Chapter 10: Proportional Reasoning - 2

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Exercise 10.1 — Dividing a Whole in a Given Ratio

Q1 A cricket coach schedules practice sessions that include different activities in a specific ratio — time for warm-up/cool-down : time for batting : time for bowling : time for fielding :: 3 : 4 : 3 : 5. If each session is 150 minutes long, how much time is spent on each activity?

SOLUTION

The ratio of activities is 3 : 4 : 3 : 5.
Sum of ratio terms = 3 + 4 + 3 + 5 = 15.

Time for warm-up/cool-down = 150 × (3/15) = 150 × (1/5) = 30 minutes.
Time for batting = 150 × (4/15) = 40 minutes.
Time for bowling = 150 × (3/15) = 30 minutes.
Time for fielding = 150 × (5/15) = 50 minutes.

Verification: 30 + 40 + 30 + 50 = 150 minutes. ✓

Answer: Warm-up/cool-down: 30 min, Batting: 40 min, Bowling: 30 min, Fielding: 50 min

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Q2 A school library has books in different languages in the following ratio — no. of Odiya books : no. of Hindi books : no. of English books :: 3 : 2 : 1. If the library has 288 Odiya books, how many Hindi and English books does it have?

SOLUTION

The ratio is Odiya : Hindi : English = 3 : 2 : 1.
The library has 288 Odiya books, which corresponds to 3 parts.

1 part = 288 ÷ 3 = 96 books.

Number of Hindi books = 2 parts = 2 × 96 = 192 books.
Number of English books = 1 part = 1 × 96 = 96 books.

Verification: 288 : 192 : 96 = 3 : 2 : 1. ✓

Answer: Hindi books: 192, English books: 96

Q3 I have 100 coins in the ratio — no. of 10 coins : no. of 5 coins : no. of 2 coins : no. of 1 coins :: 4 : 3 : 2 : 1. How much money do I have in coins?

SOLUTION

The ratio is 10 : 5 : 2 : 1 coins = 4 : 3 : 2 : 1.
Sum of ratio terms = 4 + 3 + 2 + 1 = 10.
Total coins = 100.

Number of 10 coins = 100 × (4/10) = 40 coins.
Number of 5 coins = 100 × (3/10) = 30 coins.
Number of 2 coins = 100 × (2/10) = 20 coins.
Number of 1 coins = 100 × (1/10) = 10 coins.

Total money = (40 × 10) + (30 × 5) + (20 × 2) + (10 × 1)
= 400 + 150 + 40 + 10
= 600.

Answer: 600

Q4 Construct a triangle with sidelengths in the ratio 3 : 4 : 5. Will all the triangles drawn with this ratio of sidelengths be congruent to each other? Why or why not?

SOLUTION

Step 1: Check that 3 : 4 : 5 satisfies the triangle inequality.
3 + 4 = 7 > 5 ✓, 3 + 5 = 8 > 4 ✓, 4 + 5 = 9 > 3 ✓.
Also, $3^2 + 4^2 = 9 + 16 = 25 = 5^2$, so this is a right-angled triangle.

Step 2: To construct, choose actual lengths in the ratio 3 : 4 : 5 (e.g., 3 cm, 4 cm, 5 cm). Draw the base 4 cm, then use a compass to mark arcs of radius 3 cm and 5 cm from the two ends to find the third vertex.

Step 3: Will all such triangles be congruent?
No. Triangles with sidelengths in the ratio 3 : 4 : 5 are all SIMILAR (same shape, same angles), but they will only be congruent if their actual sidelengths are also

equal.

For example, a triangle with sides 3 cm, 4 cm, 5 cm and a triangle with sides 6 cm, 8 cm, 10 cm both have the ratio 3 : 4 : 5 but are NOT congruent — they have the same shape but different sizes.

Answer: A valid right-angled triangle can be constructed. All such triangles are similar (same angles) but NOT necessarily congruent — they can have different actual sizes.

Q5 Can you construct a triangle with sidelengths in the ratio 1 : 3 : 5? Why or why not?

SOLUTION

To form a valid triangle, the sum of any two sides must be greater than the third side (triangle inequality).

Check the smallest two sides against the largest: 1 + 3 = 4, which is NOT greater than 5.

Since 1 + 3 = 4 < 5, the triangle inequality is violated.

The three line segments with lengths in the ratio 1 : 3 : 5 cannot enclose a triangle — they would lie flat on a line.

Answer: No. Since 1 + 3 = 4 < 5, the triangle inequality is not satisfied, so no triangle with sidelengths in ratio 1 : 3 : 5 can be constructed.

Exercise 10.2 — A Slice of the Pie

Q1 A group of 360 people were asked to vote for their favourite season from the three seasons — rainy, winter and summer. 90 liked the summer season, 120 liked the rainy season, and the rest liked the winter. Draw a pie chart to show this information.

SOLUTION

Step 1: Find the number who liked winter.

Winter = 360 − 90 − 120 = 150 people.

Step 2: Calculate the angle for each season. The total angle in a circle is 360°.

Angle for Summer = (90 / 360) × 360° = 90°.

Angle for Rainy = (120 / 360) × 360° = 120°.

Angle for Winter = (150 / 360) × 360° = 150°.

Verification: 90° + 120° + 150° = 360°. ✓

Step 3: Draw a circle. Draw a radius. Measure and mark 90° for Summer, then 120° from that mark for Rainy; the remaining 150° sector is Winter. Label and colour each slice.

Answer: Summer: 90°, Rainy: 120°, Winter: 150°

Q2 Draw a pie chart based on the following information about viewers' favourite type of TV channel: Entertainment — 50%, Sports — 25%, News — 15%, Information — 10%.

SOLUTION

Convert each percentage to a sector angle (multiply by 360° / 100):

Entertainment = 50% of 360° = (50/100) × 360° = 180°.

Sports = 25% of 360° = (25/100) × 360° = 90°.

News = 15% of 360° = (15/100) × 360° = 54°.

Information = 10% of 360° = (10/100) × 360° = 36°.

Verification: 180° + 90° + 54° + 36° = 360°. ✓

Draw a circle, draw a radius, and mark each sector in order: 180° for Entertainment, then 90° for Sports, then 54° for News, and the remaining 36° for Information. Label and colour each slice.

Answer: Entertainment: 180°, Sports: 90°, News: 54°, Information: 36°

Q3 Prepare a pie chart that shows the favourite subjects of the students in your class. Collect the data of the number of students for each subject (Language, Arts Education, Vocational Education, Social Science, Physical Education, Maths, Science), write these numbers in the table, and construct a pie chart.

SOLUTION

Step 1: Collect the data from your class by asking each student to choose exactly one favourite subject.

Step 2: Count the number of students for each subject and record them in the table.

Step 3: For each subject, calculate its sector angle:
Angle = (Number of students for that subject / Total students) × 360°.

Step 4: Draw a circle, draw a radius, and mark each sector using a protractor for the calculated angle. Label and colour each slice.

Example: If total students = 40 and Maths has 10 students,
Angle for Maths = (10/40) × 360° = 90°.

Answer: Answers will vary based on actual class data. Use the formula: angle = (count / total) × 360° for each subject.

Exercise 10.3 — Inverse Proportions

Q1 Which of these are in inverse proportion?
(i) x: 40, 80, 25, 16 | y: 20, 10, 32, 50
(ii) x: 40, 80, 25, 16 | y: 20, 10, 12.5, 8
(iii) x: 30, 90, 150, 10 | y: 15, 5, 3, 45

SOLUTION

Two quantities are in inverse proportion if their product x × y is constant for all pairs.

(i) Check products:
40 × 20 = 800, 80 × 10 = 800, 25 × 32 = 800, 16 × 50 = 800.
All products are equal (800). => (i) IS in inverse proportion.

(ii) Check products:
40 × 20 = 800, 80 × 10 = 800, 25 × 12.5 = 312.5, 16 × 8 = 128.
Products are NOT all equal. => (ii) is NOT in inverse proportion.

(iii) Check products:
30 × 15 = 450, 90 × 5 = 450, 150 × 3 = 450, 10 × 45 = 450.
All products are equal (450). => (iii) IS in inverse proportion.

Answer: (i) and (iii) are in inverse proportion; (ii) is not.

Q2 Fill in the empty cells if x and y are in inverse proportion.
x: 16, 12, ___, 36
y: 9, ___, 48, ___

SOLUTION

Since x and y are in inverse proportion, x × y = constant (k).

Find k from the known pair: k = 16 × 9 = 144.

For x = 12: y = 144 / 12 = 12.
For y = 48: x = 144 / 48 = 3.
For x = 36: y = 144 / 36 = 4.

Verification: 12 × 12 = 144 ✓, 3 × 48 = 144 ✓, 36 × 4 = 144 ✓.

Answer: Completed table — x: 16, 12, 3, 36 | y: 9, 12, 48, 4

Exercise 10.4 — Inverse Proportions (Applications)

Q1 Which of the following pairs of quantities are in inverse proportion?
(i) The number of taps filling a water tank and the time taken to fill it.
(ii) The number of painters hired and the days needed to paint a wall of fixed size.
(iii) The distance a car can travel and the amount of petrol in the tank.
(iv) The speed of a cyclist and the time taken to cover a fixed route.
(v) The length of cloth bought and the price paid at a fixed rate per metre.
(vi) The number of pages in a book and the time required to read it at a fixed reading speed.

SOLUTION

(i) More taps => less time to fill the same tank. One quantity increases, the other decreases by the same factor. => INVERSE proportion.

(ii) More painters => fewer days to paint the same wall. => INVERSE proportion.

(iii) More petrol => the car can travel more distance (at fixed fuel efficiency). Both increase together. => DIRECT proportion, NOT inverse.

(iv) Higher speed => less time to cover the same fixed route. => INVERSE proportion.

(v) More cloth bought => more money paid (at a fixed rate per metre). Both increase together. => DIRECT proportion, NOT inverse.

(vi) More pages => more time to read (at a fixed reading speed). Both increase together. => DIRECT proportion, NOT inverse.

Answer: (i), (ii), and (iv) are in inverse proportion. (iii), (v), and (vi) are in direct proportion.

Q2 If 24 pencils cost 120, how much will 20 such pencils cost?

SOLUTION

Cost is directly proportional to the number of pencils (at a fixed price per pencil).

Cost per pencil = 120 ÷ 24 = 5.

Cost of 20 pencils = 20 × 5 = 100.

Answer: 100

Q3 A tank on a building has enough water to supply 20 families living there for 6 days. If 10 more families move in there, how long will the water last? What assumptions do you need to make to work out this problem?

SOLUTION

If more families use the water, the supply will last fewer days => inverse proportion.

Original: 20 families × 6 days = 120 family-days of water.

New number of families = 20 + 10 = 30 families.

Using inverse proportion: 20 × 6 = 30 × x.
x = (20 × 6) / 30 = 120 / 30 = 4 days.

Assumptions needed:
- Each family uses exactly the same amount of water per day.
- No water is added to the tank during this period.
- Usage rate stays constant (no change in behaviour).

Answer: The water will last 4 days.

Q4 The pie chart shows the result of a survey on modes of transport used by children to go to school (Walk: 90°, Bus: 120°, Cycle: 60°, Car: 60°, Two-wheeler: 30°). Answer:
(i) What is the most common mode of transport?
(ii) What fraction of children travel by car?
(iii) If 18 children travel by car, how many children took part in the survey? How many children use taxis to travel to school?
(iv) By which two modes of transport are equal numbers of children travelling?

SOLUTION

(i) The most common mode is the one with the largest sector angle.
Bus has the largest angle (120°). => Bus is the most common mode.

(ii) Car sector = 60° out of total 360°.
Fraction by car = 60/360 = 1/6.

(iii) Car sector = 60°, which is 1/6 of the total.
If 18 children travel by car (= 1/6 of total),
Total children = 18 × 6 = 108 children took part in the survey.
Taxi is not shown in the pie chart, so 0 children use taxis.

(iv) Modes with equal sector angles => equal numbers of children.
Car = 60° and Cycle = 60° => equal numbers travel by Car and Cycle.
(Each corresponds to 108 × 60/360 = 18 children.)

Answer: (i) Bus. (ii) 1/6. (iii) 108 children in survey; 0 use taxis. (iv) Car and Cycle.

Q5 Three workers can paint a fence in 4 days. If one more worker joins the team, how many days will it take them to finish the work? What are the assumptions you need to make?

SOLUTION

More workers => fewer days to finish the same work => inverse proportion.

Using $x_1y_1 = x_2y_2$:
$3 \times 4 = 4 \times x_2$.
$x_2 = (3 \times 4) / 4 = 12 / 4 = 3$ days.

Assumptions needed:
- All workers work at the same rate.
- All workers work the same number of hours per day.
- No time is lost in coordination or setup.

Answer: 3 days

Q6 It takes 6 hours to fill 2 tanks of the same size with a pump. How long will it take to fill 5 such tanks with the same pump?

SOLUTION

With the same pump, filling more tanks takes more time => direct proportion.

Time to fill 1 tank = 6 ÷ 2 = 3 hours.

Time to fill 5 tanks = 5 × 3 = 15 hours.

Answer: 15 hours

Q7 A given set of chairs are arranged in 25 rows, with 12 chairs in each row. If the chairs are rearranged with 20 chairs in each row, how many rows does this new arrangement have?

SOLUTION

Total number of chairs = 25 × 12 = 300 chairs.

The total number of chairs does not change when they are rearranged.

New number of rows = 300 ÷ 20 = 15 rows.

Alternatively, using inverse proportion (more chairs per row => fewer rows):
25 × 12 = new rows × 20.
New rows = 300 / 20 = 15.

Answer: 15 rows

Q8 A school has 8 periods a day, each of 45 minutes duration. How long is each period, if the school has 9 periods a day, assuming that the number of school hours per day stays the same?

SOLUTION

Total school time per day = 8 × 45 = 360 minutes.

More periods in the same total time => each period is shorter => inverse proportion.

Duration of each period with 9 periods = 360 ÷ 9 = 40 minutes.

Verification: 9 × 40 = 360 minutes. ✓

Answer: 40 minutes

Q9 A small pump can fill a tank in 3 hours, while a large pump can fill the same tank in 2 hours. If both pumps are used together, how long will the tank take to fill?

SOLUTION

Consider the whole tank as 1 unit of work.

Rate of small pump = 1/3 tank per hour.
Rate of large pump = 1/2 tank per hour.

Combined rate = 1/3 + 1/2 = 2/6 + 3/6 = 5/6 tank per hour.

Time to fill 1 tank = 1 ÷ (5/6) = 6/5 hours.

6/5 hours = 1 hour 12 minutes.

Answer: 6/5 hours (1 hour 12 minutes)

Q10 A factory requires 42 machines to produce a given number of toys in 63 days. How many machines are required to produce the same number of toys in 54 days?

SOLUTION

Fewer days available => more machines needed (to maintain the same output) => inverse proportion.

Using $x_1y_1 = x_2y_2$:

$42 \times 63 = x_2 \times 54$.

$x_2 = (42 \times 63) / 54 = 2646 / 54 = 49$ machines.

Answer: 49 machines

Q11 A car takes 2 hours to reach a destination, travelling at a speed of 60 km/h. How long will the car take if it travels at a speed of 80 km/h?

SOLUTION

Find the distance to the destination:

Distance = speed × time = 60 × 2 = 120 km.

Higher speed => less time => inverse proportion (distance is fixed).

Time at 80 km/h = Distance / Speed = 120 / 80 = 3/2 hours = 1.5 hours.

Verification using inverse proportion: 60 × 2 = 80 × 1.5 = 120. ✓

Answer: 1.5 hours (1 hour 30 minutes)