Chapter 3: A Story of Numbers

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Exercise 3.1 — The Mechanism of Counting

Q1 Suppose you are using the number system that uses sticks to represent numbers, as in Method 1. Without using either the number names or the numerals of the Hindu number system, give a method for adding, subtracting, multiplying and dividing two numbers or two collections of sticks.

SOLUTION

We have two collections of sticks representing two numbers.

Addition: Combine both collections of sticks into one pile. The total number of sticks in the combined pile represents the sum.

Subtraction: From the larger collection, remove sticks one by one while simultaneously setting aside sticks from the smaller collection. When the smaller collection is exhausted, the remaining sticks in the larger collection represent the difference.

Multiplication: To multiply collection A by collection B, make as many copies of collection A as there are sticks in collection B. Combine all those copies. The total represents the product. Alternatively, arrange the sticks in a rectangular array — one side has as many rows as sticks in A, the other has as many columns as

sticks in B; count all sticks in the array.

Division: To divide collection A by collection B, repeatedly remove a group of B sticks from collection A. Count how many complete groups you can remove — that is the quotient. Any remaining sticks (fewer than B) form the remainder.

Answer: Add: merge piles; Subtract: remove sticks from the larger pile; Multiply: make repeated copies; Divide: repeatedly remove groups of the divisor size and count the groups.

Q2 One way of extending the number system in Method 2 is by using strings with more than one letter — for example, we could use 'aa' for 27. How can you extend this system to represent all the numbers? There are many ways of doing it!

SOLUTION

With single letters a to z, we can represent numbers 1 through 26.

One natural extension uses two-letter strings in alphabetical order: aa, ab, ac, …, az (26 more numbers = 27 to 52), then ba, bb, …, bz (53 to 78), …, zz (representing $26 + 26^2 = 702$).

But this still ends. To represent ALL numbers, we keep increasing the string length:
Length 1: a, b, c, …, z (26 numbers)
Length 2: aa, ab, …, zz ($26^2 = 676$ numbers)
Length 3: aaa, aab, …, zzz (26^3 numbers)
and so on indefinitely.

At each string length, there are finitely many strings; but since string length is unbounded, we can represent any positive integer. This is analogous to how the Hindu numeral system uses strings of digits 0–9 of increasing length.

Answer: Use strings of letters of increasing length: length 1 (a–z covers 1–26), length 2 (aa–zz covers 27–702), length 3, and so on — every positive integer is eventually covered.

Exercise 3.2 — The Roman Numerals

Q1 Represent the following numbers in the Roman system.
(i) 1222 (ii) 2999 (iii) 302 (iv) 715

SOLUTION

(i) 1222
1222 = 1000 + 100 + 100 + 10 + 10 + 1 + 1
= M + CC + XX + II
=> MCCXXII

(ii) 2999
2999 = 1000 + 1000 + 900 + 90 + 9
900 = 1000 - 100 = CM
90 = 100 - 10 = XC
9 = 10 - 1 = IX
=> MMCMXCIX
Check: 2000 + 900 + 90 + 9 = 2999 ✓

(iii) 302
302 = 100 + 100 + 100 + 1 + 1
= CCC + II
=> CCCII

(iv) 715
715 = 500 + 100 + 100 + 10 + 5
= D + CC + X + V
=> DCCXV
Check: 500 + 200 + 10 + 5 = 715 ✓

Answer: (i) MCCXXII (ii) MMCMXCIX (iii) CCCII (iv) DCCXV

Exercise 3.3 — Some Early Number Systems

Q1 A group of indigenous people in a Pacific island use different sequences of number names to count different objects. Why do you think they do this?

SOLUTION

Different objects may have been counted for different purposes in different cultural contexts — for example, counting fish, yams, people, or ritual objects might each have had its own sequence of names developed independently over time.

Possible reasons include:

1. Cultural or ritual significance: certain objects are sacred and counted differently to honour their special status.
2. Historical development: different communities within the island may have developed separate counting systems for different goods before merging into one group.
3. Grammatical categories: many languages grammatically classify objects (like the way some languages distinguish animate from inanimate), and counting words differ accordingly.
4. Practical grouping: objects that naturally come in pairs, dozens, or other natural groups might be counted with a system better suited to those groupings.

Answer: Different sequences likely arose due to cultural/ritual significance of different objects, separate historical development of counting for different goods, or grammatical classification systems in their language.

Q2 Consider the extension of the Gumulgal number system beyond 6 in the same way of counting by 2s. Come up with ways of performing the different arithmetic operations (+, –, ×, ÷) for numbers occurring in this system, without using Hindu numerals. Use this to evaluate the following:
(i) (ukasar-ukasar-ukasar-ukasar-urapon) + (ukasar-ukasar-ukasar-urapon)
(ii) (ukasar-ukasar-ukasar-ukasar-urapon) – (ukasar-ukasar-ukasar)
(iii) (ukasar-ukasar-ukasar-ukasar-urapon) × (ukasar-ukasar)
(iv) (ukasar-ukasar-ukasar-ukasar-ukasar-ukasar-ukasar-ukasar) ÷ (ukasar-ukasar)

SOLUTION

In the Gumulgal system: urapon = 1, ukasar = 2. Numbers are built by stringing these words.

Arithmetic methods (without Hindu numerals):
Addition: Concatenate the two strings, then simplify: every pair of urapon-urapon = ukasar, and every pair of ukasar-ukasar can be left as is (since the system just counts in 2s).
Subtraction: Remove from the first string as many urapon/ukasar terms as in the second string.
Multiplication: Repeated addition — make as many copies of the first string as there are components (pairs) in the second string.
Division: Repeatedly remove the divisor string from the dividend string and count how many times you can do so.

(i) (ukasar-ukasar-ukasar-ukasar-urapon) + (ukasar-ukasar-ukasar-urapon)
= 4 ukasars + 1 urapon + 3 ukasars + 1 urapon
= 7 ukasars + 2 urapon
= 7 ukasars + 1 ukasar [since 2 urapon = 1 ukasar]
= 8 ukasars
=> ukasar-ukasar-ukasar-ukasar-ukasar-ukasar-ukasar-ukasar

(ii) (ukasar-ukasar-ukasar-ukasar-urapon) – (ukasar-ukasar-ukasar)
= (4 ukasars + 1 urapon) – (3 ukasars)
= 1 ukasar + 1 urapon
=> ukasar-urapon

(iii) (ukasar-ukasar-ukasar-ukasar-urapon) × (ukasar-ukasar)
= (4 ukasars + 1 urapon) × (2 ukasars)
= 9 × 4 = 36 [converting: each ukasar = 2, each urapon = 1]
36 ÷ 2 = 18, so the result = 18 ukasars

=> ukasar repeated 18 times

(iv) (ukasar-ukasar-ukasar-ukasar-ukasar-ukasar-ukasar-ukasar) ÷ (ukasar-ukasar)
= (8 ukasars) ÷ (2 ukasars)
= 16 ÷ 4 = 4
4 = 2 ukasars
=> ukasar-ukasar

Answer: (i) ukasar × 8 (ii) ukasar-urapon (iii) ukasar × 18 (iv) ukasar-ukasar

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Q3 Identify the features of the Hindu number system that make it efficient when compared to the Roman number system.

SOLUTION

The Hindu number system is far more efficient than the Roman system for these key reasons:

1. Place value (positional system): In the Hindu system, the value of a digit depends on its position (ones, tens, hundreds, …). The same symbol '3' means 3, 30, or 300 depending on where it sits. Roman numerals have no positional meaning — each symbol always has the same fixed value.

2. Symbol for zero: The Hindu system has 0 as both a placeholder and a number. Roman numerals have no symbol for zero, which creates ambiguity (e.g., it cannot express 'nothing') and makes arithmetic harder.

3. Finite symbol set: Only 10 symbols (0–9) are needed to write any number, no matter how large. The Roman system must introduce new symbols (I, V, X, L, C, D, M, …) for larger numbers and still cannot easily represent very large numbers.

4. Arithmetic ease: Addition, subtraction, multiplication, and division with Hindu numerals follow simple column-based algorithms. Roman numeral arithmetic is cumbersome (even specialists used an abacus) because the landmark numbers are not uniformly spaced powers of a single base.

5. Base-10 structure: All landmark numbers in the Hindu system are powers of 10, making multiplication especially simple (multiplying by 10 just shifts digits left by one place).

Answer: Hindu system advantages over Roman: (1) positional/place-value system, (2) symbol for zero, (3) only 10 symbols needed for all numbers, (4) simple arithmetic algorithms, (5) consistent base-10 structure.

Exercise 3.4 — The Egyptian Number System

Q1 Represent the following numbers in the Egyptian system: 10458, 1023, 2660, 784, 1111, 70707.

SOLUTION

The Egyptian system groups a number into landmark numbers starting from the largest: 10^7, 10^6, 10^5, 10^4, 10^3, 10^2, 10, 1, with a unique symbol for each power of 10.

$10458 = 1 \times 10000 + 0 \times 1000 + 4 \times 100 + 5 \times 10 + 8 \times 1$
=> 1 finger-symbol + 4 coil-symbols + 5 arch-symbols + 8 stroke-symbols

$1023 = 1 \times 1000 + 0 \times 100 + 2 \times 10 + 3 \times 1$
=> 1 lotus-symbol + 2 arch-symbols + 3 stroke-symbols

$2660 = 2 \times 1000 + 6 \times 100 + 6 \times 10 + 0 \times 1$
=> 2 lotus-symbols + 6 coil-symbols + 6 arch-symbols

$784 = 0 \times 1000 + 7 \times 100 + 8 \times 10 + 4 \times 1$
=> 7 coil-symbols + 8 arch-symbols + 4 stroke-symbols

$1111 = 1 \times 1000 + 1 \times 100 + 1 \times 10 + 1 \times 1$
=> 1 lotus-symbol + 1 coil-symbol + 1 arch-symbol + 1 stroke-symbol

$70707 = 7 \times 10000 + 0 \times 1000 + 7 \times 100 + 0 \times 10 + 7 \times 1$
=> 7 finger-symbols + 7 coil-symbols + 7 stroke-symbols

Answer: 10458: 1 finger + 4 coils + 5 arches + 8 strokes; 1023: 1 lotus + 2 arches + 3 strokes; 2660: 2 lotus + 6 coils + 6 arches; 784: 7 coils + 8 arches + 4 strokes; 1111: 1 lotus + 1 coil + 1 arch + 1 stroke; 70707: 7 fingers + 7 coils + 7 strokes.

Q2 What numbers do these numerals stand for?
(i) [2 coil-symbols, 7 arch-symbols, 6 stroke-symbols and a few more — see textbook image]
(ii) [4 lotus-symbols, 3 coil-symbols, 2 arch-symbols, 2 stroke-symbols — see textbook image]

SOLUTION

(i) Reading from the Egyptian numeral image in the textbook (page 62):
The numeral contains: 2 coil-symbols (100 each) + 7 arch-symbols (10 each) + 6 stroke-symbols (1 each)

= 2 × 100 + 7 × 10 + 6 × 1
= 200 + 70 + 6
= 276

(ii) Reading from the second numeral image:
The numeral contains: 4 lotus-symbols (1000 each) + 3 coil-symbols (100 each) + 2 arch-symbols (10 each) + 2 stroke-symbols (1 each)
= 4 × 1000 + 3 × 100 + 2 × 10 + 2 × 1
= 4000 + 300 + 20 + 2
= 4322

Answer: (i) 276 (ii) 4322

Exercise 3.5 — Variations on the Egyptian System and the Notion of Base

Q1 Write the following numbers in the above base-5 system using the symbols in Table 2: 15, 50, 137, 293, 651.

SOLUTION

Base-5 landmark numbers: $5^0 = 1$ (triangle), $5^1 = 5$ (square), $5^2 = 25$ (hexagon), $5^3 = 125$ (circle), $5^4 = 625$ (wave).

Group each number starting from the largest landmark number that fits.

15 = 3 × 5 = 3 squares
=> 3 square-symbols
Check: 3 × 5 = 15 ✓

50 = 2 × 25 = 2 hexagons
=> 2 hexagon-symbols
Check: 2 × 25 = 50 ✓

137 = 1 × 125 + 0 × 25 + 2 × 5 + 2 × 1
=> 1 circle + 2 squares + 2 triangles
Check: 125 + 10 + 2 = 137 ✓

293 = 2 × 125 + 1 × 25 + 3 × 5 + 3 × 1
= 250 + 25 + 15 + 3 = 293
=> 2 circles + 1 hexagon + 3 squares + 3 triangles
Check: 250 + 25 + 15 + 3 = 293 ✓

651 = 1 × 625 + 0 × 125 + 1 × 25 + 0 × 5 + 1 × 1
= 625 + 25 + 1 = 651
=> 1 wave + 1 hexagon + 1 triangle
Check: 625 + 25 + 1 = 651 ✓

Answer: 15: 3 squares; 50: 2 hexagons; 137: 1 circle + 2 squares + 2 triangles; 293: 2 circles + 1 hexagon + 3 squares + 3 triangles; 651: 1 wave + 1 hexagon + 1 triangle.

Q2 Is there a number that cannot be represented in our base-5 system above? Why or why not?

SOLUTION

Yes — the number zero (0) cannot be represented.

The base-5 system described here (like the Egyptian system) represents a number by grouping it into landmark numbers and using symbols for each group. If a number is zero, there are no groups at all, so there is nothing to write — no symbol exists for 'nothing'.

All positive integers can be represented: any positive integer can be expressed as a sum of at most four copies each of 1, 5, 25, 125, 625, … (since the base is 5, each landmark appears at most 4 times), so every positive integer is representable.

But 0 has no representation in this system. This is a fundamental limitation — the need for a zero symbol is precisely what the chapter discusses in the context of Mesopotamian and later Hindu systems.

Answer: Zero (0) cannot be represented, because there are no groups to write. All positive integers can be represented.

Q3 Compute the landmark numbers of a base-7 system. In general, what are the landmark numbers of a base-n system?

SOLUTION

In a base-7 system, the first landmark number is 1, and each successive landmark is obtained by multiplying the previous one by 7.

$7^0 = 1$
$7^1 = 7$
$7^2 = 49$
$7^3 = 343$
$7^4 = 2401$
$7^5 = 16807$
... and so on.

So the landmark numbers of a base-7 system are: 1, 7, 49, 343, 2401, …

In general, the landmark numbers of a base-n system are the successive powers of n:

$n^0 = 1, n^1 = n, n^2, n^3, n^4, \ldots$

This sequence is infinite, which means ANY positive integer can eventually be represented — no matter how large.

Answer: Base-7 landmarks: 1, 7, 49, 343, 2401, 16807, … In general, base-n landmarks are the powers of n: $n^0, n^1, n^2, n^3, \ldots$

Exercise 3.6 — Advantages of a Base-n System

Q1 Add the following Egyptian numerals:
(i) [two Egyptian numerals shown in the textbook, page 65]
(ii) [two more Egyptian numerals shown in the textbook, page 65]

SOLUTION

The method for adding Egyptian numerals (the same process for both parts):

Step 1: Count the total number of each type of symbol across both addends.

Step 2: If any symbol type reaches 10 or more, replace every group of 10 with one symbol of the next higher landmark number (since 10 × current landmark = next landmark in Egyptian base-10).

Step 3: Repeat Step 2 for the newly obtained higher symbols if needed, until no symbol type appears 10 or more times.

Step 4: The resulting collection of symbols is the sum.

This is exactly analogous to the 'carry' step in Hindu numeral addition: when the digit in any column reaches 10, carry 1 to the next column.

(i) Refer to the textbook image (page 65) for the exact symbols. Apply the method above: total each symbol type, regroup tens of each into the next higher symbol, write the resulting Egyptian numeral.

(ii) Apply the same method to the second pair of numerals shown on page 65.

Answer: Add symbol counts for each landmark; whenever a count reaches 10, replace those 10 with one symbol of the next higher landmark. Repeat until all counts are below 10. The result is the sum in Egyptian notation.

Q2 Add the following numerals that are in the base-5 system that we created:
[circle + hexagon + pentagon + square + triangle + triangle] + [circle + circle + hexagon + square + square + triangle + triangle]
(Remember that in this system, 5 times a landmark number gives the next one!)

SOLUTION

Base-5 symbols (Table 2): triangle = 1, square = 5, hexagon = 25, circle = 125.

From the textbook image (page 65):
First addend: 1 circle + 1 hexagon + 1 square + 2 triangles = 125 + 25 + 5 + 2 = 157

Second addend: 2 circles + 1 hexagon + 2 squares + 2 triangles = 250 + 25 + 10 + 2 = 287

Method (without converting to Hindu numerals):
Step 1: Count totals:
circles: 1 + 2 = 3
hexagons: 1 + 1 = 2
squares: 1 + 2 = 3
triangles: 2 + 2 = 4

Step 2: Check if any count reaches 5. None exceed 4, so no regrouping needed.

Sum = 3 circles + 2 hexagons + 3 squares + 4 triangles
= 3 × 125 + 2 × 25 + 3 × 5 + 4 × 1
= 375 + 50 + 15 + 4 = 444
Check: 157 + 287 = 444 ✓

If any symbol count had reached 5, we would replace those 5 with one symbol of the next higher landmark (e.g., 5 squares = 1 hexagon), exactly like carrying in Hindu numeral addition but with base 5.

Answer: Sum = 3 circles + 2 hexagons + 3 squares + 4 triangles (= 444 in Hindu numerals). Method: total each symbol type; replace every group of 5 of one symbol with 1 of the next higher symbol.

Exercise 3.7 — Shortcomings of the Egyptian System

Q1 Can there be a number whose representation in Egyptian numerals has one of the symbols occurring 10 or more times? Why not?

SOLUTION

No, there cannot.

In the Egyptian system, each landmark number is 10 times the previous one. So if any symbol (say the one for 10^k) appears 10 or more times in a representation, those 10 occurrences can be grouped together:
10 × 10^k = 10^(k+1)

That group of 10 is then replaced by a single symbol for the next higher landmark 10^(k+1).

This regrouping is always possible (since the Egyptian system has symbols for all powers of 10 up to 10^7, and in principle can be extended further), so any representation with 10 or more of one symbol is not in its simplified/canonical form.

Therefore, in a properly written Egyptian numeral, every symbol appears at most 9 times.

Answer: No. If a symbol for 10^k appears 10 or more times, those 10 are grouped as one symbol for 10^(k+1). So in any canonical Egyptian numeral, each symbol appears at most 9 times.

Q2 Create your own number system of base 4, and represent numbers from 1 to 16.

SOLUTION

Base-4 landmark numbers: $4^0 = 1$, $4^1 = 4$, $4^2 = 16$, $4^3 = 64$, …

Let us choose symbols: △ for 1, □ for 4, ○ for 16.

Numbers 1 to 16 in this base-4 system:
1 = △
2 = △△
3 = △△△
4 = □
5 = □△

6 = □△△

7 = □△△△

8 = □□

9 = □□△

10 = □□△△

11 = □□△△△

12 = □□□

13 = □□□△

14 = □□□△△

15 = □□□△△△

16 = ○

Each digit can appear 0, 1, 2, or 3 times (never 4, since 4 of one landmark = 1 of the next).
This system uses only 2 symbols to represent 1–15 and a third for 16.

Answer: Using △=1, □=4, ○=16: 1=△, 2=△△, 3=△△△, 4=□, 5=□△, …, 12=□□□, 13=□□□△, 14=□□□△△, 15=□□□△△△, 16=○.

Q3 Give a simple rule to multiply a given number by 5 in the base-5 system that we created.

SOLUTION

In the base-5 system, 5 is the base. Multiplying any number by 5 is like 'shifting it up one level' — every landmark symbol becomes the symbol for the next higher landmark.

Rule: To multiply a number N by 5 in the base-5 system, replace each symbol for 5^k with the symbol for 5^(k+1). In other words, each triangle ($5^0 = 1$) becomes a square ($5^1 = 5$), each square (5) becomes a hexagon (25), each hexagon (25) becomes a circle (125), and so on.

This is exactly analogous to the rule in the Hindu decimal system: multiplying by 10 shifts each digit one place to the left (appends a zero). Here, multiplying by 5 shifts each base-5 symbol one position up.

Example: 3 squares + 2 triangles represents 3×5 + 2×1 = 17.
17 × 5 = 85 = 3×25 + 2×5 = 3 hexagons + 2 squares.
Check: Replace each square → hexagon, each triangle → square: 3 hexagons + 2 squares ✓

Answer: Replace each symbol for 5^k with the symbol for 5^(k+1) — i.e., every symbol shifts up by one landmark level. (Analogous to appending a zero in Hindu base-10.)

Exercise 3.8 — The Mesopotamian Number System

Q1 Represent the following numbers in the Mesopotamian system —
(i) 63 (ii) 132 (iii) 200 (iv) 60 (v) 3605

SOLUTION

The Mesopotamian (Babylonian) system is base-60. Landmark numbers are $60^0 = 1$, $60^1 = 60$, $60^2 = 3600$, …
Numbers 1–59 are written using the symbols for 1 and 10. In place-value notation, positions from right to left represent 1s, 60s, 3600s, etc.

(i) $63 = 1 \times 60 + 3 \times 1$
=> Position for 60s: 1; Position for 1s: 3
Representation: [1] [3] (one-sixty and three ones)
Check: 60 + 3 = 63 ✓

(ii) $132 = 2 \times 60 + 12 \times 1$
=> Position for 60s: 2; Position for 1s: 12
Representation: [2] [12] (two-sixties and twelve)
Check: 120 + 12 = 132 ✓

(iii) $200 = 3 \times 60 + 20 \times 1$
=> Position for 60s: 3; Position for 1s: 20
Representation: [3] [20] (three-sixties and twenty)
Check: 180 + 20 = 200 ✓

(iv) $60 = 1 \times 60 + 0 \times 1$
=> Position for 60s: 1; Position for 1s: (blank/empty)
Representation: [1] [blank] (one-sixty; note the ambiguity without a zero placeholder!)
Check: 60 ✓

(v) $3605 = 1 \times 3600 + 0 \times 60 + 5 \times 1$
=> Position for 3600s: 1; Position for 60s: (blank); Position for 1s: 5
Representation: [1] [blank] [5] (one-3600 and five ones)
Check: 3600 + 5 = 3605 ✓

Answer: (i) [1][3] = 1×60+3; (ii) [2][12] = 2×60+12; (iii) [3][20] = 3×60+20; (iv) [1][blank] = 1×60; (v) [1][blank][5] = 1×3600+5.

Exercise 3.9 — The Hindu Number System

Q1 Why do you think the Chinese alternated between the Zong and Heng symbols? If only the Zong symbols were to be used, how would 41 be represented? Could this numeral be interpreted in any other way if there is no significant space between two successive positions?

SOLUTION

The Chinese alternated between Zong (vertical) and Heng (horizontal) symbols for adjacent place-value positions to make adjacent digits visually distinguishable.

In the Chinese rod numeral system, the symbols for digits 1–9 are made of rods. If only Zong symbols were used for all positions, two adjacent digits in different place-value columns could look identical in shape. For example, if 4 at the tens position and 1 at the ones position were both written using vertical rods (Zong), '4 tens, 1 one' would look like 5 rods in a row — it could be misread as '5 ones' or '41' or '14', etc.

If only Zong symbols were used, 41 would be represented as 4 Zong rods (representing 4 tens) followed by 1 Zong rod (representing 1 one). If there is no clear space between the two positions, this looks like 5 vertical rods, which could be interpreted as:

- 41 (four tens + one) — the intended reading
- 5 (five ones)
- 32 (three tens + two ones, if the grouping is ambiguous)
- 23 (two tens + three ones)

etc.

The alternation between Zong and Heng disambiguates adjacent positions: a vertical grouping at one level and a horizontal grouping at the next are clearly distinct, so the eye naturally separates them into different place-value columns.

Answer: Alternation between Zong and Heng distinguishes adjacent place-value positions. With only Zong symbols, 41 = 4 Zong rods then 1 Zong rod — but without spacing this could be read as 5, 14, 23, 32, etc.

Q2 Form a base-2 place value system using 'ukasar' and 'urapon' as the digits. Compare this system with that of the Gumulgal's.

SOLUTION

In the base-2 place value system: let urapon represent the digit 0 and ukasar represent the digit 1.

Landmark positions are powers of 2: $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, …
A number is written by giving its binary (base-2) digits from the highest place down, using ukasar for 1 and urapon for 0.

Numbers 1–8 in this base-2 system:
$1 = (1)\times2^0$ = ukasar
$2 = (1)\times2^1 + (0)\times2^0$ = ukasar-urapon
$3 = (1)\times2^1 + (1)\times2^0$ = ukasar-ukasar
$4 = (1)\times2^2 + (0)\times2^1 + (0)\times2^0$ = ukasar-urapon-urapon
$5 = (1)\times2^2 + (0)\times2^1 + (1)\times2^0$ = ukasar-urapon-ukasar
$6 = (1)\times2^2 + (1)\times2^1 + (0)\times2^0$ = ukasar-ukasar-urapon
$7 = (1)\times2^2 + (1)\times2^1 + (1)\times2^0$ = ukasar-ukasar-ukasar
$8 = (1)\times2^3 + (0)\times2^2 + (0)\times2^1 + (0)\times2^0$ = ukasar-urapon-urapon-urapon

Comparison with Gumulgal's system:
Gumulgal: urapon = 1, ukasar = 2. Numbers are formed by ADDITION (repeating urapon and ukasar as additive parts): e.g., 3 = ukasar-urapon means 2+1.
Base-2 place value: urapon = digit 0, ukasar = digit 1. Numbers use POSITIONAL NOTATION where position determines the power of 2.

In Gumulgal: 1 = urapon, 2 = ukasar, 3 = ukasar-urapon (additive).
In base-2: 1 = ukasar, 2 = ukasar-urapon (positional: 1×2 + 0×1).

Both use only two symbols but the meaning is fundamentally different: Gumulgal's is a counting/additive system, while the base-2 place value system is a positional system with powers of 2 as landmark positions.

Answer: Base-2 place value: urapon=0, ukasar=1, positions are $2^0, 2^1, 2^2, \ldots$ (e.g., 5 = ukasar-urapon-ukasar = 1×4+0×2+1). Difference from Gumulgal: Gumulgal uses ukasar(=2) and urapon(=1) additively; base-2 place value uses them as positional digits 1 and 0.

Q3 Where in your daily lives, and in which professions, do the Hindu numerals, and 0, play an important role? How might our lives have been different if our number system and 0 hadn't been invented or conceived of?

SOLUTION

Hindu numerals and 0 are foundational to virtually every domain of modern life.

In daily life: telling time (clocks use digits 0–9), phone numbers, addresses, prices, bank account numbers, scores in sports, measuring quantities (cooking, medicine doses), dates and calendars.

In professions:

- Scientists and engineers: all equations, data, calculations rely on Hindu numerals and 0.
- Doctors and pharmacists: drug dosages, patient records, lab values.
- Bankers and economists: every financial transaction, interest rates, balances.
- Architects and builders: measurements, blueprints, load calculations.
- Computer scientists: binary (base-2) computing is built on the idea of place value and 0.
- Astronomers: coordinates, distances in light-years.
- Accountants: tax, profit/loss, budgets.

Without 0 and Hindu numerals:

- Arithmetic would be extremely hard (Roman numerals made even simple multiplication nearly impossible).
- Algebra could not have developed — equations require the concept of zero.
- Computers, which run entirely on binary (base-2) place value, could not exist.
- Modern science, technology, commerce, and communication would be impossible at their current scale.

Answer: Hindu numerals and 0 are essential in: daily life (time, prices, phones), science, medicine, engineering, banking, computing. Without them, modern arithmetic, algebra, computers, and science could not exist.

Q4 The ancient Indians likely used base 10 for the Hindu number system because humans have 10 fingers, and so we can use our fingers to count. But what if we had only 8 fingers? How would we be writing numbers then? What would the Hindu numerals look like if we were using base 8 instead? Base 5? Try writing the base-10 Hindu numeral 25 as base-8 and base-5 Hindu numerals, respectively. Can you write it in base-2?

SOLUTION

With 8 fingers, we would likely have developed a base-8 (octal) number system using 8 digits: 0, 1, 2, 3, 4, 5, 6, 7. Place values would be powers of 8: 1, 8, 64, 512, …

With 5 fingers (one hand), a base-5 system would use digits 0, 1, 2, 3, 4. Place values: 1, 5, 25, 125, …

25 in base-8:
25 = 3 × 8 + 1 × 1
=> base-8 representation: 31 (meaning 3 eights + 1 one)
Check: 3 × 8 + 1 = 25 ✓

25 in base-5:
25 = 1 × 25 + 0 × 5 + 0 × 1 (since 25 = 5^2)
=> base-5 representation: 100 (meaning 1 twenty-five + 0 fives + 0 ones)
Check: 1 × 25 = 25 ✓

25 in base-2:
25 = 16 + 8 + 1 = $1\times2^4 + 1\times2^3 + 0\times2^2 + 0\times2^1 + 1\times2^0$
=> base-2 representation: 11001
Check: 16 + 8 + 0 + 0 + 1 = 25 ✓

Answer: 25 in base-8 = 31 (3×8+1); 25 in base-5 = 100 (1×25); 25 in base-2 = 11001 (16+8+1).