Chapter 5: Number Play

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Exercise 5.1 — What Remains?

Q1 The sum of four consecutive numbers is 34. What are these numbers?

SOLUTION

Let the four consecutive numbers be n, n+1, n+2, n+3.
Their sum: n + (n+1) + (n+2) + (n+3) = 4n + 6 = 34
4n = 34 - 6 = 28
n = 7
The four consecutive numbers are 7, 8, 9, 10.

Verification: 7 + 8 + 9 + 10 = 34 ✓

Answer: 7, 8, 9, and 10

Q2 Suppose p is the greatest of five consecutive numbers. Describe the other four numbers in terms of p.

SOLUTION

If p is the greatest of five consecutive numbers, each number before it is 1 less than the next.
Going downward from p: p, p-1, p-2, p-3, p-4.
The other four numbers in descending order are (p-1), (p-2), (p-3), and (p-4).

Answer: The other four numbers are (p-1), (p-2), (p-3), and (p-4)

Q3 For each statement below, determine whether it is always true, sometimes true, or never true. Explain your answer. Mention examples and non-examples as appropriate. Justify your claim using algebra.
(i) The sum of two even numbers is a multiple of 3.
(ii) If a number is not divisible by 18, then it is also not divisible by 9.
(iii) If two numbers are not divisible by 6, then their sum is not divisible by 6.
(iv) The sum of a multiple of 6 and a multiple of 9 is a multiple of 3.
(v) The sum of a multiple of 6 and a multiple of 3 is a multiple of 9.

SOLUTION

(i) Sometimes true.
Example: 2 + 4 = 6 (multiple of 3); 4 + 8 = 12 (multiple of 3).
Non-example: 2 + 6 = 8 (not a multiple of 3); 6 + 8 = 14 (not a multiple of 3).
Algebraically: two even numbers are 2a and 2b; their sum = 2(a+b). This is a multiple of 3 only when a+b is a multiple of 3, which does not always happen.

(ii) Sometimes true.
Example: 30 is not divisible by 18 and also not divisible by 9.
Non-example: 27 is not divisible by 18 but IS divisible by 9.
Explanation: 18 = 2 × 9. A number can be divisible by 9 without being divisible by 2 (e.g., 27), so not divisible by 18 does not imply not divisible by 9.

(iii) Sometimes true.
Example: 9 and 11 are not divisible by 6; their sum 9 + 11 = 20 is also not divisible by 6.
Non-example: 8 and 10 are not divisible by 6; their sum 8 + 10 = 18 IS divisible by 6.

(iv) Always true.
Let the multiple of 6 = 6x and the multiple of 9 = 9y.
Sum = 6x + 9y = 3(2x + 3y).
This is always a multiple of 3, for any integers x and y.

(v) Sometimes true.
Example: 18 (multiple of 6) + 9 (multiple of 3) = 27 = 9 × 3 (multiple of 9).
Non-example: 12 (multiple of 6) + 9 (multiple of 3) = 21, which is NOT a multiple of 9.
Algebraically: 6m + 3n = 3(2m + n). For this to be a multiple of 9, we need 2m + n to be a multiple of 3, which only sometimes holds.

Answer: (i) Sometimes true (ii) Sometimes true (iii) Sometimes true (iv) Always true (v) Sometimes true

Q4 Find a few numbers that leave a remainder of 2 when divided by 3 and a remainder of 2 when divided by 4. Write an algebraic expression to describe all such numbers.

SOLUTION

We need a number x such that: x mod 3 = 2 and x mod 4 = 2.
This means (x - 2) is divisible by both 3 and 4.
LCM(3, 4) = 12, so (x - 2) must be a multiple of 12.
Therefore x = 12n + 2, where n is a non-negative integer.

Some such numbers: n=0: x=2; n=1: x=14; n=2: x=26; n=3: x=38; n=4: x=50.

Verification for x=14: 14 ÷ 3 = 4 remainder 2 ✓; 14 ÷ 4 = 3 remainder 2 ✓

Answer: Numbers of the form 12n + 2 (e.g., 2, 14, 26, 38, 50, ...)

Q5 "I hold some pebbles, not too many,
When I group them in 3's, one stays with me.
Try pairing them up — it simply won't do,
A stubborn odd pebble remains in my view.
Group them by 5, yet one's still around,
But grouping by seven, perfection is found.
More than one hundred would be far too bold,
Can you tell me the number of pebbles I hold?"

SOLUTION

From the riddle, let the number of pebbles = n.
Conditions: n mod 3 = 1, n mod 2 = 1 (odd), n mod 5 = 1, n mod 7 = 0, and n < 100.

LCM(3, 2, 5) = 30. So n = 30k + 1 for some positive integer k.
Values: k=1 => n=31; k=2 => n=61; k=3 => n=91.

Check which of these is divisible by 7:
31 ÷ 7 = 4 remainder 3 (no)
61 ÷ 7 = 8 remainder 5 (no)
91 ÷ 7 = 13 remainder 0 (yes!) ✓

Verification: 91 mod 3 = 1 ✓, 91 is odd ✓, 91 mod 5 = 1 ✓, 91 mod 7 = 0 ✓, 91 < 100 ✓

Answer: 91 pebbles

Q6 Tathagat has written several numbers that leave a remainder of 2 when divided by 6. He claims, "If you add any three such numbers, the sum will always be a multiple of 6." Is Tathagat's claim true?

SOLUTION

Let three such numbers be 6a+2, 6b+2, and 6c+2 (where a, b, c are non-negative integers).

Sum = (6a+2) + (6b+2) + (6c+2)

= 6a + 6b + 6c + 6

= 6(a + b + c + 1)

This is always a multiple of 6 for any non-negative integers a, b, c.

Example: 8, 14, 20 all leave remainder 2 when divided by 6.

8 + 14 + 20 = 42 = 6 × 7 ✓

Answer: Yes, Tathagat's claim is true. The sum of any three such numbers is always a multiple of 6.

Q7 When divided by 7, the number 661 leaves a remainder of 3, and 4779 leaves a remainder of 5. Without calculating, can you say what remainders the following expressions will leave when divided by 7? Show the solution both algebraically and visually.

(i) 4779 + 661

(ii) 4779 - 661

SOLUTION

Write: 661 = 7q + 3 and 4779 = 7p + 5 for some integers p, q.

(i) 4779 + 661 = (7p + 5) + (7q + 3)

= 7p + 7q + 8

= 7(p + q + 1) + 1

Remainder when divided by 7 is 1.

(ii) 4779 - 661 = (7p + 5) - (7q + 3)

= 7(p - q) + 2

Remainder when divided by 7 is 2.

Visual method: On a number line, 661 sits 3 steps past a multiple of 7, and 4779 sits 5 steps past a multiple of 7. Adding the 'extra' parts: 3 + 5 = 8 = 7 + 1, so one full group of 7 is absorbed and remainder 1 remains. Subtracting: 5 - 3 = 2, remainder 2.

Answer: (i) Remainder 1 when divided by 7; (ii) Remainder 2 when divided by 7

Q8 Find a number that leaves a remainder of 2 when divided by 3, a remainder of 3 when divided by 4, and a remainder of 4 when divided by 5. What is the smallest such number? Can you give a simple explanation of why it is the smallest?

SOLUTION

Observe the pattern: each remainder is exactly 1 less than its divisor.
Remainder 2 when divided by 3 => number is 1 less than a multiple of 3.
Remainder 3 when divided by 4 => number is 1 less than a multiple of 4.
Remainder 4 when divided by 5 => number is 1 less than a multiple of 5.

So the number must be exactly 1 less than a common multiple of 3, 4, and 5.
LCM(3, 4, 5) = 60.
The smallest such number = 60 - 1 = 59.

Verification: 59 ÷ 3 = 19 remainder 2 ✓; 59 ÷ 4 = 14 remainder 3 ✓; 59 ÷ 5 = 11 remainder 4 ✓

Why is it the smallest? Because 60 is the smallest positive common multiple of 3, 4, and 5 (LCM). Any smaller common multiple would need to be at least 60, so 59 is indeed the smallest number fitting all conditions.

Answer: 59 (the smallest such number)

Exercise 5.2 — A Shortcut for Divisibility by 9

Q1 Find, without dividing, whether the following numbers are divisible by 9.
(i) 123 (ii) 405 (iii) 8888 (iv) 93547 (v) 358095

SOLUTION

A number is divisible by 9 if and only if the sum of its digits is divisible by 9.

(i) 123: digit sum = 1 + 2 + 3 = 6. 6 is not divisible by 9. Not divisible by 9.
(ii) 405: digit sum = 4 + 0 + 5 = 9. 9 is divisible by 9. Divisible by 9. ✓
(iii) 8888: digit sum = 8 + 8 + 8 + 8 = 32. 32 is not divisible by 9. Not divisible by 9.
(iv) 93547: digit sum = 9 + 3 + 5 + 4 + 7 = 28. 28 is not divisible by 9. Not divisible by 9.
(v) 358095: digit sum = 3 + 5 + 8 + 0 + 9 + 5 = 30. 30 is not divisible by 9. Not divisible by 9.

Answer: Only 405 (ii) is divisible by 9

Q2 Find the smallest multiple of 9 with no odd digits.

SOLUTION

Only even digits are 0, 2, 4, 6, 8.
The digit sum must be a multiple of 9.
The smallest multiple of 9 that is achievable with even digits:
A sum of 9 is impossible using only even digits (since even + even = even, and 9 is odd).
The next multiple of 9 is 18. Can we form 18 with even digits?
Yes: 2 + 8 + 8 = 18.
The smallest number with these digits is 288 (arranging 2, 8, 8 in ascending order).

Verification: 2 + 8 + 8 = 18, divisible by 9 ✓; digits 2, 8, 8 are all even ✓; 288 ÷ 9 = 32 ✓

Answer: 288

Q3 Find the multiple of 9 that is closest to the number 6000.

SOLUTION

Divide 6000 by 9: 6000 = 9 × 666 + 6.
So 9 × 666 = 5994 and 9 × 667 = 6003.
Distance from 6000: |6000 - 5994| = 6; |6003 - 6000| = 3.
6003 is closer to 6000 than 5994 is.

Answer: 6003

Q4 How many multiples of 9 are there between the numbers 4300 and 4400?

SOLUTION

Find the smallest multiple of 9 greater than 4300:
4300 ÷ 9 = 477 remainder 7, so next multiple = 9 × 478 = 4302.
Find the largest multiple of 9 less than 4400:
4400 ÷ 9 = 488 remainder 8, so last multiple = 9 × 488 = 4392.

Count of multiples = 488 - 478 + 1 = 11.

The multiples are: 4302, 4311, 4320, 4329, 4338, 4347, 4356, 4365, 4374, 4383, 4392.

Answer: 11 multiples of 9

Exercise 5.3 — Digital Roots

Q1 The digital root of an 8-digit number is 5. What will be the digital root of 10 more than that number?

SOLUTION

The digital root of a number equals its remainder when divided by 9 (with digital root 9 when the remainder is 0).
If digital root = 5, then the number ≡ 5 (mod 9).
Adding 10: the new number ≡ 5 + 10 = 15 ≡ 6 (mod 9).
So the digital root of the new number is 6.

Example: 40000001 has digit sum 4+0+0+0+0+0+0+1 = 5 (digital root 5).
40000001 + 10 = 40000011; digit sum = 4+0+0+0+0+0+1+1 = 6 (digital root 6). ✓

Answer: 6

Q2 Write any number. Generate a sequence of numbers by repeatedly adding 11. What would be the digital roots of this sequence of numbers? Share your observations.

SOLUTION

Adding 11 changes a number by 11. Since 11 ≡ 2 (mod 9), each addition increases the digital root by 2 (mod 9).

Example starting with 10:
10 -> digital root 1
21 -> digital root 3
32 -> digital root 5
43 -> digital root 7
54 -> digital root 9
65 -> digital root 2
76 -> digital root 4
87 -> digital root 6
98 -> digital root 8
109 -> digital root 1 (cycle repeats)

Observation: the digital roots form a repeating pattern with period 9, cycling through odd numbers then even numbers: 1, 3, 5, 7, 9, 2, 4, 6, 8, then back to 1, 3, 5, ... The digital roots increase by 2 each time (mod 9).

Answer: Digital roots form a repeating cycle of period 9, increasing by 2 each step (mod 9). E.g., starting from 10: 1, 3, 5, 7, 9, 2, 4, 6, 8, 1, 3, ...

Q3 What will be the digital root of the number 9a + 36b + 13?

SOLUTION

Rewrite the expression by separating multiples of 9:

$9a + 36b + 13 = 9a + 36b + 9 + 4$

$= 9(a + 4b + 1) + 4$

The term 9(a + 4b + 1) is a multiple of 9, so its digital root is 9.

Adding 4: digital root of [multiple of 9] + 4 => digital root = 4 + 9 = 13 => 1 + 3 = 4.

Verification: a=1, b=1: 9(1) + 36(1) + 13 = 58; digital root = 5+8=13 => 4 ✓

a=2, b=3: 18 + 108 + 13 = 139; digital root = 1+3+9=13 => 4 ✓

Answer: 4

Q4 Make conjectures by examining if there are any patterns or relations between
(i) the parity of a number and its digital root.
(ii) the digital root of a number and the remainder obtained when the number is divided by 3 or 9.

SOLUTION

(i) Parity and digital root:

There is no consistent relationship between a number's parity (even/odd) and its digital root.

For example: 10 (even) has digital root 1; 19 (odd) has digital root 1.

12 (even) has digital root 3; 21 (odd) has digital root 3.

Even and odd numbers can share the same digital root, and vice versa.

Conjecture: parity and digital root are independent — knowing one tells you nothing about the other.

(ii) Digital root and remainder when divided by 3 or 9:

Relationship with 9: the digital root equals the remainder when the number is divided by 9 (except when the remainder is 0, in which case the digital root is 9).

Relationship with 3: if the digital root is 1, 4, or 7 => remainder when divided by 3 is 1.

If the digital root is 2, 5, or 8 => remainder when divided by 3 is 2.

If the digital root is 3, 6, or 9 => remainder when divided by 3 is 0 (divisible by 3). Conjecture: the digital root determines the remainder mod 9 exactly, and the remainder mod 3 is determined by whether the digital root falls in {1,4,7}, {2,5,8}, or {3,6,9}.

Answer: (i) No consistent pattern between parity and digital root. (ii) Digital root = remainder mod 9 (9 when remainder is 0); digital root mod 3 = remainder of number mod 3.

Exercise 5.4 — Digits in Disguise

Q1 If 31z5 is a multiple of 9, where z is a digit, what is the value of z? Explain why there are two answers to this problem.

SOLUTION

For 31z5 to be a multiple of 9, the sum of its digits must be a multiple of 9.
Digit sum = 3 + 1 + z + 5 = 9 + z.
We need 9 + z to be a multiple of 9.

Case 1: 9 + z = 9 => z = 0.
Case 2: 9 + z = 18 => z = 9.
Case 3: 9 + z = 27 => z = 18, not a single digit.

There are two answers because z = 0 gives digit sum 9, and z = 9 gives digit sum 18 — both multiples of 9.

Verification: 3105 ÷ 9 = 345 ✓; 3195 ÷ 9 = 355 ✓

Answer: z = 0 or z = 9 (two values because both give digit sums that are multiples of 9)

Q2 "I take a number that leaves a remainder of 8 when divided by 12. I take another number which is 4 short of a multiple of 12. Their sum will always be a multiple of 8", claims Snehal. Examine his claim and justify your conclusion.

SOLUTION

Let the first number be a = 12n + 8 (leaves remainder 8 when divided by 12).
Let the second number be b = 12m - 4 (4 short of a multiple of 12).

Sum = a + b = (12n + 8) + (12m - 4) = 12(n + m) + 4.

Is 12k + 4 always a multiple of 8? Let us check:
k=1: 12+4 = 16, divisible by 8 ✓
k=2: 24+4 = 28, NOT divisible by 8 (28 ÷ 8 = 3 rem 4) ✗

The sum 12k + 4 = 4(3k + 1). For this to be a multiple of 8, we need 3k+1 to be even, i.e., k must be odd. But k can be any integer, so the claim fails when k is even.

Snehal's claim is FALSE.

Answer: Snehal's claim is false. The sum is 12(n+m)+4 = 4(3(n+m)+1), which is not always divisible by 8.

Q3 When is the sum of two multiples of 3, a multiple of 6 and when is it not? Explain the different possible cases, and generalise the pattern.

SOLUTION

Let the two multiples of 3 be 3m and 3n.
Sum = 3m + 3n = 3(m + n).
For this sum to be a multiple of 6, we need 3(m+n) divisible by 6, i.e., (m+n) must be even.

Case 1: m and n both even => m+n even => sum = 6 × (even/2) × ... => multiple of 6.
Example: 6 + 12 = 18 = 6 × 3 ✓

Case 2: m and n both odd => m+n even => sum is a multiple of 6.
Example: 3 + 9 = 12 = 6 × 2 ✓

Case 3: m even and n odd (or vice versa) => m+n odd => sum is NOT a multiple of 6.
Example: 3 + 6 = 9 (not divisible by 6) ✗

Generalisation: the sum of two multiples of 3 is a multiple of 6 if and only if m and n have the same parity (both even or both odd), i.e., when m+n is even.

Answer: The sum 3m + 3n = 3(m+n) is a multiple of 6 when m and n have the same parity (both even or both odd); not a multiple of 6 when m and n have different parity.

Q4 Sreelatha says, "I have a number that is divisible by 9. If I reverse its digits, it will still be divisible by 9".
(i) Examine if her conjecture is true for any multiple of 9.
(ii) Are any other digit shuffles possible such that the number formed is still a multiple of 9?

SOLUTION

(i) A number is divisible by 9 if and only if its digit sum is divisible by 9.
When we reverse the digits, the digit sum remains exactly the same (same digits, just in different order).

Therefore, the reversed number has the same digit sum, which is still divisible by 9.
Sreelatha's conjecture is TRUE for any multiple of 9.
Example: 9 × 123 = 1107; reversed = 7011; 7+0+1+1 = 9 ✓; 7011 ÷ 9 = 779 ✓

(ii) Any rearrangement (shuffle) of the digits of a multiple of 9 gives another multiple of 9.
Reason: any rearrangement keeps the digit sum the same, and divisibility by 9 depends only on the digit sum.
Example: 189 (div by 9); rearrangements 198, 819, 891, 918, 981 are all divisible by 9 ✓

Answer: (i) True — reversing digits does not change the digit sum, so divisibility by 9 is preserved. (ii) Yes — any digit shuffle preserves divisibility by 9 for the same reason.

Q5 If 48a23b is a multiple of 18, list all possible pairs of values for a and b.

SOLUTION

18 = 2 × 9, so 48a23b must be divisible by both 2 and 9.

Divisibility by 2: the units digit b must be even, so b ∈ {0, 2, 4, 6, 8}.

Divisibility by 9: digit sum must be divisible by 9.
Digit sum = 4 + 8 + a + 2 + 3 + b = 17 + a + b.
Need 17 + a + b ≡ 0 (mod 9), i.e., a + b ≡ 1 (mod 9) (since 18 ≡ 0, 27 ≡ 0, etc.).
So 17 + a + b = 18 or 27 (since a and b are digits, max a+b = 9+8=17, so 17+17=34 < 36).

If 17 + a + b = 18: a + b = 1. With b even: b=0,a=1.
If 17 + a + b = 27: a + b = 10. With b even: b=0 => a=10 (invalid); b=2 => a=8; b=4 => a=6; b=6 => a=4; b=8 => a=2.

Possible pairs (a, b): (1,0), (8,2), (6,4), (4,6), (2,8).

Verification: 481230 ÷ 18 = 26735 ✓; 488232 ÷ 18 = 27124 ✓

Answer: (a, b) = (1, 0), (8, 2), (6, 4), (4, 6), or (2, 8)

Q6 If 3p7q8 is divisible by 44, list all possible pairs of values for p and q.

SOLUTION

44 = 4 × 11, so 3p7q8 must be divisible by both 4 and 11.

Divisibility by 4: last two digits 'q8' must form a number divisible by 4.
q8 divisible by 4: try q = 0 (08=8 ✓), q=2 (28 ÷ 4 = 7 ✓), q=4 (48 ÷ 4=12 ✓), q=6 (68 ÷ 4=17 ✓), q=8 (88 ÷ 4=22 ✓).
So q ∈ {0, 2, 4, 6, 8} — but we also need divisibility by 11.

Divisibility by 11: alternating digit sum (odd positions minus even positions, from right) must be 0 or ±11.
Number: 3 p 7 q 8 (positions from right: 8 is pos 1, q is pos 2, 7 is pos 3, p is pos 4, 3 is pos 5).
Odd positions (1,3,5): 8 + 7 + 3 = 18.
Even positions (2,4): q + p.
Difference = 18 - (p + q) must equal 0 or ±11.

If 18 - (p+q) = 0: p+q = 18. With q ∈ {0,2,4,6,8} and p a digit (0-9): max p+q = 9+8=17 < 18. Impossible.
If 18 - (p+q) = 11: p+q = 7. Pairs with q ∈ {0,2,4,6}: (p=7,q=0), (p=5,q=2), (p=3,q=4), (p=1,q=6).
Note q=8 would give p = -1, invalid.

Verification: 37708 ÷ 44 = 857 ✓; 35728 ÷ 44 = 812 ✓; 33748 ÷ 44 = 767 ✓; 31768 ÷ 44 = 722 ✓

Answer: (p, q) = (7, 0), (5, 2), (3, 4), or (1, 6)

Q7 Find three consecutive numbers such that the first number is a multiple of 2, the second number is a multiple of 3, and the third number is a multiple of 4. Are there more such numbers? How often do they occur?

SOLUTION

Let the three consecutive numbers be n, n+1, n+2.
Conditions: n divisible by 2; (n+1) divisible by 3; (n+2) divisible by 4.

Since n is divisible by 2, let n = 2k.
n+2 must be divisible by 4, so n+2 = 4j => n = 4j-2.
Thus n ≡ 2 (mod 4) (i.e., n is even but not divisible by 4).

Also n+1 ≡ 0 (mod 3) => n ≡ 2 (mod 3).

Using Chinese Remainder Theorem: n ≡ 2 (mod 4) and n ≡ 2 (mod 3).
=> n ≡ 2 (mod 12).

Smallest: n = 2, giving 2, 3, 4. (2 div by 2 ✓; 3 div by 3 ✓; 4 div by 4 ✓)
Next: n = 14, giving 14, 15, 16. (14 div by 2 ✓; 15 div by 3 ✓; 16 div by 4 ✓)
Then: 26, 27, 28 and 38, 39, 40 etc.

Such triples occur every 12 numbers.

Answer: Smallest triple: 2, 3, 4. More exist: 14, 15, 16; 26, 27, 28; etc. They occur every 12 numbers.

Q8 Write five multiples of 36 between 45,000 and 47,000. Share your approach with the class.

SOLUTION

36 = 4 × 9, so a multiple of 36 must be divisible by both 4 and 9.

Approach: find 45000 ÷ 36 = 1250 exactly. So 36 × 1250 = 45000.
Consecutive multiples: 36 × 1251 = 45036, 36 × 1252 = 45072, 36 × 1253 = 45108,
36 × 1254 = 45144, 36 × 1255 = 45180.

Verify each is between 45,000 and 47,000: 45036, 45072, 45108, 45144, 45180. ✓

Verification check: 45036 ÷ 36 = 1251 ✓; 45036 ÷ 9: digit sum = 4+5+0+3+6=18 ✓; 45036 ÷ 4: last two digits 36 ÷ 4 = 9 ✓

Answer: Five multiples of 36 between 45,000 and 47,000: 45036, 45072, 45108, 45144, 45180

Q9 The middle number in the sequence of 5 consecutive even numbers is 5p. Express the other four numbers in sequence in terms of p.

SOLUTION

In a sequence of 5 consecutive even numbers, consecutive terms differ by 2.
If the middle (3rd) number is 5p, the sequence is:
5p - 4, 5p - 2, 5p, 5p + 2, 5p + 4.
The other four numbers are 5p-4, 5p-2, 5p+2, and 5p+4.

Answer: The other four numbers are 5p - 4, 5p - 2, 5p + 2, and 5p + 4

Q10 Write a 6-digit number that is divisible by 15, such that when the digits are reversed, it is divisible by 6.

SOLUTION

Divisibility by 15 requires divisibility by both 3 and 5.
Divisibility by 5: units digit f must be 0 or 5.
If f = 0: reversing gives a number starting with 0, which is not a 6-digit number. So f = 5.

Let the 6-digit number be abcde5 (divisible by 15, units digit = 5).
When reversed: 5edcba. For this to be divisible by 6, it must be divisible by 2 and 3.
Divisibility by 2: the units digit of the reversed number = a must be even. So a $\in$ {2, 4, 6, 8}.
Divisibility by 3: same digit sum as original, which is already divisible by 3 (since 15 | original => 3 | original). ✓

Example: choose a=2, f=5. Need 2+b+c+d+e+5 = b+c+d+e+7 divisible by 3.
Choose b=0,c=0,d=0,e=2: digit sum = 2+0+0+0+2+5 = 9 ✓ => 200025.
Reversed: 520002; ends in 2 (even) ✓; digit sum 9 ✓; 520002 ÷ 6 = 86667 ✓

Other examples: 200055, 200085, 202005, etc.

Answer: Example: 200025 (reversed 520002 is divisible by 6). Other examples: 200055, 400035, etc.

Q11 Deepak claims, "There are some multiples of 11 which, when doubled, are still multiples of 11. But other multiples of 11 don't remain multiples of 11 when doubled". Examine if his conjecture is true; explain your conclusion.

SOLUTION

Let n be any multiple of 11, so n = 11k for some integer k.
When doubled: 2n = 2 × 11k = 22k = 11 × (2k).
2n is always a multiple of 11.

Therefore, doubling ANY multiple of 11 always gives another multiple of 11.
Deepak's conjecture is FALSE — there is no multiple of 11 that fails to remain a multiple when doubled.

Answer: Deepak's conjecture is false. Doubling any multiple of 11 always gives a multiple of 11 (since 2 × 11k = 11 × 2k).

Q12 Determine whether the statements below are 'Always True', 'Sometimes True', or 'Never True'. Explain your reasoning.
(i) The product of a multiple of 6 and a multiple of 3 is a multiple of 9.
(ii) The sum of three consecutive even numbers will be divisible by 6.
(iii) If abcdef is a multiple of 6, then badcef will be a multiple of 6.
(iv) 8(7b - 3) - 4(11b + 1) is a multiple of 12.

SOLUTION

(i) Always True.
Let the multiple of 6 = 6m and the multiple of 3 = 3n.
Product = 6m × 3n = 18mn = 9 × (2mn).
This is always a multiple of 9. ✓

(ii) Always True.
Three consecutive even numbers: 2n, 2n+2, 2n+4.
Sum = 6n + 6 = 6(n+1).
This is always a multiple of 6. ✓

(iii) Always True.
abcdef and badcef are rearrangements of the same digits a, b, c, d, e, f.
Divisibility by 3: both have the same digit sum => same divisibility by 3.
Divisibility by 2: both have the same units digit f => same divisibility by 2.
Since abcdef is divisible by 6 (by 2 and 3), badcef also satisfies both conditions. ✓

(iv) Never True.
8(7b-3) - 4(11b+1) = 56b - 24 - 44b - 4 = 12b - 28.
12b - 28 = 4(3b - 7).
For this to be a multiple of 12: 12 | 4(3b-7) => 3 | (3b-7) => 3 | (-7) => 3 | 7. False!
So 12b - 28 is NEVER a multiple of 12 for any integer b.
Check: b=0: -28, -28 mod 12 = 8 (not 0); b=1: -16, -16 mod 12 = 8 (not 0). ✓

Answer: (i) Always True (ii) Always True (iii) Always True (iv) Never True

Q13 Choose any 3 numbers. When is their sum divisible by 3? Explore all possible cases and generalise.

SOLUTION

Any integer leaves a remainder of 0, 1, or 2 when divided by 3.
Let the three numbers have remainders R1, R2, R3 respectively when divided by 3.
The sum is divisible by 3 iff R1 + R2 + R3 ≡ 0 (mod 3).

Case 1: All three remainders are equal (R1 = R2 = R3 = r).
R1+R2+R3 = 3r ≡ 0 (mod 3). Sum is always divisible by 3.
Examples: (3,6,9): all rem 0, sum=18 ✓; (1,4,7): all rem 1, sum=12 ✓; (2,5,8): all rem 2, sum=15 ✓

Case 2: The three remainders are {0, 1, 2} in some order.
R1+R2+R3 = 0+1+2 = 3 ≡ 0 (mod 3). Sum is divisible by 3.
Example: (3,1,2): sum=6 ✓

Case 3: Two remainders are equal, the third is different (but not the case {0,1,2}).
E.g., remainders {0,0,1}: sum = 1, NOT divisible by 3.
E.g., remainders {1,1,0}: sum = 2, NOT divisible by 3.
E.g., remainders {0,1,1}: sum = 2, NOT divisible by 3.

Generalisation: the sum of three numbers is divisible by 3 if and only if either all three have the same remainder when divided by 3, or the remainders are {0, 1, 2} (one of each type).

Answer: Sum is divisible by 3 when all three numbers leave the same remainder mod 3, or when their remainders are 0, 1, and 2 (one of each). Otherwise not.

Q14 Is the product of two consecutive integers always a multiple of 2? Why? What about the product of three consecutive integers? Is it always a multiple of 6? Why or why not? What can you say about the product of 4 consecutive integers? What about the product of five consecutive integers?

SOLUTION

Product of 2 consecutive integers: Always a multiple of 2.
Among any 2 consecutive integers, one is even and one is odd. The product contains the even number as a factor, so it is divisible by 2.
Example: 3 × 4 = 12 ✓; 7 × 8 = 56 ✓

Product of 3 consecutive integers: Always a multiple of 6.
Among any 3 consecutive integers, at least one is divisible by 2 and at least one is divisible by 3 (in any 3 consecutive numbers, exactly one is a multiple of 3, and at least one is even).
So the product is divisible by 2 × 3 = 6.
Example: 4 × 5 × 6 = 120 ✓; 7 × 8 × 9 = 504 ✓

Product of 4 consecutive integers: Always a multiple of 24.
Among any 4 consecutive integers, exactly 2 are even. Of those two even numbers, exactly one is divisible by 4 (since every other even number is a multiple of 4). So the even numbers together contribute a factor of 4 × 2 = 8. Additionally, exactly one of the four integers is a multiple of 3. Therefore the product is divisible by 8 × 3 = 24.
Example: 1 × 2 × 3 × 4 = 24 ✓; 2 × 3 × 4 × 5 = 120 ✓

Product of 5 consecutive integers: Always a multiple of 120.
Among any 5 consecutive integers, the product includes factors of 2, 3, 4, and 5, giving at least 2 × 3 × 4 × 5 = 120.
Example: 1 × 2 × 3 × 4 × 5 = 120 ✓; 2 × 3 × 4 × 5 × 6 = 720 ✓

Answer: 2 consecutive: always multiple of 2. 3 consecutive: always multiple of 6. 4 consecutive: always multiple of 24. 5 consecutive: always multiple of 120.

Q15 Solve the cryptarithms —
(i) EF × E = GGG
(ii) WOW × 5 = MEOW

SOLUTION

(i) EF × E = GGG:
GGG is a 3-digit number with all digits equal: GGG = G × 111 = G × 3 × 37.
So EF × E = 3 × 37 × G.
Try E = 3: EF × 3 = GGG => EF = GGG ÷ 3 = 37G.
EF must be a 2-digit number: 37 × 1 = 37 ✓, giving G=1.
Check: 37 × 3 = 111 = GGG with G=1. E=3, F=7, G=1, all different ✓

(ii) WOW × 5 = MEOW:
WOW is a 3-digit palindrome: W×100 + O×10 + W = 101W + 10O.
MEOW: M×1000 + E×100 + O×10 + W.
Try W=5 (units digit of WOW × 5 must match W: 5×5=25, units digit 5 = W ✓).

WOW = 505 + 10O; product = 5 × (505 + 10O) = 2525 + 50O.
MEOW must have W=5 in units: units digit of 2525+50O is 5 ✓ (since 50O ends in 0).
Try O=7: 2525 + 350 = 2875. Check MEOW: M=2, E=8, O=7, W=5 — all different ✓
Verification: 575 × 5 = 2875 ✓

Answer: (i) E = 3, F = 7, G = 1 (37 × 3 = 111) (ii) W = 5, O = 7, M = 2, E = 8 (575 × 5 = 2875)

Q16 Which of the following Venn diagrams captures the relationship between the multiples of 4, 8, and 32?

SOLUTION

Analyse the divisibility relationships:
32 = 8 × 4, so every multiple of 32 is also a multiple of 8.
8 = 2 × 4, so every multiple of 8 is also a multiple of 4.
Therefore: multiples of 32 ⊂ multiples of 8 ⊂ multiples of 4.

This is a strict nesting (proper containment):
- 4 is a multiple of 4, but NOT of 8 or 32.
- 8 is a multiple of both 4 and 8, but NOT of 32.
- 32 is a multiple of 4, 8, AND 32.

The correct Venn diagram shows three concentric circles, with multiples of 4 as the outermost, multiples of 8 in the middle, and multiples of 32 as the innermost circle.
This corresponds to diagram (iv).

Answer: Diagram (iv) — three concentric circles with multiples of 4 outermost, multiples of 8 in the middle, and multiples of 32 innermost