Chapter 2: Power Play

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Exercise 2.1 — Exponential Notation and Operations

Q1 Express the following in exponential form:
(i) 6 × 6 × 6 × 6
(ii) y × y
(iii) b × b × b × b
(iv) 5 × 5 × 7 × 7 × 7
(v) 2 × 2 × a × a
(vi) a × a × a × c × c × c × c × d

SOLUTION

Exponential form means writing repeated multiplication as a base raised to a power.

(i) 6 × 6 × 6 × 6
6 is multiplied by itself 4 times.
=> 6^4

(ii) y × y
y is multiplied by itself 2 times.
=> y^2

(iii) b × b × b × b
b is multiplied by itself 4 times.
=> b^4

(iv) 5 × 5 × 7 × 7 × 7
5 appears 2 times, 7 appears 3 times.
=> $5^2 \times 7^3$

(v) 2 × 2 × a × a
2 appears 2 times, a appears 2 times.
=> $2^2 \times a^2$

(vi) a × a × a × c × c × c × c × d

a appears 3 times, c appears 4 times, d appears once.
=> $a^3 \times c^4 \times d$

Answer: (i) 6^4 (ii) y^2 (iii) b^4 (iv) $5^2 \times 7^3$ (v) $2^2 \times a^2$ (vi) $a^3 \times c^4 \times d$

Q2 Express each of the following as a product of powers of their prime factors in exponential form.
(i) 648 (ii) 405 (iii) 540 (iv) 3600

SOLUTION

(i) 648
648 ÷ 2 = 324
324 ÷ 2 = 162
162 ÷ 2 = 81
81 ÷ 3 = 27
27 ÷ 3 = 9
9 ÷ 3 = 3
3 ÷ 3 = 1
=> $648 = 2^3 \times 3^4$

(ii) 405
405 ÷ 3 = 135
135 ÷ 3 = 45
45 ÷ 3 = 15
15 ÷ 3 = 5
5 is prime.
=> $405 = 3^4 \times 5$

(iii) 540
540 ÷ 2 = 270
270 ÷ 2 = 135
135 ÷ 3 = 45
45 ÷ 3 = 15
15 ÷ 3 = 5
5 is prime.
=> $540 = 2^2 \times 3^3 \times 5$

(iv) 3600
3600 ÷ 2 = 1800

1800 ÷ 2 = 900
900 ÷ 2 = 450
450 ÷ 2 = 225
225 ÷ 3 = 75
75 ÷ 3 = 25
25 ÷ 5 = 5
5 is prime.
=> 3600 = $2^4 \times 3^2 \times 5^2$

Answer: (i) $2^3 \times 3^4$ (ii) $3^4 \times 5$ (iii) $2^2 \times 3^3 \times 5$ (iv) $2^4 \times 3^2 \times 5^2$

Q3 Write the numerical value of each of the following:
(i) 2×10^3
(ii) $7^2 \times 2^3$
(iii) 3×4^4
(iv) $(-3)^2 \times (-5)^2$
(v) $3^2 \times 10^4$
(vi) $(-2)^5 \times (-10)^6$

SOLUTION

(i) 2×10^3
$10^3 = 10 \times 10 \times 10 = 1000$
$2 \times 1000 = 2000$

(ii) $7^2 \times 2^3$
$7^2 = 49$, $2^3 = 8$
$49 \times 8 = 392$

(iii) 3×4^4
$4^4 = 4 \times 4 \times 4 \times 4 = 256$
$3 \times 256 = 768$

(iv) $(-3)^2 \times (-5)^2$
$(-3)^2 = 9$ (negative × negative = positive)
$(-5)^2 = 25$
$9 \times 25 = 225$

(v) $3^2 \times 10^4$
$3^2 = 9$, $10^4 = 10000$
$9 \times 10000 = 90000$

(vi) $(-2)^5 \times (-10)^6$

$(-2)^5 = -32$ (odd power of negative = negative)

$(-10)^6 = 1000000$ (even power of negative = positive)

$-32 \times 1000000 = -32000000$

Answer: (i) 2000 (ii) 392 (iii) 768 (iv) 225 (v) 90000 (vi) –32000000

Exercise 2.2 — A Pinch of History — Powers of 10 and Large Numbers

Q1 Find out the units digit in the value of $2^{224} \div 4^{32}$? [Hint: $4 = 2^2$]

SOLUTION

Using the hint, $4 = 2^2$, so $4^{32} = (2^2)^{32} = 2^{64}$.

$2^{224} \div 4^{32} = 2^{224} \div 2^{64} = 2^{(224 - 64)} = 2^{160}$.

Find the units digit of 2^{160}.
Powers of 2 have a units digit cycle of period 4: $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$ (units digit 6), then repeats.
Cycle: 2, 4, 8, 6 (period 4).

160 ÷ 4 = 40 remainder 0.
A remainder of 0 corresponds to the 4th position in the cycle, which has units digit 6.
(Verify: $2^4 = 16$ ends in 6; $2^8 = 256$ ends in 6; the pattern holds.)

Answer: Units digit is 6

Q2 There are 5 bottles in a container. Every day, a new container is brought in. How many bottles would be there after 40 days?

SOLUTION

Each day, one new container with 5 bottles is brought in.
After 40 days, 40 containers have been brought in.

Total bottles = 40 × 5 = 200.

In exponential form: $200 = 2 \times 10^2$.

Answer: 200 bottles ($= 2 \times 10^2$)

Q3 Write the given number as the product of two or more powers in three different ways. The powers can be any integers.
(i) 64^3
(ii) 192^8
(iii) $32^{(-5)}$

SOLUTION

(i) 64^3

$64 = 2^6$, so $64^3 = (2^6)^3 = 2^{18}$.

Three ways to write 2^{18} as a product of powers:

Way 1: $2^{10} \times 2^8$ (10 + 8 = 18)

Way 2: $4^5 \times 4^4$ (since $4 = 2^2$, so $4^5 = 2^{10}$ and $4^4 = 2^8$; 10 + 8 = 18)

Way 3: $8^3 \times 8^3$ (since $8 = 2^3$, so $8^3 = 2^9$; 9 + 9 = 18)

(ii) 192^8

$192 = 2^6 \times 3$, so $192^8 = (2^6)^8 \times 3^8 = 2^{48} \times 3^8$.

Three ways:

Way 1: $2^{48} \times 3^8$

Way 2: $2^{40} \times 2^8 \times 3^8$ (40 + 8 = 48)

Way 3: $2^{40} \times 6^8$ (since $6 = 2 \times 3$, $6^8 = 2^8 \times 3^8$; so $2^{40} \times 6^8 = 2^{48} \times 3^8$)

(iii) 32^(–5)

$32 = 2^5$, so 32^(–5) = (2^5)^(–5) = 2^(–25).

Three ways to write 2^(–25):

Way 1: 2^(–10) × 2^(–15) (–10 + (–15) = –25)

Way 2: 2^(–5) × 2^(–20) (–5 + (–20) = –25)

Way 3: 4^(–12) × 2^(–1) (since 4^(–12) = 2^(–24); –24 + (–1) = –25)

Answer: (i) Three ways: $2^{10} \times 2^8$; $4^5 \times 4^4$; $8^3 \times 8^3$ (ii) Three ways: $2^{48} \times 3^8$; $2^{40} \times 2^8 \times 3^8$; $2^{40} \times 6^8$ (iii) Three ways: 2^(–10) × 2^(–15); 2^(–5) × 2^(–20); 4^(–12) × 2^(–1)

Q4 Examine each statement below and find out if it is 'Always True', 'Only Sometimes True', or 'Never True'. Explain your reasoning.

(i) Cube numbers are also square numbers.

(ii) Fourth powers are also square numbers.

(iii) The fifth power of a number is divisible by the cube of that number.

(iv) The product of two cube numbers is a cube number.

(v) q^{46} is both a 4th power and a 6th power (q is a prime number).

SOLUTION

(i) Cube numbers are also square numbers — Only Sometimes True.

A cube number n^3 is also a perfect square only when n itself is a perfect square.

If $n = k^2$, then $n^3 = k^6 = (k^3)^2$, which is a perfect square.

In general, n^3 is a square exactly when n is a 6th power (like $n = m^6$), but the simplest case is: $n^3 = (n^{(3/2)})^2$ only when 3/2 is an integer power of n — i.e., n must be a perfect square.

Example: $8 = 2^3$ is a cube but not a square. However $64 = 4^3 = 8^2$ is both.

The numbers that are both squares and cubes are exactly the 6th powers: $n^6 = (n^3)^2 = (n^2)^3$.

(ii) Fourth powers are also square numbers — Always True.
For any integer n: $n^4 = (n^2)^2$, which is always a perfect square.

(iii) The fifth power of a number is divisible by the cube of that number — Always True.
$n^5 \div n^3 = n^{(5-3)} = n^2$, which is always an integer for any non-zero n.

(iv) The product of two cube numbers is a cube number — Always True.
Let the two cube numbers be a^3 and b^3.
$a^3 \times b^3 = (a \times b)^3$, which is the cube of $(a \times b)$.

(v) q^{46} is both a 4th power and a 6th power (q is prime) — Never True.
For q^{46} to be a 4th power, we need 46 to be divisible by 4: $46 \div 4 = 11.5$ — not an integer.
For q^{46} to be a 6th power, we need 46 to be divisible by 6: $46 \div 6 \approx 7.67$ — not an integer.
Since q is prime, q^{46} can only be expressed as a perfect k-th power if 46 is divisible by k.
46 is divisible by neither 4 nor 6, so q^{46} is never both a 4th power and a 6th power.

Answer: (i) Only Sometimes True — only 6th powers are both cubes and squares (ii) Always True — $n^4 = (n^2)^2$ (iii) Always True — $n^5 \div n^3 = n^2$ (iv) Always True — $a^3 \times b^3 = (ab)^3$ (v) Never True — 46 is not divisible by 4 or 6

Q5 Simplify and write these in the exponential form.
(i) 10^(–2) × 10^(–5)
(ii) $5^7 \div 5^4$
(iii) 9^(–7) ÷ 9^4
(iv) (13^(–2))^(–3)
(v) $m^5n^{12}(mn)^9$

SOLUTION

(i) 10^(–2) × 10^(–5)
Using n^a × n^b = n^(a+b):
10^(–2) × 10^(–5) = 10^(–2 + (–5)) = 10^(–7)

(ii) $5^7 \div 5^4$

Using n^a ÷ n^b = n^(a–b):
$5^7 \div 5^4 = 5^{(7-4)} = 5^3$

(iii) 9^(–7) ÷ 9^4
9^(–7) ÷ 9^4 = 9^(–7 – 4) = 9^(–11)

(iv) (13^(–2))^(–3)
Using (n^a)^b = n^(a×b):
(13^(–2))^(–3) = 13^(–2 × –3) = 13^6

(v) $m^5n^{12}(mn)^9$
Expand $(mn)^9 = m^9n^9$:
$m^5 \times n^{12} \times m^9 \times n^9$
Group like bases: m^(5+9) × n^(12+9) = $m^{14} \times n^{21}$

Answer: (i) 10^(–7) (ii) 5^3 (iii) 9^(–11) (iv) 13^6 (v) $m^{14} \times n^{21}$

Q6 If $12^2 = 144$ what is
(i) $(1.2)^2$
(ii) $(0.12)^2$
(iii) $(0.012)^2$
(iv) 120^2

SOLUTION

Key idea: scale the base, then scale the result by the same factor squared.

(i) $(1.2)^2$
$1.2 = 12 \div 10$, so $(1.2)^2 = (12 \div 10)^2 = 12^2 \div 10^2 = 144 \div 100 = 1.44$

(ii) $(0.12)^2$
$0.12 = 12 \div 100$, so $(0.12)^2 = 12^2 \div 100^2 = 144 \div 10000 = 0.0144$

(iii) $(0.012)^2$
$0.012 = 12 \div 1000$, so $(0.012)^2 = 12^2 \div 1000^2 = 144 \div 1000000 = 0.000144$

(iv) 120^2
$120 = 12 \times 10$, so $120^2 = (12 \times 10)^2 = 12^2 \times 10^2 = 144 \times 100 = 14400$

Answer: (i) 1.44 (ii) 0.0144 (iii) 0.000144 (iv) 14400

Q7 Circle the numbers that are the same —
$2^4 \times 3^6$ $6^4 \times 3^2$ 6^{10} $18^2 \times 6^2$ 6^{24}

SOLUTION

Compute each expression:
$2^4 \times 3^6 = 16 \times 729 = 11664$
$6^4 \times 3^2 = 1296 \times 9 = 11664$
$6^{10} = 60466176$
$18^2 \times 6^2 = 324 \times 36 = 11664$
6^{24} is a very large number (not 11664)

Verify algebraically why three are equal:
$2^4 \times 3^6 = 2^4 \times 3^4 \times 3^2 = (2 \times 3)^4 \times 3^2 = 6^4 \times 3^2$ ✓
$18^2 \times 6^2 = (2 \times 3^2)^2 \times (2 \times 3)^2 = 2^2 \times 3^4 \times 2^2 \times 3^2 = 2^4 \times 3^6$ ✓

Answer: $2^4 \times 3^6$, $6^4 \times 3^2$, and $18^2 \times 6^2$ are the same (all equal 11664)

Q8 Identify the greater number in each of the following —
(i) 4^3 or 3^4
(ii) 2^8 or 8^2
(iii) 100^2 or 2^{100}

SOLUTION

(i) 4^3 or 3^4
$4^3 = 64$
$3^4 = 81$
$81 > 64$, so 3^4 is greater.

(ii) 2^8 or 8^2
$2^8 = 256$
$8^2 = 64$
$256 > 64$, so 2^8 is greater.

(iii) 100^2 or 2^{100}
$100^2 = 10000$
$2^{100} = 2^{10} \times 2^{90} = 1024 \times 2^{90}$, which is astronomically large (approximately 1.27×10^{30}).
$2^{100} >> 100^2$, so 2^{100} is greater.

Answer: (i) 3^4 is greater (ii) 2^8 is greater (iii) 2^{100} is greater

Q9 A dairy plans to produce 8.5 billion packets of milk in a year. They want a unique ID (identifier) code for each packet. If they choose to use the digits 0–9, how many digits should the code consist of?

SOLUTION

Total packets = 8.5 billion = 8,500,000,000.

With n digits (each from 0–9), the number of unique codes = 10^n.
We need 10^n >= 8,500,000,000.

Check n = 9: 10^9 = 1,000,000,000 < 8,500,000,000. Not enough.
Check n = 10: 10^{10} = 10,000,000,000 > 8,500,000,000. Sufficient.

So n = 10 is the minimum.

Answer: At least 10 digits are required

Q10 64 is a square number (8^2) and a cube number (4^3). Are there other numbers that are both squares and cubes? Is there a way to describe such numbers in general?

SOLUTION

64 = 8^2 = 4^3 = 2^6. It is a 6th power.

In general, a number is both a perfect square and a perfect cube if and only if it is a perfect 6th power.
Reason: $n^6 = (n^3)^2$ — so it is a perfect square, and $n^6 = (n^2)^3$ — so it is a perfect cube.

Examples of numbers that are both squares and cubes:
1^6 = 1 (= 1^2 = 1^3)
2^6 = 64 (= 8^2 = 4^3)
3^6 = 729 (= 27^2 = 9^3)
4^6 = 4096 (= 64^2 = 16^3)

There are infinitely many such numbers — one for each positive integer n.

Answer: Yes, infinitely many. In general they are 6th powers: 1, 64, 729, 4096, ... (i.e., n^6 for positive integers n)

Q11 A digital locker has an alphanumeric (it can have both digits and letters) passcode of length 5. Some example codes are G89P0, 38098, BRJKW, and 003AZ. How many such codes are possible?

SOLUTION

Available characters: 26 letters (A–Z) + 10 digits (0–9) = 36 characters.

Each of the 5 positions can independently be any of the 36 characters.
Total codes = $36 \times 36 \times 36 \times 36 \times 36 = 36^5$.

$36^5 = 60{,}466{,}176$.

Answer: $36^5 = 60{,}466{,}176$ codes

Q12 The worldwide population of sheep (2024) is about 10^9, and that of goats is also about the same. What is the total population of sheep and goats?
(i) 20^9 (ii) 10^{11} (iii) 10^{10} (iv) 10^{18} (v) 2×10^9 (vi) $10^9 + 10^9$

SOLUTION

Population of sheep $\approx 10^9$.
Population of goats $\approx 10^9$.
Total = $10^9 + 10^9$.

This is most directly written as $10^9 + 10^9$ (option vi), which also equals 2×10^9.

Check the incorrect options:
20^9 is a different number ($20^9 \neq 2 \times 10^9$).
10^{11}, 10^{10}, 10^{18} are all far too large.
The correct answer is (vi): $10^9 + 10^9$.

Answer: (vi) $10^9 + 10^9$ (which equals 2×10^9)

Q13 Calculate and write the answer in scientific notation:
(i) If each person in the world had 30 pieces of clothing, find the total number of pieces of clothing.
(ii) There are about 100 million bee colonies in the world. Find the number of honeybees if each colony has about 50,000 bees.
(iii) The human body has about 38 trillion bacterial cells. Find the bacterial population residing in all humans in the world.
(iv) Total time spent eating in a lifetime in seconds.

SOLUTION

(i) Pieces of clothing
World population ≈ 8.2×10^9 people.
Each person has 30 pieces.
Total = $8.2 \times 10^9 \times 30 = 246 \times 10^9 = 2.46 \times 10^{11}$.

(ii) Honeybees
Bee colonies = 100 million = 10^8.
Bees per colony = 50,000 = 5×10^4.
Total bees = $10^8 \times 5 \times 10^4 = 5 \times 10^{12}$.

(iii) Bacterial cells in all humans
Bacterial cells per human body ≈ 38 trillion = 38×10^{12}.
World population ≈ 8.2×10^9.
Total = $38 \times 10^{12} \times 8.2 \times 10^9 = (38 \times 8.2) \times 10^{21} = 311.6 \times 10^{21} \approx 3.116 \times 10^{23}$.

(iv) Total eating time in a lifetime (seconds)
Assumed lifetime: 70 years.
Assumed eating time per day: 1 hour = 3600 seconds.
Total seconds = $3600 \times 365 \times 70 = 91{,}980{,}000 \approx 9.198 \times 10^7$ seconds.

Answer: (i) 2.46×10^{11} pieces (ii) 5.0×10^{12} bees (iii) $\approx 3.116 \times 10^{23}$ bacterial cells (iv) $\approx 9.198 \times 10^7$ seconds

Q14 What was the date 1 arab/1 billion seconds ago?

SOLUTION

1 arab = 1 billion = 10^9 seconds.

Convert 10^9 seconds to days:
$10^9 \div (60 \times 60 \times 24) = 10^9 \div 86400 \approx 11{,}574$ days.

Convert to years:
11,574 ÷ 365.25 ≈ 31.7 years.

So 1 billion seconds ago is approximately 31 years and 8 months before today.
Example calculation (today = July 2026): 31.7 years before July 2026 ≈ late 1994.

General method: subtract 10^9 seconds (≈ 11,574 days ≈ 31.7 years) from the current date.

Answer: Approximately 31 years and 8 months ago (≈ late 1994 if calculated in mid-2026); subtract $10^9 \div 86400 \approx 11{,}574$ days from today's date