Chapter 4: Quadrilaterals

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Exercise 4.1 — Rectangles and Squares

Q1 Find all the other angles inside the following rectangles.
(i) Rectangle with vertices D(top-left), C(top-right), A(bottom-left), B(bottom-right). The diagonal AC makes an angle of 30° with the base AB at vertex A, i.e. ∠CAB = 30°.
(ii) Rectangle with vertices Q(top-left), R(top-right), P(bottom-left), S(bottom-right). The diagonals QS and RP meet at O, and the angle at O opening towards the top side QR is 110°, i.e. ∠QOR = 110°.

SOLUTION

Part (i) — Rectangle DCBA, ∠CAB = 30°:

Vertices: D(top-left), C(top-right), A(bottom-left), B(bottom-right).
All angles of a rectangle = 90°. Diagonals AC and BD are equal and bisect each other at O.
The 30° in the figure is at vertex A, between the diagonal AC and the base AB, so ∠CAB = 30°.

At vertex A (∠DAB = 90°): ∠CAD = 90° − ∠CAB = 90° − 30° = 60°.

Since AB ∥ DC, the diagonal AC is a transversal, so alternate interior angles give ∠ACD = ∠CAB = 30°, and ∠ACB = 90° − ∠ACD = 60° (∠DCB = 90° at vertex C).

Find ∠ABD using the diagonals. Diagonals of a rectangle are equal and bisect each other, so OA = OB.
Triangle OAB is isosceles, so ∠OBA = ∠OAB = ∠CAB = 30°; hence ∠ABD = 30°.
At vertex B (∠ABC = 90°): ∠DBC = 90° − ∠ABD = 90° − 30° = 60°.
Since AB ∥ DC with BD a transversal, ∠BDC = ∠ABD = 30° (alternate interior angles).
At vertex D (∠ADC = 90°): ∠ADB = 90° − ∠BDC = 90° − 30° = 60°.

Summary for (i):

∠CAB = 30°, ∠CAD = 60°, ∠ACB = 60°, ∠ACD = 30°,
∠ABD = 30°, ∠DBC = 60°, ∠ADB = 60°, ∠BDC = 30°.

Part (ii) — Rectangle QRSP, ∠QOR = 110°:

Vertices: Q(top-left), R(top-right), P(bottom-left), S(bottom-right). Diagonals QS and RP intersect at O.
Diagonals of a rectangle are equal and bisect each other: OQ = OR = OP = OS.
The 110° in the figure is the angle at O opening towards the top side QR, so ∠QOR = 110°.

Vertically opposite: ∠POS = 110°.
Adjacent supplementary angles: ∠QOP = ∠ROS = 180° − 110° = 70°.

Triangle OQR (isosceles, OQ = OR), apex ∠QOR = 110°:
∠OQR = ∠ORQ = (180° − 110°) ÷ 2 = 35°.

Triangle OQP (isosceles, OQ = OP), apex ∠QOP = 70°:
∠OQP = ∠OPQ = (180° − 70°) ÷ 2 = 55°.

Triangle ORS (isosceles, OR = OS), apex ∠ROS = 70°:
∠ORS = ∠OSR = (180° − 70°) ÷ 2 = 55°.

Triangle OPS (isosceles, OP = OS), apex ∠POS = 110°:
∠OPS = ∠OSP = (180° − 110°) ÷ 2 = 35°.

Verify rectangle corner angles: at Q = ∠OQR + ∠OQP = 35° + 55° = 90° ✓; at R = 35° + 55° = 90° ✓.

Summary for (ii):
∠QOR = ∠POS = 110°, ∠QOP = ∠ROS = 70°.
∠OQR = ∠ORQ = 35°, ∠OQP = ∠OPQ = 55°,
∠ORS = ∠OSR = 55°, ∠OPS = ∠OSP = 35°.

Answer: (i) ∠CAB = 30°, ∠CAD = 60°, ∠ACB = 60°, ∠ACD = 30°, ∠ABD = 30°, ∠DBC = 60°, ∠ADB = 60°, ∠BDC = 30°. (ii) ∠QOR = ∠POS = 110°; ∠QOP = ∠ROS = 70°; ∠OQR = ∠ORQ = 35°, ∠OQP = ∠OPQ = 55°, ∠ORS = ∠OSR = 55°, ∠OPS = ∠OSP = 35° (each 90° corner is split by the diagonal into 35° and 55°).

Q2 Draw a quadrilateral whose diagonals have equal lengths of 8 cm that bisect each other, and intersect at an angle of (i) 30° (ii) 40° (iii) 90° (iv) 140°.

SOLUTION

A quadrilateral whose diagonals are equal, bisect each other is a rectangle (if diagonals are at 90° it becomes a square).

Construction method (same for all four cases):
Step 1: Draw a line segment AB = 8 cm.
Step 2: Mark the midpoint O of AB.
Step 3: At O, draw line l at the required angle (30°, 40°, 90°, or 140°) to AB.
Step 4: On line l, mark points C and D such that OC = OD = 4 cm (so CD = 8 cm).
Step 5: Join AC, CB, BD, and DA.
Quadrilateral ACBD (or ADBC) is the required figure.

(i) 30°: Diagonals AB and CD, each 8 cm, bisect each other at O with angle 30° between them.
(ii) 40°: Same construction with 40° at O.
(iii) 90°: When the angle is 90°, the quadrilateral ACBD is a square (diagonals equal, perpendicular, bisecting each other).
(iv) 140°: Same construction with 140° at O.

Note: For angles other than 90°, the resulting quadrilateral is a rectangle (if diagonals are along the perpendicular bisectors of each other and equal).

Answer: Draw AB = 8 cm, mark midpoint O, draw line l at the given angle at O, mark C and D on l with OC = OD = 4 cm, join AC, CB, BD, DA. The quadrilateral ACBD has diagonals of 8 cm bisecting each other at the required angle.

Q3 Consider a circle with centre O. Line segments PL and AM are two perpendicular diameters of the circle. What is the figure APML? Reason and/or experiment to figure this out.

SOLUTION

PL and AM are two diameters of the circle with centre O, perpendicular to each other.
So O is the midpoint of both PL and AM, and PL ⊥ AM.

Step 1 — All sides are equal:
A, P, M, L are all on the circle, so AP = PM = ML = LA = chord subtending 90° arc.

(Each side subtends an arc of 90° since the two diameters divide the circle into 4 equal arcs.)
By the chord-length formula, all four chords are equal: AP = PM = ML = LA = r√2.

Step 2 — Diagonals are equal and perpendicular:
The diagonals of APML are AM and PL — these are the two given diameters.
AM = PL = 2r (both are diameters, hence equal).
They bisect each other at O (centre = midpoint of each diameter).
They are perpendicular (given).

Step 3 — All angles are 90°:
At vertex A: diagonal AM bisects the angle at A.
In triangle OAP: OA = OP = r, angle AOP = 90° => ∠OAP = ∠OPA = 45°.
Similarly ∠OAL = 45°. So ∠PAL = 45° + 45° = 90°.
By symmetry, all four angles of APML are 90°.

Conclusion: APML has all four sides equal (= r√2), all four angles = 90°, and diagonals equal and perpendicular bisecting each other.
Therefore APML is a SQUARE.

Answer: APML is a square. All four sides are equal (each = r√2), all four angles = 90°, and the diagonals AM and PL are equal diameters that bisect each other perpendicularly at the centre O.

Q4 We have seen how to get 90° using paper folding. Now, suppose we do not have any paper but two sticks of equal length, and a thread. How do we make an exact 90° using these?

SOLUTION

This is a Math Talk problem (open reasoning/discussion), but the answer has a determinate method:

Step 1: Take the two sticks of equal length and cross them at their midpoints.
Step 2: Thread the four endpoints together to form a quadrilateral.

Why this gives 90°:
The two sticks form the diagonals of the quadrilateral.
Since the sticks are equal (diagonals are equal) and we cross them at their midpoints (diagonals bisect each other), the quadrilateral is a rectangle.

For the diagonals to be perpendicular, we adjust the angle at the crossing point.

When the two equal sticks cross at their midpoints AND at right angles to each other, the four endpoints form a SQUARE — every angle = 90°.

To achieve exactly 90°: balance the sticks so that all four segments from the centre to the endpoints are equal (which is guaranteed since each stick is equal, and we use the midpoints). Then tighten the thread until the crossing angle is 90° — this is confirmed when all four sides of the formed quadrilateral are equal, making it a square.

Answer: Cross the two equal-length sticks at their midpoints and connect the four endpoints with thread. The diagonals (sticks) being equal and bisecting each other makes it a rectangle. Adjusting the crossing angle to 90° makes it a square, giving exact 90° angles.

Q5 We saw that one of the properties of a rectangle is that its opposite sides are parallel. Can this be chosen as a definition of a rectangle? In other words, is every quadrilateral that has opposite sides parallel and equal, a rectangle?

SOLUTION

This is a Try This problem. The question asks whether 'opposite sides parallel and equal' is sufficient to define a rectangle.

Answer: NO — this condition alone is not sufficient to define a rectangle.

A parallelogram has opposite sides parallel and equal by definition, but a parallelogram need NOT have right angles.
For example, a rhombus (not a square) has all sides equal (so opposite sides equal) and opposite sides parallel, but its angles are not 90°.
More generally, any non-rectangular parallelogram has opposite sides parallel and equal, yet is not a rectangle.

Therefore, 'opposite sides parallel and equal' describes a PARALLELOGRAM, which is a broader set than rectangles.
A rectangle requires the additional condition that all angles are 90° (or equivalently, that the diagonals are equal).

Answer: No. A quadrilateral with opposite sides parallel and equal is a parallelogram, which need not be a rectangle. An additional condition (all angles = 90°, or diagonals equal) is needed to guarantee a rectangle.

Exercise 4.2 — More Quadrilaterals with Parallel Opposite Sides (Parallelograms and Rh

Q1 Find the remaining angles in the following quadrilaterals.
(i) Parallelogram PEAR: ∠P = 40°.
(ii) Parallelogram SPQR: ∠P = 110°.
(iii) Rhombus XWVU: the diagonal at V makes 30° with each side (∠XVW = ∠XVU = 30°, so full ∠V = 60°).
(iv) Rhombus OIAE: the diagonal at E makes 20° with each side (∠OEI = 20°, so full ∠E = 40°).

SOLUTION

Part (i) — Parallelogram PEAR, ∠P = 40°:
In a parallelogram, adjacent angles are supplementary and opposite angles are equal.
∠P + ∠E = 180° => ∠E = 180° − 40° = 140°.
∠A = ∠P = 40° (opposite angles).
∠R = ∠E = 140° (opposite angles).
Check: 40° + 140° + 40° + 140° = 360° ✓

Part (ii) — Parallelogram SPQR, ∠P = 110°:
Going around S-P-Q-R: adjacent angles are supplementary.
∠P + ∠Q = 180° => ∠Q = 70°.
∠R = ∠P = 110° (opposite angles).
∠S = ∠Q = 70° (opposite angles).
Check: 110° + 70° + 110° + 70° = 360° ✓

Part (iii) — Rhombus XWVU, 30° marked at V:
In a rhombus, the diagonals bisect the vertex angles.
The 30° shown is the angle between the diagonal XV and side VW at vertex V.
Since the diagonal bisects angle V: ∠XVW = ∠XVU = 30° => full ∠V = 60°.
Opposite angle: ∠X = ∠V = 60°.
Adjacent angles: ∠W = ∠U = 180° − 60° = 120°.
Check: 60° + 120° + 60° + 120° = 360° ✓

Part (iv) — Rhombus OIAE, 20° marked at E:
The 20° shown is the angle between diagonal IE and side EO at vertex E.
Diagonal bisects angle E: ∠OEI = ∠IEA = 20° => full ∠E = 40°.
In rhombus OIAE (vertices in order), I and E are opposite: ∠I = ∠E = 40°.
Adjacent angles: ∠O = ∠A = 180° − 40° = 140°.

Check: 40° + 140° + 40° + 140° = 360° ✓

Answer: (i) ∠E = 140°, ∠A = 40°, ∠R = 140°. (ii) ∠Q = 70°, ∠R = 110°, ∠S = 70°. (iii) ∠V = 60°, ∠X = 60°, ∠W = ∠U = 120°. (iv) ∠E = 40°, ∠I = 40°, ∠O = ∠A = 140°.

Q2 Using the diagonal properties, construct a parallelogram whose diagonals are of lengths 7 cm and 5 cm, and intersect at an angle of 140°.

SOLUTION

In a parallelogram, diagonals bisect each other.
So each diagonal is divided into two equal halves at the intersection point O.

Step 1: Draw a line segment AB = 7 cm (one diagonal).
Step 2: Mark the midpoint O of AB.
Step 3: At O, draw a ray making an angle of 140° with AB.
Step 4: On this ray (line l), mark points C and D on opposite sides of O such that OC = OD = 2.5 cm (so CD = 5 cm, the second diagonal).
Step 5: Join AC, CB, BD, and DA.
ADBC (or ACBD) is the required parallelogram with diagonals AB = 7 cm and CD = 5 cm intersecting at 140°.

The four angles between the diagonals at O are: 140°, 40°, 140°, 40° (supplementary pairs).

Answer: Draw diagonal AB = 7 cm, mark midpoint O, draw line l at 140° at O, mark C and D on l with OC = OD = 2.5 cm, join AC, CB, BD, DA. Quadrilateral ACBD is the required parallelogram.

Q3 Using the diagonal properties, construct a rhombus whose diagonals are of lengths 4 cm and 5 cm.

SOLUTION

In a rhombus, the diagonals bisect each other at right angles (90°).

Step 1: Draw a line segment AB = 5 cm (one diagonal).
Step 2: Mark the midpoint O of AB.
Step 3: At O, draw a line perpendicular to AB (i.e., at 90°).
Step 4: On this perpendicular, mark points C and D on opposite sides of O such that OC = OD = 2 cm (so CD = 4 cm, the second diagonal).
Step 5: Join AC, CB, BD, and DA.

ADBC is the required rhombus with diagonals of 5 cm and 4 cm.

Verification: All four sides equal?
AC = sqrt($OA^2 + OC^2$) = sqrt($2.5^2 + 2^2$) = sqrt(6.25 + 4) = sqrt(10.25) ≈ 3.2 cm.
All four sides = sqrt(10.25) cm ✓ (all equal, confirming it is a rhombus).

Answer: Draw diagonal AB = 5 cm, mark midpoint O, draw perpendicular at O, mark C and D on the perpendicular with OC = OD = 2 cm, join AC, CB, BD, DA. ADBC is the required rhombus.

Exercise 4.3 — Kite and Trapezium

Q1 Find all the sides and the angles of the quadrilateral obtained by joining two equilateral triangles with sides 4 cm.

SOLUTION

Take two equilateral triangles PQR and PQS, each with sides 4 cm, joined along the common side PQ.
The resulting quadrilateral has vertices R, Q, S, P (going around).

Sides:
All four sides of the quadrilateral = the sides of the equilateral triangles = 4 cm each.
So it is a rhombus with all sides = 4 cm.

Angles:
Each interior angle of an equilateral triangle = 60°.
At vertex R (or S): only one triangle contributes => angle R = angle S = 60°.
At vertex P (or Q): two triangles meet, each contributing 60° on the same side of PQ.
angle at P = angle RPQ + angle SPQ = 60° + 60° = 120°.
angle at Q = angle RQP + angle SQP = 60° + 60° = 120°.

Check: 60° + 120° + 60° + 120° = 360° ✓

The quadrilateral RPSQ (or RQSP) is a rhombus with:
All sides = 4 cm, angles at R and S = 60°, angles at P and Q = 120°.

Answer: The quadrilateral is a rhombus with all sides = 4 cm; angles at R and S = 60°, angles at P and Q = 120°.

Q2 Construct a kite whose diagonals are of lengths 6 cm and 8 cm.

SOLUTION

In a kite, one diagonal (the axis of symmetry) perpendicularly bisects the other diagonal.
Here take the 8 cm diagonal PQ as the axis of symmetry; it perpendicularly bisects the 6 cm diagonal RS.

Step 1: Draw the diagonal PQ = 8 cm (the axis of symmetry).
Step 2: Choose the point T on PQ where the second diagonal will cross. For a symmetric figure take T at the midpoint of PQ, i.e. PT = TQ = 4 cm.

Step 3: At T, draw a line perpendicular to PQ.
Step 4: On this perpendicular, mark R and S on opposite sides of PQ with RT = TS = 3 cm (so RS = 6 cm; PQ bisects RS at right angles).
Step 5: Join PR, RQ, QS, and SP.
PRQS is the required kite with diagonals PQ = 8 cm and RS = 6 cm.

Verification: PQ is the perpendicular bisector of RS, so PR = PS and QR = QS — the kite property (two pairs of adjacent equal sides) holds.

Answer: Draw PQ = 8 cm, mark midpoint T, draw perpendicular at T, mark R and S on it with RT = TS = 3 cm, join PR, RQ, QS, SP. PRQS is the required kite with diagonals 8 cm and 6 cm.

Q3 Find the remaining angles in the following trapeziums —
(i) A trapezium with bottom angles 135° and 105°.
(ii) An isosceles trapezium with one angle of 100°.

SOLUTION

Part (i) — Trapezium with top ∥ bottom (one pair of parallel sides):
Given bottom-left angle = 135°, bottom-right angle = 105°.

Since the top side ∥ bottom side, co-interior (same-side interior) angles on the same transversal are supplementary.
Left transversal: bottom-left + top-left = 180°
=> top-left = 180° − 135° = 45°.
Right transversal: bottom-right + top-right = 180°
=> top-right = 180° − 105° = 75°.

Check: 135° + 105° + 45° + 75° = 360° ✓

Part (ii) — Isosceles trapezium with ∠ = 100° at one of the top vertices:
In an isosceles trapezium, the two legs (non-parallel sides) are equal.
Property: angles adjacent to the same parallel side are equal.
So the two top angles are equal: both = 100°.

Since top ∥ bottom: co-interior angles are supplementary.
Bottom angle = 180° − 100° = 80°.
Both bottom angles = 80° (equal in isosceles trapezium).

Check: 100° + 100° + 80° + 80° = 360° ✓

Answer: (i) Top-left = 45°, top-right = 75°. (ii) Both top angles = 100°, both bottom angles = 80°.

Q4 Draw a Venn diagram showing the set of parallelograms, kites, rhombuses, rectangles, and squares. Then answer:
(i) What is the quadrilateral that is both a kite and a parallelogram?
(ii) Can there be a quadrilateral that is both a kite and a rectangle?
(iii) Is every kite a rhombus? If not, what is the correct relationship between these two types of quadrilaterals?

SOLUTION

Venn diagram structure (from outermost to innermost):
- Parallelogram (largest set)
  - Rectangle (subset of Parallelogram)
    - Square (subset of both Rectangle and Rhombus)
  - Rhombus (subset of Parallelogram)
    - Square (at intersection of Rectangle and Rhombus)
- Kite (overlaps with Rhombus at Square; mostly separate from Parallelogram)

Part (i) — Quadrilateral that is both a kite and a parallelogram:
A kite has two pairs of adjacent equal sides (AB = BC, CD = DA).
A parallelogram has two pairs of opposite equal sides (AB = CD, BC = DA).
For both conditions to hold: AB = BC = CD = DA — all sides equal.
A quadrilateral with all sides equal AND both pairs of opposite sides parallel is a RHOMBUS (and specifically a SQUARE when angles = 90°).
Answer: A rhombus (and its special case, a square) is both a kite and a parallelogram.

Part (ii) — Can a quadrilateral be both a kite and a rectangle?
A rectangle has all angles = 90°. A kite that is also a parallelogram must be a rhombus.
A rhombus that is also a rectangle has all sides equal AND all angles = 90° => it is a SQUARE.
So the only quadrilateral that is both a kite and a rectangle is a SQUARE.
Answer: Yes, but only a square can be both a kite and a rectangle.

Part (iii) — Is every kite a rhombus?
No. A general kite has sides AB = BC and CD = DA (two pairs of adjacent equal sides), but AB need not equal CD.

A rhombus requires ALL four sides to be equal.
So a kite is a rhombus only when all its sides are equal (i.e., when it is actually a rhombus).
Correct relationship: Every rhombus is a kite, but NOT every kite is a rhombus. (Rhombus is a special case of kite with all sides equal.)

Answer: (i) A rhombus (and square) is both a kite and a parallelogram. (ii) Only a square can be both a kite and a rectangle. (iii) No; every rhombus is a kite, but a kite need not be a rhombus (a rhombus is a special kite with all sides equal).

Q5 If PAIR and RODS are two rectangles, find ∠IOD. (In rectangle PAIR: P(top-left), A(top-right), I(bottom-right), R(bottom-left); the left side PR = 5 cm. The vertex O of rectangle RODS lies on the side AI, and RO makes an angle of 30° with the base RI, i.e. ∠ORI = 30°. In rectangle RODS the vertices are R, O, D, S in order, with the side RS = 5 cm.)

SOLUTION

Rectangle PAIR: P(top-left), A(top-right), I(bottom-right), R(bottom-left).
The vertex O of rectangle RODS sits on the side AI of PAIR, and ∠ORI = 30° is given (angle at R between RO and the base RI).

Step 1: Find ∠ROI in triangle ROI.
Since PAIR is a rectangle, its side AI is perpendicular to its side RI, so ∠OIR = 90° (O lies on AI, directly above I).
In right triangle ROI: ∠ROI = 90° − ∠ORI = 90° − 30° = 60°.

Step 2: Use the right angle of rectangle RODS at O.
In rectangle RODS the vertices go R → O → D → S, so the sides RO and OD meet at O at a right angle: ∠ROD = 90°.

Step 3: Find ∠IOD.
The ray OI lies inside ∠ROD (O–I is between O–R and O–D), so
∠IOD = ∠ROD − ∠ROI = 90° − 60° = 30°.

Therefore ∠IOD = 30°.

Answer: ∠IOD = 30°. (Since AI ⊥ RI in rectangle PAIR, ∠ROI = 90° − 30° = 60°; since ∠ROD = 90° in rectangle RODS, ∠IOD = 90° − 60° = 30°.)

Q6 Construct a square with diagonal 6 cm without using a protractor.

SOLUTION

In a square, the diagonals are equal, perpendicular to each other, and bisect each other.

Step 1: Draw a line segment AB = 6 cm (one diagonal).
Step 2: Find the midpoint O of AB using a compass (draw arcs of equal radius from A and B; the intersection points give the perpendicular bisector; mark O where it crosses AB).
Step 3: Draw the perpendicular bisector of AB through O (this is done with compass only — no protractor needed).
Step 4: On this perpendicular bisector, mark points C and D on opposite sides of O such that OC = OD = 3 cm (half of 6 cm).
Step 5: Join A-C, C-B, B-D, and D-A.
ACBD is the required square with diagonal 6 cm.

Verification: All sides = sqrt($3^2 + 3^2$) = sqrt(18) = $3\sqrt{2}$ cm (equal). All angles = 90° (diagonals perpendicular). ✓

Answer: Draw AB = 6 cm, find midpoint O with compass, draw perpendicular bisector at O (no protractor), mark C and D on it with OC = OD = 3 cm, join AC, CB, BD, DA. Square ACBD has all sides = $3\sqrt{2}$ cm and all angles = 90°.

Q7 CASE is a square. The points U, V, W and X are the midpoints of the sides of the square. What type of quadrilateral is UVWX? Find this by using geometric reasoning, as well as by construction and measurement.

SOLUTION

Let CASE be a square with side length s. From the figure the vertices are C(top-left), A(top-right), S(bottom-right), E(bottom-left).
Midpoints (as marked in the figure): U = midpoint of CA (top side), V = midpoint of CE (left side), W = midpoint of ES (bottom side), X = midpoint of AS (right side).

Set up coordinates with E at the origin: E = (0, 0), S = (s, 0), A = (s, s), C = (0, s).
Then U = (s/2, s), V = (0, s/2), W = (s/2, 0), X = (s, s/2).

Step 1 — Find length of UV:
$UV^2 = (s/2 - 0)^2 + (s - s/2)^2 = (s/2)^2 + (s/2)^2 = s^2/4 + s^2/4 = s^2/2$.
$UV = s/\sqrt{2} = s\sqrt{2}/2$.

By symmetry: UV = VW = WX = XU = s√2/2. All four sides are equal.

Step 2 — Check angles:
Vector VU = U − V = (s/2 − 0, s − s/2) = (s/2, s/2).
Vector VW = W − V = (s/2 − 0, 0 − s/2) = (s/2, −s/2).
Dot product: (s/2)(s/2) + (s/2)(−s/2) = s²/4 − s²/4 = 0.
=> VU ⊥ VW, so ∠UVW = 90°.
By symmetry, all four angles = 90°.

Conclusion: UVWX has all four sides equal (= s√2/2) and all four angles = 90°.
Therefore UVWX is a SQUARE.

For part (b): other ways to get a square within CASE — place the inner square's vertices at positions other than the midpoints, as long as the construction is symmetric. For example, divide each side in ratio 1:3 (quarter-points) and join those points.

Answer: UVWX is a square. Each side = s√2/2 (where s is the side of CASE), all angles = 90°. Proof: UV² = (s/2)² + (s/2)² = s²/2, and adjacent sides are perpendicular (dot product = 0).

Q8 If a quadrilateral has four equal sides and one angle of 90°, will it be a square? Find the answer using geometric reasoning as well as by construction and measurement.

SOLUTION

Given: A quadrilateral ABCD with AB = BC = CD = DA (all sides equal) and ∠A = 90°.

Step 1: A quadrilateral with all four sides equal is a rhombus.
In a rhombus, opposite angles are equal (property of rhombus, since it is a parallelogram).
=> ∠C = ∠A = 90°.

Step 2: In a rhombus (parallelogram), adjacent angles are supplementary.
=> ∠A + ∠B = 180° => ∠B = 180° − 90° = 90°.
=> ∠D = ∠B = 90° (opposite angles in parallelogram are equal).

Step 3: All four angles = 90° and all four sides are equal.
=> The quadrilateral is a SQUARE.

Yes, a quadrilateral with four equal sides and one angle of 90° must be a square.

Answer: Yes. Since all sides are equal it is a rhombus; in a rhombus opposite angles are equal (so the angle opposite 90° is also 90°) and adjacent angles are supplementary (so both remaining angles = 90°). All angles = 90° and all sides equal => it is a square.

Q9 What type of a quadrilateral is one in which the opposite sides are equal? Justify your answer.
Hint: Draw a diagonal and check for congruent triangles.

SOLUTION

Given: Quadrilateral ABCD with AB = CD and BC = DA (opposite sides equal).

Step 1: Draw diagonal AC, dividing ABCD into triangles ABC and CDA.

Step 2: In triangles ABC and CDA:
AB = CD (given), BC = DA (given), AC = AC (common side).
By SSS congruence: $\triangle ABC \cong \triangle CDA$.

Step 3: From congruence, corresponding angles are equal:
$\angle BAC = \angle DCA$ (these are alternate interior angles for lines AB and CD with transversal AC).
=> AB ∥ CD.
$\angle BCA = \angle DAC$ (alternate interior angles for lines BC and DA with transversal AC).
=> BC ∥ DA.

Step 4: Since both pairs of opposite sides are parallel, ABCD is a PARALLELOGRAM.

Answer: A quadrilateral with opposite sides equal is a parallelogram. Proof: diagonal AC gives $\triangle ABC \cong \triangle CDA$ (SSS), yielding alternate angle equalities that prove AB ∥ CD and BC ∥ DA.

Q10 Will the sum of the angles in a quadrilateral such as the following one (a non-convex/concave quadrilateral, where one vertex D lies inside triangle ABC) also be 360°? Find the answer using geometric reasoning as well as by constructing this figure and measuring.

SOLUTION

The figure shows a non-convex (concave) quadrilateral ABDC where vertex D is indented inward.

Geometric reasoning:
Draw diagonal AC, dividing the quadrilateral into triangles ABC and ACD.
In △ABC: ∠BAC + ∠ABC + ∠BCA = 180°.
In △ACD: ∠DAC + ∠ACD + ∠ADC = 180°.
Adding all six angles: sum = 360°.

The angles ∠BAC + ∠DAC = ∠BAD (full angle at A in the quadrilateral).
The angles ∠BCA + ∠ACD = ∠BCD (full angle at C in the quadrilateral).
∠ABC and ∠ADC are the full interior angles at B and D.
So the sum of interior angles of the quadrilateral = 360°.

Note: For a concave quadrilateral, the 'interior angle' at the reflex vertex (D) is measured as the interior angle of the quadrilateral (not the reflex angle). With this convention, the sum is still 360°.

Yes, the sum of angles is 360° for any simple (non-self-intersecting) quadrilateral, convex or concave.

Answer: Yes. The sum of interior angles of any simple quadrilateral (convex or concave) is 360°. Drawing diagonal AC splits it into two triangles, each contributing 180°, totaling 360°.

Q11 State whether the following statements are true or false. Justify your answers.
(i) A quadrilateral whose diagonals are equal and bisect each other must be a square.
(ii) A quadrilateral having three right angles must be a rectangle.
(iii) A quadrilateral whose diagonals bisect each other must be a parallelogram.
(iv) A quadrilateral whose diagonals are perpendicular to each other must be a rhombus.
(v) A quadrilateral in which the opposite angles are equal must be a parallelogram.
(vi) A quadrilateral in which all the angles are equal is a rectangle.
(vii) Isosceles trapeziums are parallelograms.

SOLUTION

(i) FALSE.
A rectangle also has diagonals that are equal and bisect each other, yet a rectangle is not (in general) a square.
The additional condition needed for a square is that the diagonals also bisect each other at right angles (or equivalently, that all sides are equal).

(ii) TRUE.
If three angles of a quadrilateral are 90° each, the fourth angle = 360° − 90° − 90° − 90° = 90°.
All four angles = 90° => the quadrilateral is a rectangle.

(iii) TRUE.
If the diagonals of a quadrilateral bisect each other (each diagonal is cut into two equal halves at the intersection), then the quadrilateral is a parallelogram.
Proof: if diagonals AC and BD bisect each other at O, then OA = OC and OB = OD. Triangles AOB and COD are congruent (SAS), giving AB = CD and AB ∥ CD. Similarly BC = DA and BC ∥ DA. => Parallelogram.

(iv) FALSE.
A kite has diagonals that are perpendicular to each other, but a kite is not (in general) a rhombus (its sides are not all equal).

(v) TRUE.
If ∠A = ∠C and ∠B = ∠D (opposite angles equal), then ∠A + ∠B + ∠C + ∠D = 360° => 2∠A + 2∠B = 360° => ∠A + ∠B = 180°.
Since adjacent angles are supplementary, opposite sides are parallel => it is a parallelogram.

(vi) TRUE.

If all four angles are equal, each angle = 360° ÷ 4 = 90°. All angles 90° => the quadrilateral is a rectangle.

(vii) FALSE.
An isosceles trapezium has only ONE pair of parallel opposite sides. A parallelogram requires BOTH pairs of opposite sides to be parallel. So an isosceles trapezium is not a parallelogram.

Answer: (i) False — a rectangle also satisfies this. (ii) True — fourth angle = 90° too. (iii) True — diagonals bisecting each other is the defining property of a parallelogram. (iv) False — a kite has perpendicular diagonals but is not a rhombus. (v) True — opposite angles equal => adjacent angles supplementary => parallelogram. (vi) True — each angle = 90° => rectangle. (vii) False — isosceles trapezium has only one pair of parallel sides.