - 3s (g) 2x + 1 = 6 - (2x - 3) (h) 10 - 5x = 3(x - 4) - 2(x - 7)

SOLUTION

(a) 5(r + 2) = 10 => r + 2 = 2 => r = 0
Check: 5(0+2)=10 ✓

(b) -3(u + 2) = 2(u - 1) => -3u - 6 = 2u - 2 => -5u = 4 => u = -4/5
Check: -3(-4/5+2)=2(-4/5-1) => -3(6/5)=2(-9/5) => -18/5=-18/5 ✓

(c) 2(7 - 2n) = -6 => 14 - 4n = -6 => 4n = 20 => n = 5
Check: 2(7-10)=2(-3)=-6 ✓

(d) 2(x - 4) = -16 => x - 4 = -8 => x = -4
Check: 2(-4-4)=2(-8)=-16 ✓

(e) 6(x - 1) = 2(x - 1) - 4 => 4(x - 1) = -4 => x - 1 = -1 => x = 0
Check: 6(-1)=-6, 2(-1)-4=-6 ✓

(f) 3 - 7s = 7 - 3s => -4s = 4 => s = -1
Check: 3-7(-1)=10, 7-3(-1)=10 ✓

(g) 2x + 1 = 6 - (2x - 3) = 6 - 2x + 3 = 9 - 2x => 4x = 8 => x = 2
Check: 2(2)+1=5, 6-(4-3)=6-1=5 ✓

(h) 10 - 5x = 3(x-4) - 2(x-7) = 3x - 12 - 2x + 14 = x + 2
10 - 5x = x + 2 => 8 = 6x => x = 4/3
Check: 10-5(4/3)=10-20/3=10/3, (4/3)+2=10/3 ✓

Answer: (a) r=0 (b) u=-4/5 (c) n=5 (d) x=-4 (e) x=0 (f) s=-1 (g) x=2 (h) x=4/3