Chapter 6: We Distribute, Yet Things Multiply

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Exercise 6.1 — Some Properties of Multiplication

Q1 Observe the multiplication grid below. Each number inside the grid is formed by multiplying two numbers. If the middle number of a 3 × 3 frame is given by the expression pq, as shown in the figure, write the expressions for the other numbers in the grid.

SOLUTION

The multiplication grid has row values and column values as factors. The middle number of the 3 × 3 frame is pq, meaning the row factor is p and the column factor is q.
Each entry in the grid = (row factor) × (column factor).

The 3 × 3 frame is arranged around the centre pq as follows:
The row above uses factor (p − 1), the current row uses p, and the row below uses (p + 1).
The column to the left uses factor (q − 1), the current column uses q, and the column to the right uses (q + 1).

So the eight surrounding expressions are:
Top-left: (p − 1)(q − 1)
Top-centre: (p − 1)q
Top-right: (p − 1)(q + 1)
Middle-left: p(q − 1)
Centre: pq
Middle-right: p(q + 1)
Bottom-left: (p + 1)(q − 1)
Bottom-centre: (p + 1)q
Bottom-right: (p + 1)(q + 1)

Verification with the sample frame in the figure (p = 4, q = 6, so centre = pq = 24):

the 3 × 3 frame reads 15, 18, 21 / 20, 24, 28 / 25, 30, 35, i.e. 3×5, 3×6, 3×7 / 4×5, 4×6, 4×7 / 5×5, 5×6, 5×7, which matches the grid.

Answer: The nine expressions in the 3 × 3 frame: (p−1)(q−1), (p−1)q, (p−1)(q+1) | p(q−1), pq, p(q+1) | (p+1)(q−1), (p+1)q, (p+1)(q+1)

Q2 Expand the following products.

(i) (3 + u)(v − 3)
(ii) (2/3)(15 + 6a)
(iii) (10a + b)(10c + d)
(iv) (3 − x)(x − 6)
(v) (−5a + b)(c + d)
(vi) (5 + z)(y + 9)

SOLUTION

(i) (3 + u)(v − 3)
= 3 × v + 3 × (−3) + u × v + u × (−3)
= 3v − 9 + uv − 3u
= 3v − 3u + uv − 9

(ii) (2/3)(15 + 6a)
= (2/3) × 15 + (2/3) × 6a
= 10 + 4a

(iii) (10a + b)(10c + d)
= 10a × 10c + 10a × d + b × 10c + b × d
= 100ac + 10ad + 10bc + bd

(iv) (3 − x)(x − 6)
= 3 × x + 3 × (−6) + (−x) × x + (−x) × (−6)
= 3x − 18 − x^2 + 6x
= −x^2 + 9x − 18

(v) (−5a + b)(c + d)
= (−5a) × c + (−5a) × d + b × c + b × d
= −5ac − 5ad + bc + bd

(vi) (5 + z)(y + 9)
= 5 × y + 5 × 9 + z × y + z × 9
= 5y + 45 + yz + 9z

Answer: (i) $3v - 3u + uv - 9$; (ii) $10 + 4a$; (iii) $100ac + 10ad + 10bc + bd$; (iv) $-x^2 + 9x - 18$; (v) $-5ac - 5ad + bc + bd$; (vi) $5y + 45 + yz + 9z$

Q3 Find 3 examples where the product of two numbers remains unchanged when one of them is increased by 2 and the other is decreased by 4.

SOLUTION

We need $(a + 2)(b - 4) = ab$.
Expanding the left side: $(a + 2)(b - 4) = ab - 4a + 2b - 8$.
Setting this equal to ab: $-4a + 2b - 8 = 0$, so $2b = 4a + 8$, giving $b = 2a + 4$.

Choosing three values of a and computing $b = 2a + 4$:
$a = 1$: $b = 2(1) + 4 = 6$. Check: $1 \times 6 = 6$ and $3 \times 2 = 6$. ✓
$a = 2$: $b = 2(2) + 4 = 8$. Check: $2 \times 8 = 16$ and $4 \times 4 = 16$. ✓
$a = 3$: $b = 2(3) + 4 = 10$. Check: $3 \times 10 = 30$ and $5 \times 6 = 30$. ✓

Answer: Three examples: (1, 6), (2, 8), (3, 10) — in each case the product is unchanged when one number is increased by 2 and the other is decreased by 4.

Q4 Expand (i) $(a + ab - 3b^2)(4 + b)$, and (ii) $(4y + 7)(y + 11z - 3)$.

SOLUTION

(i) $(a + ab - 3b^2)(4 + b)$
$= 4(a + ab - 3b^2) + b(a + ab - 3b^2)$
$= 4a + 4ab - 12b^2 + ab + ab^2 - 3b^3$
$= 4a + 5ab + ab^2 - 12b^2 - 3b^3$

Verification ($a = 2$, $b = 1$): LHS $= (2 + 2 - 3)(5) = 1 \times 5 = 5$; RHS $= 8 + 10 + 2 - 12 - 3 = 5$. ✓

(ii) $(4y + 7)(y + 11z - 3)$
$= 4y(y + 11z - 3) + 7(y + 11z - 3)$
$= 4y^2 + 44yz - 12y + 7y + 77z - 21$
$= 4y^2 + 44yz - 5y + 77z - 21$

Verification ($y = 1$, $z = 1$): LHS $= 11 \times 9 = 99$; RHS $= 4 + 44 - 5 + 77 - 21 = 99$. ✓

Answer: (i) $4a + 5ab + ab^2 - 12b^2 - 3b^3$; (ii) $4y^2 + 44yz - 5y + 77z - 21$

Q5 Expand (i) $(a - b)(a + b)$, (ii) $(a - b)(a^2 + ab + b^2)$ and (iii) $(a - b)(a^3 + a^2b + ab^2 + b^3)$. Do you see a pattern? What would be the next identity in the pattern that you see? Can you check it by expanding?

SOLUTION

(i) $(a - b)(a + b)$
$= a^2 + ab - ab - b^2$
$= a^2 - b^2$

(ii) $(a - b)(a^2 + ab + b^2)$
$= a(a^2 + ab + b^2) - b(a^2 + ab + b^2)$
$= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$
$= a^3 - b^3$

(iii) $(a - b)(a^3 + a^2b + ab^2 + b^3)$
$= a(a^3 + a^2b + ab^2 + b^3) - b(a^3 + a^2b + ab^2 + b^3)$
$= a^4 + a^3b + a^2b^2 + ab^3 - a^3b - a^2b^2 - ab^3 - b^4$
$= a^4 - b^4$

Pattern: $(a - b)(a^{(n-1)} + a^{(n-2)}b + \ldots + ab^{(n-2)} + b^{(n-1)}) = a^n - b^n$.

Next identity in the pattern:
$(a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4) = a^5 - b^5$

Verification ($a = 3$, $b = 2$):
LHS $= (3 - 2)(81 + 54 + 36 + 24 + 16) = 1 \times 211 = 211$.
RHS $= 3^5 - 2^5 = 243 - 32 = 211$. ✓

Answer: (i) $a^2 - b^2$; (ii) $a^3 - b^3$; (iii) $a^4 - b^4$. Pattern: $(a - b)$(sum of n terms $a^{(n-1)} \ldots b^{(n-1)}$) $= a^n - b^n$. Next: $(a-b)(a^4+a^3b+a^2b^2+ab^3+b^4) = a^5 - b^5$.

Exercise 6.2 — Special Cases of the Distributive Property

Q1 Which is greater: $(a - b)^2$ or $(b - a)^2$? Justify your answer.

SOLUTION

Note that $(b - a) = -(a - b)$.
When we square both sides: $(b - a)^2 = (-(a - b))^2 = (a - b)^2$.
Therefore $(a - b)^2 = (b - a)^2$ — they are equal.

Alternative: expand both.
$(a - b)^2 = a^2 - 2ab + b^2$.
$(b - a)^2 = b^2 - 2ba + a^2 = a^2 - 2ab + b^2$.
Both expressions are identical.

Answer: Neither is greater — $(a - b)^2 = (b - a)^2$ always.

Q2 Express 100 as the difference of two squares.

SOLUTION

We need $a^2 - b^2 = 100$, i.e., $(a + b)(a - b) = 100$.
Choose factor pairs of 100: one convenient pair is 50×2.
Set $a + b = 50$ and $a - b = 2$ => $2a = 52$ => $a = 26$, $b = 24$.
Check: $26^2 - 24^2 = 676 - 576 = 100$. ✓

Another pair: 100×1 gives $a = 50.5$ (not integer, skip).
Another valid integer solution: $a = 10$, $b = 0$ (since $10^2 - 0^2 = 100$).
Other examples can be found by exploring other factor pairs of 100.

Answer: $100 = 26^2 - 24^2$ (since $676 - 576 = 100$). Also $100 = 10^2 - 0^2$.

Q3 Find 406^2, 72^2, 145^2, 1097^2, and 124^2 using the identities you have learnt so far.

SOLUTION

406^2 using Identity 1A: $(a + b)^2 = a^2 + 2ab + b^2$, with $a = 400$, $b = 6$.
$406^2 = 400^2 + 2 \times 400 \times 6 + 6^2 = 160000 + 4800 + 36 = 164836$.

72^2 using Identity 1A: $a = 70$, $b = 2$.
$72^2 = 70^2 + 2 \times 70 \times 2 + 2^2 = 4900 + 280 + 4 = 5184$.

145^2 using Identity 1B: $(a - b)^2 = a^2 - 2ab + b^2$, with $a = 150$, $b = 5$.
$145^2 = 150^2 - 2 \times 150 \times 5 + 5^2 = 22500 - 1500 + 25 = 21025$.

1097^2 using Identity 1B: a = 1100, b = 3.
$1097^2 = 1100^2 - 2 \times 1100 \times 3 + 3^2 = 1210000 - 6600 + 9 = 1203409$.

124^2 using Identity 1A: a = 120, b = 4.
$124^2 = 120^2 + 2 \times 120 \times 4 + 4^2 = 14400 + 960 + 16 = 15376$.

Answer: $406^2 = 164836$; $72^2 = 5184$; $145^2 = 21025$; $1097^2 = 1203409$; $124^2 = 15376$.

Q4 Do Patterns 1 and 2 hold only for counting numbers? Do they hold for negative integers as well? What about fractions? Justify your answer.

SOLUTION

Pattern 1 is $2(a^2 + b^2) = (a + b)^2 + (a - b)^2$.
Pattern 2 is $a^2 - b^2 = (a + b)(a - b)$.

Both identities were derived purely using the distributive property of multiplication and the rules of algebra. They hold for any values of a and b — negative integers or fractions — because the distributive property itself holds for all integers and rational numbers.

Example with negative integers: a = −3, b = 2.
Pattern 1: $2(9 + 4) = 26$; $(-3 + 2)^2 + (-3 - 2)^2 = 1 + 25 = 26$. ✓
Pattern 2: $(-3)^2 - 2^2 = 9 - 4 = 5$; $(-3 + 2)(-3 - 2) = (-1)(-5) = 5$. ✓

Example with fractions: a = 1/2, b = 1/3.
Pattern 2: $(1/2)^2 - (1/3)^2 = 1/4 - 1/9 = 5/36$; $(1/2 + 1/3)(1/2 - 1/3) = (5/6)(1/6) = 5/36$. ✓

Answer: Both patterns hold for all integers (including negatives) and fractions, since they follow from the distributive property which is valid for all rational numbers.

Exercise 6.3 — This Way or That Way, All Ways Lead to the Bay

Q1 Compute these products using the suggested identity.
(i) 46^2 using Identity 1A for $(a + b)^2$
(ii) 397×403 using Identity 1C for $(a + b)(a - b)$
(iii) 91^2 using Identity 1B for $(a - b)^2$
(iv) 43×45 using Identity 1C for $(a + b)(a - b)$

SOLUTION

(i) 46^2 using $(a + b)^2 = a^2 + 2ab + b^2$, with $a = 40$, $b = 6$:
$46^2 = (40 + 6)^2 = 40^2 + 2 \times 40 \times 6 + 6^2 = 1600 + 480 + 36 = 2116$.

(ii) 397×403 using $(a + b)(a - b) = a^2 - b^2$, with $a = 400$, $b = 3$:
$397 \times 403 = (400 - 3)(400 + 3) = 400^2 - 3^2 = 160000 - 9 = 159991$.

(iii) 91^2 using $(a - b)^2 = a^2 - 2ab + b^2$, with $a = 100$, $b = 9$:
$91^2 = (100 - 9)^2 = 100^2 - 2 \times 100 \times 9 + 9^2 = 10000 - 1800 + 81 = 8281$.

(iv) 43×45 using $(a + b)(a - b) = a^2 - b^2$, with $a = 44$, $b = 1$:
$43 \times 45 = (44 - 1)(44 + 1) = 44^2 - 1^2 = 1936 - 1 = 1935$.

Answer: (i) $46^2 = 2116$; (ii) $397 \times 403 = 159991$; (iii) $91^2 = 8281$; (iv) $43 \times 45 = 1935$.

Q2 Use either a suitable identity or the distributive property to find each of the following products.
(i) $(p - 1)(p + 11)$
(ii) $(3a - 9b)(3a + 9b)$
(iii) $-(2y + 5)(3y + 4)$
(iv) $(6x + 5y)^2$
(v) $(2x - 1/2)^2$
(vi) $(7p) \times (3r) \times (p + 2)$

SOLUTION

(i) $(p - 1)(p + 11)$
Use Identity 1 with $a = p$, $m = -1$, $n = 11$:
$(p - 1)(p + 11) = p^2 + 11p - p - 11 = p^2 + 10p - 11$.

(ii) $(3a - 9b)(3a + 9b)$
Use Identity 1C: $(A - B)(A + B) = A^2 - B^2$ with $A = 3a$, $B = 9b$:
$(3a - 9b)(3a + 9b) = (3a)^2 - (9b)^2 = 9a^2 - 81b^2$.

(iii) $-(2y + 5)(3y + 4)$

First expand $(2y + 5)(3y + 4) = 6y^2 + 8y + 15y + 20 = 6y^2 + 23y + 20$.
Then negate: $-(2y + 5)(3y + 4) = -6y^2 - 23y - 20$.

(iv) $(6x + 5y)^2$
Use Identity 1A: $(a + b)^2 = a^2 + 2ab + b^2$ with $a = 6x$, $b = 5y$:
$(6x + 5y)^2 = (6x)^2 + 2(6x)(5y) + (5y)^2 = 36x^2 + 60xy + 25y^2$.

(v) $(2x - 1/2)^2$
Use Identity 1B: $(a - b)^2 = a^2 - 2ab + b^2$ with $a = 2x$, $b = 1/2$:
$(2x - 1/2)^2 = (2x)^2 - 2(2x)(1/2) + (1/2)^2 = 4x^2 - 2x + 1/4$.

(vi) $(7p) \times (3r) \times (p + 2)$
First combine: $7p \times 3r = 21pr$.
Then: $21pr \times (p + 2) = 21p^2r + 42pr$.

Answer: (i) $p^2 + 10p - 11$; (ii) $9a^2 - 81b^2$; (iii) $-6y^2 - 23y - 20$; (iv) $36x^2 + 60xy + 25y^2$; (v) $4x^2 - 2x + 1/4$; (vi) $21p^2r + 42pr$.

Q3 For each statement identify the appropriate algebraic expression(s).
(i) Two more than a square number.
Options: $2 + s$, $(s + 2)^2$, $s^2 + 2$, $s^2 + 4$, $2s^2$, 2^2s
(ii) The sum of the squares of two consecutive numbers.
Options: $m^2 + n^2$, $(m + n)^2$, $m^2 + 1$, $m^2 + (m + 1)^2$, $m^2 + (m - 1)^2$, $(m + (m + 1))^2$, $(2m)^2 + (2m + 1)^2$

SOLUTION

(i) A square number is s^2. Two more than a square number is $s^2 + 2$.
Check each option:
$2 + s$ — this is 2 more than s (not a square number in general).
$(s + 2)^2$ — this is the square of $(s + 2)$, not $s^2 + 2$.
$s^2 + 2$ — this is s^2 (a square number) plus 2. ✓
$s^2 + 4$ — this is 4 more than a square number.
$2s^2$ — this is twice a square number.
$2^2s = 4s$ — this is 4 times s.
Correct answer: $s^2 + 2$.

(ii) Two consecutive numbers can be written as m and $m + 1$. Their sum of squares = $m^2 + (m + 1)^2$.
Check each option:
$m^2 + n^2$ — uses two unrelated variables; not specific to consecutive numbers.

$(m + n)^2$ — square of sum, not sum of squares.
$m^2 + 1$ — only works if the second consecutive number is 1.
$m^2 + (m + 1)^2$ — sum of squares of m and m + 1 (consecutive). ✓
$m^2 + (m - 1)^2$ — consecutive numbers going backwards (also valid as a form).
$(m + (m + 1))^2 = (2m + 1)^2$ — square of sum, not sum of squares.
$(2m)^2 + (2m + 1)^2$ — sum of squares of consecutive even and odd (special case).
The most general answer for any two consecutive numbers m, m+1 is: $m^2 + (m + 1)^2$.

Answer: (i) $s^2 + 2$; (ii) $m^2 + (m + 1)^2$.

Q4 Consider any 2 by 2 square of numbers in a calendar. Find products of numbers lying along each diagonal — 4 × 12 = 48, 5 × 11 = 55. Do this for the other 2 by 2 squares. What do you observe about the diagonal products? Explain why this happens.
Hint: Label the numbers in each 2 by 2 square as: a, (a+1) / (a+7), (a+8).

SOLUTION

Label the 2 × 2 calendar square as:

a	(a + 1)
(a + 7)	(a + 8)

Diagonal 1 (top-left × bottom-right): $a \times (a + 8) = a^2 + 8a$.
Diagonal 2 (top-right × bottom-left): $(a + 1)(a + 7) = a^2 + 8a + 7$.

Difference = $(a + 1)(a + 7) - a(a + 8) = (a^2 + 8a + 7) - (a^2 + 8a) = 7$.

Observation: The product along one diagonal is always 7 more than the product along the other diagonal.

Verification with the given example: a = 4.
4 × 12 = 48 and 5 × 11 = 55. Difference = 55 − 48 = 7. ✓

Another example: a = 11.
11 × 19 = 209 and 12 × 18 = 216. Difference = 7. ✓

This always holds because consecutive numbers in a week differ by 7 (days), so in a calendar column the gap is always 7.

Answer: The product (a+1)(a+7) is always exactly 7 more than a(a+8), because (a+1)(a+7) − a(a+8) = 7 for any value of a.

Q5 Verify which of the following statements are true.
(i) $(k + 1)(k + 2) - (k + 3)$ is always 2.
(ii) $(2q + 1)(2q - 3)$ is a multiple of 4.
(iii) Squares of even numbers are multiples of 4, and squares of odd numbers are 1 more than multiples of 8.
(iv) $(6n + 2)^2 - (4n + 3)^2$ is 5 less than a square number.

SOLUTION

(i) Expand $(k + 1)(k + 2) - (k + 3)$:
$= (k^2 + 3k + 2) - (k + 3)$
$= k^2 + 2k - 1$.
At $k = 1$: $1 + 2 - 1 = 2$. At $k = 2$: $4 + 4 - 1 = 7$. Not always 2.
Statement (i) is FALSE.

(ii) Expand $(2q + 1)(2q - 3)$:
$= 4q^2 - 6q + 2q - 3 = 4q^2 - 4q - 3 = 4(q^2 - q) - 3$.
$4(q^2 - q)$ is a multiple of 4, but subtracting 3 means the result leaves remainder 1 when divided by 4 (since $-3 \equiv 1 \bmod 4$).
At $q = 0$: $(1)(-3) = -3$, which is not a multiple of 4.
Statement (ii) is FALSE.

(iii) Let an even number be $2k$. Then $(2k)^2 = 4k^2$, which is a multiple of 4. ✓
Let an odd number be $2k + 1$. Then $(2k + 1)^2 = 4k^2 + 4k + 1 = 4k(k + 1) + 1$.
Since k and $k + 1$ are consecutive integers, one of them is even, so $k(k + 1)$ is always even.
Therefore $4k(k + 1)$ is a multiple of 8, and $(2k + 1)^2 = 8 \times [k(k+1)/2] + 1$ is 1 more than a multiple of 8. ✓
Statement (iii) is TRUE.

(iv) Expand $(6n + 2)^2 - (4n + 3)^2$:
$(6n + 2)^2 = 36n^2 + 24n + 4$.
$(4n + 3)^2 = 16n^2 + 24n + 9$.
Difference $= 20n^2 - 5 = 5(4n^2 - 1)$.
For this to be 5 less than a perfect square, we need $5(4n^2 - 1) + 5 = 20n^2$ to be a perfect square.
$20n^2 = 4 \times 5 \times n^2$. This is a perfect square only if $5n^2$ is a perfect square, which requires n to be a multiple of 5 — not true for all n.
At $n = 1$: $20(1) = 20$, not a perfect square. Statement (iv) is FALSE.

Answer: Only statement (iii) is true. Statements (i), (ii), and (iv) are false.

Q6 A number leaves a remainder of 3 when divided by 7, and another number leaves a remainder of 5 when divided by 7. What is the remainder when their sum, difference, and product are divided by 7?

SOLUTION

Let the two numbers be $n_1 = 7a + 3$ and $n_2 = 7b + 5$, where a and b are non-negative integers.

Sum: $n_1 + n_2 = (7a + 3) + (7b + 5) = 7(a + b) + 8 = 7(a + b + 1) + 1$.
Remainder when sum is divided by 7 = 1.

Difference $(n_2 - n_1)$: $(7b + 5) - (7a + 3) = 7(b - a) + 2$.
Remainder when difference is divided by 7 = 2.

Product: $n_1 \times n_2 = (7a + 3)(7b + 5) = 49ab + 35a + 21b + 15 = 7(7ab + 5a + 3b + 2) + 1$.
Remainder when product is divided by 7 = 1.

Verification (a = 2, b = 3, so $n_1 = 17$, $n_2 = 26$):
Sum = 43; 43 = 7 × 6 + 1; remainder 1. ✓
Difference = 9; 9 = 7 × 1 + 2; remainder 2. ✓
Product = 442; 442 = 7 × 63 + 1; remainder 1. ✓

Answer: Sum leaves remainder 1; difference leaves remainder 2; product leaves remainder 1 — all when divided by 7.

Q7 Choose three consecutive numbers, square the middle one, and subtract the product of the other two. Repeat the same with other sets of numbers. What pattern do you notice? How do we write this as an algebraic equation? Expand both sides of the equation to check that it is a true identity.

SOLUTION

Let the three consecutive numbers be $(n - 1)$, n, $(n + 1)$.
Square the middle: n^2.
Product of the other two: $(n - 1)(n + 1) = n^2 - 1$ (by Identity 1C).
Difference: $n^2 - (n - 1)(n + 1) = n^2 - (n^2 - 1) = 1$.

The result is always 1.

Algebraic identity: $n^2 - (n - 1)(n + 1) = 1$.

Verification by expansion:
LHS = $n^2 - (n^2 - n + n - 1) = n^2 - n^2 + 1 = 1 = RHS$. ✓

Numerical checks:
n = 5: $25 - 4 \times 6 = 25 - 24 = 1$. ✓
n = 10: $100 - 9 \times 11 = 100 - 99 = 1$. ✓

Answer: The result is always 1. Identity: $n^2 - (n - 1)(n + 1) = 1$.

Q8 What is the algebraic expression describing the following steps — add any two numbers. Multiply this by half of the sum of the two numbers? Prove that this result will be half of the square of the sum of the two numbers.

SOLUTION

Let the two numbers be a and b.
Step 1 — Add them: a + b.
Step 2 — Half of the sum: (a + b)/2.
Step 3 — Multiply the sum by half the sum: $(a + b) \times (a + b)/2 = (a + b)^2/2$.

Proof that this equals half the square of the sum:
Half of the square of the sum = $(a + b)^2/2$.
The result of step 3 = $(a + b) \times (a + b)/2 = (a + b)^2/2$.
They are identical. ✓

Algebraic expression for the result: $(a + b)^2/2$.

Numerical check (a = 3, b = 5): sum = 8, half = 4, product = $8 \times 4 = 32$; $(a+b)^2/2 = 64/2 = 32$. ✓

Answer: The result is $(a + b)^2/2$, which equals half of the square of the sum of the two numbers.

Q9 Which is larger? Find out without fully computing the product.
(i) 14×26 or 16×24
(ii) 25×75 or 26×74

SOLUTION

(i) Compare 14×26 and 16×24.
Write both in terms of distance from 20:
$14 \times 26 = (20 - 6)(20 + 6) = 20^2 - 6^2 = 400 - 36 = 364$.

$16 \times 24 = (20 - 4)(20 + 4) = 20^2 - 4^2 = 400 - 16 = 384$.
Since $4^2 < 6^2$, we subtract less, so $16 \times 24 > 14 \times 26$.
16×24 is larger.

(ii) Compare 25×75 and 26×74.
Write in terms of distance from 50:
$25 \times 75 = (50 - 25)(50 + 25) = 50^2 - 25^2 = 2500 - 625 = 1875$.
$26 \times 74 = (50 - 24)(50 + 24) = 50^2 - 24^2 = 2500 - 576 = 1924$.
Since $24^2 < 25^2$, we subtract less, so $26 \times 74 > 25 \times 75$.
26×74 is larger.

Answer: (i) 16×24 is larger ($384 > 364$); (ii) 26×74 is larger ($1924 > 1875$).

Q10 A tiny park is coming up in Dhauli. The plan is shown in the figure. The two square plots, each of area g^2 sq. ft., will have a green cover. All the remaining area is a walking path w ft. wide that needs to be tiled. Write an expression for the area that needs to be tiled.

SOLUTION

From the plan, the two $g \times g$ square plots sit side by side in a rectangular park, with a w-ft-wide path along every outside edge and a 2w-ft-wide path (w on each side) separating the two squares.

Length of the rectangular park = w (left path) + g + 2w (path between the squares) + g + w (right path) = $2g + 4w$ ft.
Breadth of the rectangular park = w (top path) + g + w (bottom path) = $g + 2w$ ft.

Total area of rectangular park = $(2g + 4w)(g + 2w)$.
Expanding: $= 2g^2 + 4gw + 4gw + 8w^2 = 2g^2 + 8gw + 8w^2$.

Area of both green square plots = $2g^2$.

Area to be tiled = $(2g + 4w)(g + 2w) - 2g^2$
$= 2g^2 + 8gw + 8w^2 - 2g^2$
$= 8gw + 8w^2$
$= 8w(g + w)$ sq. ft.

Verification ($g = 5$, $w = 1$): $8 \times 1 \times (5 + 1) = 48$ sq. ft; direct = $(14)(7) - 50 = 98 - 50 = 48$. ✓

Answer: Area to be tiled = $8w(g + w)$ sq. ft. (equivalently, $(2g + 4w)(g + 2w) - 2g^2$).

Q11 For each pattern shown below,
(i) Draw the next figure in the sequence.
(ii) How many basic units are there in Step 10?
(iii) Write an expression to describe the number of basic units in Step y.
[Pattern (a): staircase of square tiles growing at each step. Pattern (b): rectangular grid of square tiles growing at each step.]

SOLUTION

Pattern (a) — staircase tile pattern:
Step 1: 9 tiles = $(2 + 1)^2 = 3^2$
Step 2: 16 tiles = $(2 + 2)^2 = 4^2$
Step 3: 25 tiles = $(2 + 3)^2 = 5^2$
Step 4 (next figure): 36 tiles = $(2 + 4)^2 = 6^2$

(ii) Step 10: $(2 + 10)^2 = 12^2 = 144$ tiles.
(iii) Expression for Step y: $(y + 2)^2$.

Pattern (b) — rectangular grid pattern:
Step 1: $2^2 + 1 = 5$ tiles
Step 2: $3^2 + 2 = 11$ tiles
Step 3: $4^2 + 3 = 19$ tiles
Step 4 (next figure): $5^2 + 4 = 29$ tiles

(ii) Step 10: $11^2 + 10 = 121 + 10 = 131$ tiles.
(iii) Expression for Step y: $(y + 1)^2 + y$.

Answer: Pattern (a): Step 10 = 144 units; expression = $(y + 2)^2$. Pattern (b): Step 10 = 131 units; expression = $(y + 1)^2 + y$.