Chapter 7: Proportional Reasoning - 1

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Exercise 7.1 — Problem Solving with Proportional Reasoning

Q1 Circle the following statements of proportion that are true.
(i) 4 : 7 :: 12 : 21
(ii) 8 : 3 :: 24 : 6
(iii) 7 : 12 :: 12 : 7
(iv) 21 : 6 :: 35 : 10
(v) 12 : 18 :: 28 : 12
(vi) 24 : 8 :: 9 : 3

SOLUTION

Two ratios a : b and c : d are proportional if ad = bc (cross multiplication).

(i) 4 : 7 :: 12 : 21 — Check: 4 × 21 = 84 and 7 × 12 = 84. Equal, so TRUE.

(ii) 8 : 3 :: 24 : 6 — Check: 8 × 6 = 48 and 3 × 24 = 72. Not equal, so FALSE.

(iii) 7 : 12 :: 12 : 7 — Check: 7 × 7 = 49 and 12 × 12 = 144. Not equal, so FALSE.

(iv) 21 : 6 :: 35 : 10 — Check: 21 × 10 = 210 and 6 × 35 = 210. Equal, so TRUE. Alternatively, 21 : 6 in simplest form = 7 : 2, and 35 : 10 in simplest form = 7 : 2. Same, so TRUE.

(v) 12 : 18 :: 28 : 12 — Check: 12 × 12 = 144 and 18 × 28 = 504. Not equal, so FALSE.

(vi) 24 : 8 :: 9 : 3 — Check: 24 × 3 = 72 and 8 × 9 = 72. Equal, so TRUE. Alternatively, both ratios simplify to 3 : 1. TRUE.

Answer: (i), (iv), and (vi) are true.

Q2 Give 3 ratios that are proportional to 4 : 9.

SOLUTION

A ratio proportional to 4 : 9 is obtained by multiplying (or dividing) both terms by the same non-zero number.

Multiply both terms by 2: 4 × 2 : 9 × 2 = 8 : 18
Multiply both terms by 3: 4 × 3 : 9 × 3 = 12 : 27
Multiply both terms by 4: 4 × 4 : 9 × 4 = 16 : 36

Verify (e.g. 8 : 18): cross multiply with 4 : 9 → 4 × 18 = 72, 9 × 8 = 72. ✓

Answer: Three ratios proportional to 4 : 9 are 8 : 18, 12 : 27, and 16 : 36 (or any other multiples of 4 : 9).

Q3 Fill in the missing numbers for these ratios that are proportional to 18 : 24.
3 : ___ 12 : ___ 20 : ___ 27 : ___

SOLUTION

First, find the simplest form of 18 : 24.
HCF(18, 24) = 6. So 18 : 24 = 3 : 4 in simplest form.

For 3 : __
Factor of change: 3 ÷ 18 = 1/6. Missing term = 24 × (1/6) = 4.
Check: 18 × 4 = 72, 24 × 3 = 72. ✓ => 3 : 4

For 12 : __
Factor of change: 12 ÷ 18 = 2/3. Missing term = 24 × (2/3) = 16.
Check: 18 × 16 = 288, 24 × 12 = 288. ✓ => 12 : 16

For 20 : __
Factor of change: 20 ÷ 18 = 10/9. Missing term = 24 × (10/9) = 240/9 = 80/3.
Check: 18 × (80/3) = 480, 24 × 20 = 480. ✓ => 20 : 80/3

For 27 : __
Factor of change: 27 ÷ 18 = 3/2. Missing term = 24 × (3/2) = 36.
Check: 18 × 36 = 648, 24 × 27 = 648. ✓ => 27 : 36

Answer: 3 : 4, 12 : 16, 20 : 80/3, 27 : 36

Q4 Look at the following rectangles. Which rectangles are similar to each other? You can verify this by measuring the width and height using a scale and comparing their ratios. (Rectangles A, B, C, D, E shown in the figure.)

SOLUTION

Two rectangles are similar when the ratio of their width to height is the same in simplest form.

Measure the width and height of each rectangle A, B, C, D, E using a ruler.
Compute width : height for each rectangle and reduce to simplest form.
Rectangles whose ratios are equal (same simplest form) are similar to each other.

Note: The answer depends on the actual printed measurements of the rectangles in your textbook. Measure carefully and compare the reduced ratios.
For example, if A has ratio 2 : 1 and D also gives 2 : 1 after reduction, then A and D are similar.

Answer: Measure widths and heights with a ruler, reduce each ratio to simplest form, and group the rectangles with the same simplified ratio — those groups are similar to each other.

Q5 Look at the following rectangle. Can you draw a smaller rectangle and a bigger rectangle with the same width to height ratio in your notebooks? Compare your rectangles with your classmates' drawings. Are all of them the same? If they are different from yours, can you think why? Are they wrong?

SOLUTION

Measure the width and height of the given rectangle. Compute the ratio width : height.
Reduce the ratio to its simplest form, say m : n.

For a SMALLER rectangle: choose a scale factor k < 1. Draw a rectangle of width km and height kn.
For a BIGGER rectangle: choose a scale factor k > 1. Draw a rectangle of width km and height kn.

All correctly drawn rectangles will have ratio m : n (the same simplest form), so they are all similar — just different sizes.
Classmates' drawings may differ in size but if their ratios are proportional (same simplest form as the original), they are all correct.

Answer: Any rectangle whose width : height ratio reduces to the same simplest form as the given rectangle is correct — smaller or bigger versions are all valid.

Q6 The following figure shows a small portion of a long brick wall with patterns made using coloured bricks. Each wall continues this pattern throughout the wall. What is the ratio of grey bricks to coloured bricks? Try to give the ratios in their simplest form.
(a) [wall pattern with grey and red coloured bricks]
(b) [wall pattern with grey and brown coloured bricks]

SOLUTION

(a) Count one repeating unit of the pattern:
Grey bricks in one pattern block: rows of 2, 3, 4 grey bricks = 2 + 3 + 4 = 9 grey bricks.
Coloured bricks in one pattern block: rows of 3, 2, 1 coloured bricks = 3 + 2 + 1 = 6 coloured bricks.
Ratio = 9 : 6.
HCF(9, 6) = 3. Simplest form = 9/3 : 6/3 = 3 : 2.

(b) Count one repeating unit of the pattern:
Grey bricks in one pattern block = 16.
Coloured bricks in one pattern block = 12.
Ratio = 16 : 12.
HCF(16, 12) = 4. Simplest form = 16/4 : 12/4 = 4 : 3.

Answer: (a) Grey : Coloured = 3 : 2
(b) Grey : Coloured = 4 : 3

Q7 Let us draw some human figures. Measure your friend's body — the lengths of their head, torso, arms, and legs. Write the ratios as mentioned below:
head : torso = ___ : ___
torso : arms = ___ : ___
torso : legs = ___ : ___
Now, draw a figure with head, torso, arms, and legs with equivalent ratios as above. Does the drawing look more realistic if the ratios are proportional? Why? Why not?

SOLUTION

This is a measurement and drawing activity.

Step 1: Measure your friend's head length, torso length, arm length, and leg length in cm.
Step 2: Record the ratios head : torso, torso : arms, and torso : legs.
Step 3: Reduce each ratio to its simplest form.
Step 4: Draw a figure using any convenient scale, keeping all three ratios proportional to the measured ones.

Example: If head = 20 cm, torso = 50 cm, arms = 60 cm, legs = 80 cm, then:
head : torso = 20 : 50 = 2 : 5
torso : arms = 50 : 60 = 5 : 6
torso : legs = 50 : 80 = 5 : 8

A drawing with these proportional ratios looks more realistic because it maintains the natural body proportions of the person measured.

Answer: Measure and record the three ratios for your friend; draw using proportional measurements. The figure looks more realistic when ratios are maintained because proportional drawings match real body shape.

Exercise 7.2 — Trairasika — The Rule of Three

Q1 The Earth travels approximately 940 million kilometres around the Sun in a year. How many kilometres will it travel in a week?

SOLUTION

1 year = 52 weeks.
Distance in 1 year = 940 million km = 940,000,000 km.

Set up proportion: the ratio of time to distance must be proportional.
52 weeks : 940,000,000 km :: 1 week : x km

By cross multiplication:
52 × x = 940,000,000 × 1
x = 940,000,000 ÷ 52
x = 18,076,923 km (approximately, since 940,000,000 / 52 = 235,000,000 / 13 ≈ 18,076,923.08)

Answer: The Earth travels approximately 18,076,923 km in a week.

Q2 A mason is building a house in the shape shown in the diagram. He needs to construct both the outer walls and the inner wall that separates two rooms. To build a wall of 10-feet, he requires approximately 1450 bricks. How many bricks would he need to build the house? Assume all walls are of the same height and thickness. [Diagram shows an L-shaped house: outer top wall = 15 ft, outer left wall = 12 ft, inner horizontal = 9 ft, lower right section height = 9 ft, lower right section width = 6 ft]

SOLUTION

Identify all wall segments from the floor plan (outer perimeter + inner dividing wall):
The L-shaped house has outer walls and one inner dividing wall.
Tracing all wall segments:
12 ft + 12 ft + 12 ft + 15 ft + 9 ft + 15 ft + 9 ft + 9 ft + 9 ft + 6 ft
Total wall length = 12 + 12 + 12 + 15 + 9 + 15 + 9 + 9 + 9 + 6 = 108 ft.

Set up the proportion:
10 ft of wall requires 1450 bricks.
108 ft of wall requires x bricks.

10 : 1450 :: 108 : x

$10 \times x = 1450 \times 108$

$x = (1450 \times 108) \div 10$

$x = 156{,}600 \div 10 = 15{,}660$ bricks.

Answer: The mason would need 15,660 bricks to build the house.

Exercise 7.3 — Sharing, but Not Equally!

Q1 Divide 4,500 into two parts in the ratio 2 : 3.

SOLUTION

Sum of ratio terms = 2 + 3 = 5.
Size of each group = 4500 ÷ 5 = 900.

First part = 2 × 900 = 1,800.
Second part = 3 × 900 = 2,700.

Check: 1800 + 2700 = 4500 ✓
Check ratio: 1800 : 2700 = 2 : 3 ✓

Answer: The two parts are 1,800 and 2,700.

Q2 In a science lab, acid and water are mixed in the ratio of 1 : 5 to make a solution. In a bottle that has 240 mL of the solution, how much acid and water does the solution contain?

SOLUTION

Acid : Water = 1 : 5.
Sum of ratio terms = 1 + 5 = 6.
Total solution = 240 mL.

Size of each part = 240 ÷ 6 = 40 mL.

Amount of acid = 1 × 40 = 40 mL.
Amount of water = 5 × 40 = 200 mL.

Check: 40 + 200 = 240 mL ✓

Answer: The bottle contains 40 mL of acid and 200 mL of water.

Q3 Blue and yellow paints are mixed in the ratio of 3 : 5 to produce green paint. To produce 40 mL of green paint, how much of these two colours are needed? To make the paint a lighter shade of green, I added 20 mL of yellow to the mixture. What is the new ratio of blue and yellow in the paint?

SOLUTION

Part 1: How much blue and yellow for 40 mL of green?
Blue : Yellow = 3 : 5. Sum = 3 + 5 = 8.
Each part = 40 ÷ 8 = 5 mL.
Blue needed = 3 × 5 = 15 mL.
Yellow needed = 5 × 5 = 25 mL.
Check: 15 + 25 = 40 mL ✓

Part 2: Add 20 mL of yellow to the 40 mL mixture.
New blue amount = 15 mL (unchanged).
New yellow amount = 25 + 20 = 45 mL.
New ratio = 15 : 45.
HCF(15, 45) = 15.
Simplest form = 15/15 : 45/15 = 1 : 3.

Answer: 15 mL of blue and 25 mL of yellow are needed for 40 mL of green. After adding 20 mL yellow, the new ratio of blue : yellow = 1 : 3.

Q4 To make soft idlis, you need to mix rice and urad dal in the ratio of 2 : 1. If you need 6 cups of this mixture to make idlis tomorrow morning, how many cups of rice and urad dal will you need?

SOLUTION

Rice : Urad dal = 2 : 1. Sum = 2 + 1 = 3.
Total mixture needed = 6 cups.
Each part = 6 ÷ 3 = 2 cups.

Rice needed = 2 × 2 = 4 cups.
Urad dal needed = 1 × 2 = 2 cups.

Check: 4 + 2 = 6 cups ✓

Answer: 4 cups of rice and 2 cups of urad dal are needed.

Q5 I have one bucket of orange paint that I made by mixing red and yellow paints in the ratio of 3 : 5. I added another bucket of yellow paint to this mixture. What is the ratio of red paint to yellow paint in the new mixture?

SOLUTION

In the original bucket (1 bucket total), red : yellow = 3 : 5. Sum = 8.
Red paint in the bucket = 3/8 of a bucket.
Yellow paint in the bucket = 5/8 of a bucket.

Added: 1 full bucket of yellow paint.
New yellow amount = 5/8 + 1 = 5/8 + 8/8 = 13/8 buckets.
Red amount remains = 3/8 bucket.

New ratio = (3/8) : (13/8) = 3 : 13.
Check: cross multiply with original fractions: 3 × 8 = 24 vs 8 × 3 = 24... ratio 3:13 is already in simplest form since HCF(3,13) = 1.

Answer: The new ratio of red paint to yellow paint is 3 : 13.

Exercise 7.4 — Unit Conversions

Q1 Anagh mixes 600 mL of orange juice with 900 mL of apple juice to make a fruit drink. Write the ratio of orange juice to apple juice in its simplest form.

SOLUTION

Ratio of orange juice to apple juice = 600 : 900.
HCF(600, 900) = 300.
Simplest form = 600/300 : 900/300 = 2 : 3.

Answer: Orange juice : Apple juice = 2 : 3.

Q2 Last year, we hired 3 buses for the school trip. We had a total of 162 students and teachers who went on that trip and all the buses were full. This year we have 204 students. How many buses will we need? Will all the buses be full?

SOLUTION

Capacity of each bus = 162 ÷ 3 = 54 persons.

Buses needed for 204 persons:
3 buses : 162 persons :: x buses : 204 persons
x = 3 × 204 ÷ 162 = 612 ÷ 162 = 3.78 (approximately).

Since we cannot have a fractional number of buses, we need to round up.
Number of buses needed = 4 buses.

Total seats in 4 buses = 4 × 54 = 216.
Total students/teachers this year = 204.
Empty seats = 216 - 204 = 12.
So all 4 buses will NOT be full — 12 seats will remain vacant.

Answer: 4 buses are needed. No, all the buses will not be full — 12 seats will remain empty.

Q3 The area of Delhi is 1,484 sq. km and the area of Mumbai is 550 sq. km. The population of Delhi is approximately 30 million and that of Mumbai is 20 million people. Which city is more crowded? Why do you say so?

SOLUTION

To compare crowding, calculate population density (persons per sq. km) for each city.

Delhi: 30,000,000 ÷ 1,484 ≈ 20,216 persons per sq. km.

Mumbai: 20,000,000 ÷ 550 ≈ 36,364 persons per sq. km.

Mumbai has more persons per sq. km than Delhi.
Therefore, Mumbai is more crowded.

Answer: Mumbai is more crowded — it has about 36,364 persons per sq. km compared to Delhi's 20,216 persons per sq. km.

Q4 A crane of height 155 cm has its neck and the rest of its body in the ratio 4 : 6. For your height, if your neck and the rest of the body also had this ratio, how tall would your neck be?

SOLUTION

For the crane: neck : rest of body = 4 : 6. Sum = 4 + 6 = 10.
Crane's neck length = (4/10) × 155 = 62 cm.
Crane's body (rest) = (6/10) × 155 = 93 cm. Check: 62 + 93 = 155 cm ✓

For a student of height H cm:
Neck length = (4/10) × H = (2/5) × H cm.

Example: if your height is 150 cm, your neck would be (4/10) × 150 = 60 cm.
Measure your actual height and apply this formula.

Answer: Neck length = (4/10) × your height = (2/5) × your height. For example, at 150 cm tall, neck = 60 cm.

Q5 Let us try an ancient problem from Lilavati. At that time weights were measured in a unit named palas and niskas was a unit of money. "If 2½ palas of saffron costs 3/7 niskas, O expert businessman! tell me quickly what quantity of saffron can be bought for 9 niskas?"

SOLUTION

Given: 2½ palas (= 5/2 palas) of saffron costs 3/7 niskas.
Find: quantity of saffron for 9 niskas.

Set up proportion (palas : niskas are proportional):
(5/2) : (3/7) :: x : 9

By cross multiplication:
(5/2) × 9 = (3/7) × x
45/2 = (3/7) × x
x = (45/2) × (7/3)
x = (45 × 7) / (2 × 3)
x = 315 / 6
x = 52.5 palas.

Answer: 52.5 palas of saffron can be bought for 9 niskas.

Q6 Harmain is a 1-year-old girl. Her elder brother is 5 years old. What will be Harmain's age when the ratio of her age to her brother's age is 1 : 2?

SOLUTION

Let x be the number of years from now when the ratio becomes 1 : 2.
Harmain's age then = (1 + x) years.
Brother's age then = (5 + x) years.

Set up the proportion: (1 + x) : (5 + x) = 1 : 2
=> 2(1 + x) = 1(5 + x)
=> 2 + 2x = 5 + x
=> 2x - x = 5 - 2
=> x = 3

After 3 years: Harmain will be 1 + 3 = 4 years old, brother will be 5 + 3 = 8 years old.
Check ratio: 4 : 8 = 1 : 2 ✓

Answer: Harmain's age will be 4 years (3 years from now) when the ratio of her age to her brother's age is 1 : 2.

Q7 The mass of equal volumes of gold and water are in the ratio 37 : 2. If 1 litre of water is 1 kg in mass, what is the mass of 1 litre of gold?

SOLUTION

Given: mass of equal volumes, gold : water = 37 : 2.
Mass of 1 litre of water = 1 kg.

Set up proportion:
37 : 2 :: x : 1 (where x = mass of 1 litre of gold in kg)

By cross multiplication:
37 × 1 = 2 × x
x = 37 / 2 = 18.5 kg.

Answer: The mass of 1 litre of gold is 18.5 kg.

Q8 It is good farming practice to apply 10 tonnes of cow manure for 1 acre of land. A farmer is planning to grow tomatoes in a plot of size 200 ft by 500 ft. How much manure should he buy? (Please refer to the section on Unit Conversions earlier in this chapter.)

SOLUTION

Area of the plot = 200 ft × 500 ft = 1,00,000 sq ft.

From the unit conversion table: 1 acre = 43,560 sq ft.

Set up proportion: 10 tonnes per 43,560 sq ft :: x tonnes per 1,00,000 sq ft.
43,560 : 10 :: 1,00,000 : x

By cross multiplication:
43,560 × x = 10 × 1,00,000
x = 10,00,000 ÷ 43,560
x ≈ 22.957 tonnes

Converting to kg: 22.957 × 1000 ≈ 22,957 kg.

Answer: The farmer should buy approximately 22.957 tonnes (about 22,957 kg) of cow manure.

Q9 A tap takes 15 seconds to fill a mug of water. The volume of the mug is 500 mL. How much time does the same tap take to fill a bucket of water if the bucket has a 10-litre capacity?

SOLUTION

Volume of bucket = 10 litres = 10 × 1000 mL = 10,000 mL.
(Using: 1 litre = 1,000 mL from the unit conversion table.)

Tap fills 500 mL in 15 seconds.
Set up proportion: time is proportional to volume.
15 seconds : 500 mL :: x seconds : 10,000 mL

By cross multiplication:
500 × x = 15 × 10,000
x = 1,50,000 ÷ 500 = 300 seconds.

Convert to minutes: 300 ÷ 60 = 5 minutes.

Answer: The tap takes 300 seconds (5 minutes) to fill the 10-litre bucket.

Q10 One acre of land costs 15,00,000. What is the cost of 2,400 square feet of the same land?

SOLUTION

From the unit conversion table: 1 acre = 43,560 sq ft.
Cost of 43,560 sq ft = 15,00,000.

Set up proportion:
43,560 : 15,00,000 :: 2,400 : x

By cross multiplication:
43,560 × x = 15,00,000 × 2,400
x = (15,00,000 × 2,400) ÷ 43,560
x = 36,00,00,000 ÷ 43,560
x ≈ 82,645 (approximately).

Answer: The cost of 2,400 square feet of land is approximately 82,645.

Q11 A tractor can plough the same area of a field 4 times faster than a pair of oxen. A farmer wants to plough his 20-acre field. A pair of oxen takes 6 hours to plough an acre of land. How much time would it take if the farmer used a pair of oxen to plough the field? How much time would it take him if he decides to use a tractor instead?

SOLUTION

Time for a pair of oxen to plough 1 acre = 6 hours.
Time for a pair of oxen to plough 20 acres:
6 hours/acre × 20 acres = 120 hours.

A tractor is 4 times faster than oxen.
Time for tractor to plough 1 acre = 6 ÷ 4 = 3/2 = 1.5 hours.
Time for tractor to plough 20 acres:
1.5 hours/acre × 20 acres = 30 hours.

Alternatively, since tractor is 4x faster:
Tractor time = Oxen time ÷ 4 = 120 ÷ 4 = 30 hours.

Answer: Using oxen: 120 hours. Using a tractor: 30 hours.

Q12 The 10 coin is an alloy of copper and nickel called 'cupro-nickel'. Copper and nickel are mixed in a 3 : 1 ratio to get this alloy. The mass of the coin is 7.74 grams. If the cost of copper is 906 per kg and the cost of nickel is 1,341 per kg, what is the cost of these metals in a 10 coin?

SOLUTION

Copper : Nickel = 3 : 1. Sum = 4.
Total mass of coin = 7.74 g.

Mass of copper = (3/4) × 7.74 = 23.22/4 = 5.805 g.
Mass of nickel = (1/4) × 7.74 = 7.74/4 = 1.935 g.
Check: 5.805 + 1.935 = 7.74 g ✓

Cost of copper:
906 per kg = 906 per 1000 g = 0.906 per g.
Cost of 5.805 g copper = 5.805 × 0.906 ≈ 5.26.

Cost of nickel:
1,341 per kg = 1.341 per g.
Cost of 1.935 g nickel = 1.935 × 1.341 ≈ 2.59.

Total cost of metals in the 10 coin = 5.26 + 2.59 ≈ 7.85.

Answer: Cost of copper in the coin ≈ 5.26; cost of nickel ≈ 2.59; total metal cost ≈ 7.85.