Chapter 12: Tales by Dots and Lines

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Exercise 12.1 — The Balancing Act (Mean and Median)

Q1 Find the mean of the following data and share your observations: (i) The first 50 natural numbers. (ii) The first 50 odd numbers. (iii) The first 50 multiples of 4.

SOLUTION

(i) First 50 natural numbers: 1, 2, 3, …, 50.
Sum = 50 × 51 / 2 = 1275.
Mean = 1275 / 50 = 25.5.

(ii) First 50 odd numbers: 1, 3, 5, …, 99.
Sum = 50^2 = 2500.
Mean = 2500 / 50 = 50.
Observation: the mean equals the 25th and 26th odd numbers (49 and 51), i.e., the middle of the sequence. It also equals n = 50, the count of terms.

(iii) First 50 multiples of 4: 4, 8, 12, …, 200.
Sum = 4 × (1 + 2 + … + 50) = 4 × 1275 = 5100.
Mean = 5100 / 50 = 102.

Observation across all three: in each case the mean equals the average of the first and last term, confirming that for evenly-spaced (arithmetic) sequences the mean lies exactly at the midpoint of the sequence.

Answer: (i) 25.5 (ii) 50 (iii) 102

Q2 The dot plot below shows a collection of data and its average; but one dot is missing. Mark the missing value so that the mean is 9.

SOLUTION

From the dot plot the existing 10 values are: 4, 7, 8, 8, 9, 9, 9, 9, 9, 11 (one dot at 4, one at 7, two at 8, five at 9, one at 11).
Sum of existing values = 4 + 7 + 8 + 8 + 9 + 9 + 9 + 9 + 9 + 11 = 83.
For the mean of all 11 values (10 existing + 1 missing) to be 9:
Required total sum = 9 × 11 = 99.
Missing value = 99 − 83 = 16.
Verify: (4 + 7 + 8 + 8 + 9 + 9 + 9 + 9 + 9 + 11 + 16) / 11 = 99 / 11 = 9. ✓
Place the missing dot at 16 on the dot plot.

Answer: The missing value is 16.

Q3 Sudhakar, the class teacher, asks Shreyas to measure the heights of all 24 students in his class and calculate the average height. Shreyas informs the teacher that the average height is 150.2 cm. Sudhakar discovers that the students were wearing uniform shoes when the measurements were taken and the shoes add 1 cm to the height. (i) Should the teacher get all the heights measured again without the shoes to find the correct average height? Or is there a simpler way? (ii) What is the correct average height of the class? (a) 174.2 cm (b) 126.2 cm (c) 150.2 cm (d) 149.2 cm (e) 151.2 cm (f) None of the above (g) Insufficient information

SOLUTION

(i) No, the teacher does not need to re-measure all heights.
When every value in a data set decreases by the same fixed amount (here, 1 cm), the mean also decreases by that same amount.
Since each student's true height = measured height − 1 cm, the correct average = measured average − 1 cm.

(ii) Correct average height = 150.2 − 1 = 149.2 cm.
This matches option (d).

Answer: (i) No re-measurement needed; subtract 1 from the mean. (ii) 149.2 cm — option (d).

Q4 The three dot plots below show the lengths, in minutes, of songs of different albums. Which of these has a mean of 5.57 minutes? Explain how you arrived at the answer.

SOLUTION

Examine each dot plot visually:
Plot B has dots spread widely from about 1 to 6.5 minutes — its mean is around 3.5 to 4 minutes (low).
Plot C has dots clustered between 4 and 5 minutes — its mean is around 4.5 to 5 minutes (medium).
Plot A has dots clustered on the right, between 5 and 6.5 minutes — its mean is the highest.

For Plot A, reading the dot positions (7 dots: two at 5, and one each at 5.25, 5.5, 5.75, 6, 6.5):
Sum = 5 + 5 + 5.25 + 5.5 + 5.75 + 6 + 6.5 = 39.
Mean = 39 / 7 = 5.571 ≈ 5.57 minutes.

Plots B and C have visibly lower means and cannot reach 5.57.
Therefore, Plot A has a mean of 5.57 minutes.

Answer: Plot A has a mean of 5.57 minutes.

Q5 Find the median of 8, 10, 19, 23, 26, 34, 40, 41, 41, 48, 51, 55, 70, 84, 91, 92. (i) If we include one value to the data (in the given list) without affecting the median, what could that value be? (ii) If we include two values to the data without affecting the median what could the two values be? (iii) If we remove one value from the data without affecting the median what could the value be?

SOLUTION

Sorted data: 8, 10, 19, 23, 26, 34, 40, 41, 41, 48, 51, 55, 70, 84, 91, 92.
n = 16 values.
Median = average of 8th and 9th values = (41 + 41) / 2 = 41.

(i) Adding 1 value gives 17 values; median = 9th value.
Currently 7 values are less than 41 and 7 are greater than 41.
If we add any value v ≤ 41: the two 41s shift to 9th and 10th positions, so 9th = 41. Median stays 41.
If we add any value v > 41: the 41s remain at 8th and 9th; 9th = 41. Median stays 41.

Conclusion: adding any value keeps the median at 41.
Example: add 41 itself, or add 30, or add 100.

(ii) Adding 2 values gives 18 values; median = (9th + 10th) / 2.
For median = 41 we need both the 9th and 10th values to be 41.
This holds when we add one value ≤ 41 and one value ≥ 41 (keeping the two existing 41s at positions 9 and 10).
Example pairs: (20, 50), (41, 60), (30, 41).
If both values < 41: 9th value < 41, making median < 41. Avoid this.
If both values > 41: 9th = 41 but 10th = 48, making median = 44.5. Avoid this.

(iii) Removing 1 value gives 15 values; median = 8th value.
Currently 8th = 41.
Removing any value leaves at least one 41 in the list.
If we remove a value < 41: 6 values remain below 41, two 41s remain; 8th = 41. Median stays.
If we remove a value > 41: 7 below 41, two 41s; 8th = 41. Median stays.
If we remove one of the 41s: 7 below, one 41, 7 above; 8th = 41. Median stays.
Example: remove 8, or remove 92, or remove one 41.

Answer: Median = 41. (i) Any value keeps the median at 41. (ii) Add one value ≤ 41 and one ≥ 41 (e.g. 20 and 50). (iii) Any one value can be removed (median stays 41).

Q6 Examine the statements below and justify if the statement is always true, sometimes true, or never true. (i) Removing a value less than the median will decrease the median. (ii) Including a value less than the mean will decrease the mean. (iii) Including any 4 values will not affect the median. (iv) Including 4 values less than the median will increase the median.

SOLUTION

(i) NEVER TRUE.
Removing a value that is less than the median takes a value away from the LEFT side of the sorted data, so the middle position can only shift toward the larger values — the median increases or stays the same, but can never decrease.
Example: data = 1, 3, 5, 7, 9 (median = 5). Remove 1 (< 5). New data = 3, 5, 7, 9, median = (5+7)/2 = 6. Median INCREASED.
Example: data = 1, 2, 3, 4, 5 (median = 3). Remove 1 (< 3). New data = 2, 3, 4, 5, median = (3+4)/2 = 3.5. Median INCREASED.

In every case, removing a below-median value keeps the median the same or raises it — so the statement 'it will decrease the median' is never true.

(ii) ALWAYS TRUE.
If a new value v < mean m is added to n existing values with total sum S = nm:
New mean = (S + v) / (n + 1) = (nm + v) / (n + 1).
Since v < m: nm + v < nm + m = m(n + 1), so (nm + v)/(n+1) < m.
The mean always decreases when a value below the mean is added.

(iii) SOMETIMES TRUE.
Example where median is unchanged: data = 1,2,3,4,5 (median=3). Add 1,2,4,5 (two each side). New data = 1,1,2,2,3,4,4,5,5, median = 3. Unchanged.
Example where median changes: add 6,7,8,9 (all > median). New data = 1,2,3,4,5,6,7,8,9, median = 5. Changed.

(iv) NEVER TRUE.
Adding 4 values all less than the median pushes the median down (or keeps it the same), but never increases it.
When 4 values < median are added: the left side of the distribution gains 4 extra values; the balance point (median) shifts to the left or stays, never to the right.
Example: data = 1,3,5,7,9 (median = 5). Add 1,2,3,4 (all < 5). Sorted: 1,1,2,3,3,4,5,7,9. Median (5th of 9) = 3 < 5. Median decreased.

Answer: (i) Never true. (ii) Always true. (iii) Sometimes true. (iv) Never true.

Q7 The mean of the numbers 8, 13, 10, 4, 5, 20, y, 10 is 10.375. Find the value of y.

SOLUTION

There are 8 numbers in the list.
Required sum = mean × count = 10.375 × 8 = 83.
Sum of known numbers = 8 + 13 + 10 + 4 + 5 + 20 + 10 = 70.
y = 83 − 70 = 13.
Verify: (8+13+10+4+5+20+13+10) / 8 = 83/8 = 10.375. ✓

Answer: y = 13

Q8 The mean of a set of data with 15 values is 134. Find the sum of the data.

SOLUTION

Mean = Sum / Number of values.
Sum = Mean × Number of values = 134 × 15 = 2010.

Answer: Sum of the data = 2010

Q9 Consider the data: 12, 47, 8, 73, 18, 35, 39, 8, 29, 25, p. Which of the following number(s) could be p if the median of this data is 29? (i) 10 (ii) 25 (iii) 40 (iv) 100 (v) 29 (vi) 47 (vii) 30

SOLUTION

Sort the 10 known values (without p): 8, 8, 12, 18, 25, 29, 35, 39, 47, 73.
With p included there are 11 values. Median = 6th value in sorted order.

Check each option by inserting p and finding the 6th value:

(i) p = 10: sorted = 8, 8, 10, 12, 18, 25, 29, 35, 39, 47, 73. 6th = 25. Median = 25. NOT 29.
(ii) p = 25: sorted = 8, 8, 12, 18, 25, 25, 29, 35, 39, 47, 73. 6th = 25. Median = 25. NOT 29.
(iii) p = 40: sorted = 8, 8, 12, 18, 25, 29, 35, 39, 40, 47, 73. 6th = 29. Median = 29. YES.
(iv) p = 100: sorted = 8, 8, 12, 18, 25, 29, 35, 39, 47, 73, 100. 6th = 29. Median = 29. YES.
(v) p = 29: sorted = 8, 8, 12, 18, 25, 29, 29, 35, 39, 47, 73. 6th = 29. Median = 29. YES.
(vi) p = 47: sorted = 8, 8, 12, 18, 25, 29, 35, 39, 47, 47, 73. 6th = 29. Median = 29. YES.
(vii) p = 30: sorted = 8, 8, 12, 18, 25, 29, 30, 35, 39, 47, 73. 6th = 29. Median = 29. YES.

When p ≤ 25, it inserts before position 6, shifting 25 into the 6th spot (median = 25 ≠ 29).
When p ≥ 29, the value 29 stays at position 6 (median = 29).

Answer: p can be (iii) 40, (iv) 100, (v) 29, (vi) 47, or (vii) 30. Options (i) 10 and (ii) 25 do NOT give median 29.

Q10 The number of times students rode their cycles in a week is shown in the dot plot below. Four students rode their cycles twice in that week. (i) Find the average number of times students rode their cycles. (ii) Find the median number of times students rode their cycles. (iii) Which of the following statements are valid? Why? (a) Everyone used their cycle at least once. (b) Almost everyone used their cycle a few times. (c) There are some students who cycled more than once on some days. (d) Exactly 5 students have used their cycles more than once on some days. (e) The following week, if all of them cycled 1 more time than they did the previous week, what would be the average and median of the next week's data?

SOLUTION

Reading the dot plot (4 students at 2 rides confirms the scale):
0 rides: 3 students, 1 ride: 1 student, 2 rides: 4 students, 3 rides: 7 students,
4 rides: 7 students, 5 rides: 5 students, 6 rides: 4 students, 7 rides: 6 students,
8 rides: 3 students.
Total students n = 3+1+4+7+7+5+4+6+3 = 40.

(i) Total rides = 0×3 + 1×1 + 2×4 + 3×7 + 4×7 + 5×5 + 6×4 + 7×6 + 8×3
= 0 + 1 + 8 + 21 + 28 + 25 + 24 + 42 + 24 = 173.
Mean = 173 / 40 = 4.325 ≈ 4.33 rides.

(ii) With n = 40 (even), median = average of the 20th and 21st values.
Cumulative counts: up to 0 → 3, up to 1 → 4, up to 2 → 8, up to 3 → 15, up to 4 → 22.
Both the 20th and 21st values fall in the '4 rides' group (positions 16 to 22).
Median = (4 + 4) / 2 = 4 rides.

(iii)
(a) INVALID. 3 students have 0 rides — they did not use their cycles at all.
(b) VALID in spirit. 37 out of 40 students (about 92%) rode their cycles, and most rode a moderate number of times (between 2 and 7).
(c) VALID. The 3 students with 8 rides in a 7-day week must have ridden more than once on at least one day (8 > 7).
(d) INVALID. We can confirm only that the 3 students with 8 rides definitely had multi-ride days; students with 7 or fewer rides may or may not have, so we cannot claim it is exactly 5.

(e) If every student rides 1 more time next week:
Every value increases by 1. By the property of mean, new mean = old mean + 1 = 4.325 + 1 = 5.325 ≈ 5.33 rides.

Median also shifts by 1: new median = 4 + 1 = 5 rides.

Answer: (i) Mean ≈ 4.33 rides (= 173/40). (ii) Median = 4 rides. (iii) (a) Invalid, (b) valid, (c) valid, (d) invalid. (e) Next week: mean ≈ 5.33, median = 5.

Q11 A dart-throwing competition was organised in a school. The number of throws participants took to hit the bull's eye (the centre circle) is given in the table below. Describe the data using its minimum, maximum, mean and median.

No. of trials	1	2	3	4	5	6	7	8	9	10
No. of students	1	0	0	1	4	9	12	15	10	10

SOLUTION

Total students n = 1+0+0+1+4+9+12+15+10+10 = 62.

Minimum: 1 trial (1 student hit the bull's eye on the very first throw).
Maximum: 10 trials (10 students needed 10 throws).

Mean:
Sum = 1×1 + 2×0 + 3×0 + 4×1 + 5×4 + 6×9 + 7×12 + 8×15 + 9×10 + 10×10
= 1 + 0 + 0 + 4 + 20 + 54 + 84 + 120 + 90 + 100 = 473.
Mean = 473 / 62 ≈ 7.63 trials.

Median:
n = 62 (even). Median = average of 31st and 32nd values.
Cumulative frequencies:
Up to 1 trial: 1 student.
Up to 4 trials: 1+1 = 2 students.
Up to 5 trials: 2+4 = 6 students.
Up to 6 trials: 6+9 = 15 students.
Up to 7 trials: 15+12 = 27 students.
Up to 8 trials: 27+15 = 42 students.
The 31st and 32nd values both fall in the '8 trials' group (positions 28 to 42).
Median = (8 + 8) / 2 = 8 trials.

Most participants needed 7–9 throws (37 out of 62 = 60%), and only 1 person hit it in the first try.

Answer: Minimum = 1, Maximum = 10, Mean ≈ 7.63 trials, Median = 8 trials.

Exercise 12.2 — Visualising and Interpreting Data (Line Graphs)

Q1 The average number of customers visiting a shop and the average number of customers actually purchasing items over different days of the week is shown in the table below. Visualise this data on a line graph.

Day:	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Visiting:	16	19	10	14	20	22	35
Purchasing:	10	8	7	11	12	16	26

SOLUTION

Draw a line graph with the x-axis showing the days of the week (Mon to Sun) and the y-axis showing number of customers (scale 0 to 35).

Plot two lines:
Line 1 (Visiting): (Mon,16), (Tue,19), (Wed,10), (Thu,14), (Fri,20), (Sat,22), (Sun,35).
Line 2 (Purchasing): (Mon,10), (Tue,8), (Wed,7), (Thu,11), (Fri,12), (Sat,16), (Sun,26).

Connect consecutive points on each line with straight segments. Use different colours or markers for Visiting and Purchasing.

Key observations from the graph:
Both lines show a dip on Wednesday and a strong rise toward Sunday.
Sunday has the highest footfall and the most purchases.
The gap between Visiting and Purchasing (non-purchasers) is largest on Tuesday and smallest on Thursday and Sunday as a proportion.

Answer: Two-line graph with Visiting and Purchasing plotted across Mon–Sun; both peak on Sunday and dip on Wednesday.

Q2 The average number of days of rainfall in each month for a few cities is shown in the table below. (i) What could be the possible method to compile this data? (ii) Mark the data for Mangaluru, Port Blair, and Rameswaram in the line graph shown. You can round off values to the nearest integer. (iii) Based on the line for New Delhi in the graph fill the data in the table. (iv) Which city among these receives the most number of days of rainfall per year? Which city gets the least number of days of rainfall per year? (v) Looking at the table, when is the rainy season in New Delhi and Rameswaram?

SOLUTION

(i) Data collection method: rainfall is recorded daily at weather stations in each city over several years. For each month (e.g., June) the number of rainy days across all years is totalled and divided by the number of years to get the monthly average. This is repeated for all months and all cities.

(ii) Round each value to the nearest integer and plot the three lines on the same graph (x-axis = months Jan–Dec, y-axis = number of rainy days, scale 0–30).
Mangaluru: 0, 0, 0, 2, 6, 24, 28, 25, 14, 9, 4, 1.
Port Blair: 2, 1, 1, 3, 16, 19, 17, 19, 17, 14, 11, 5.
Rameswaram: 3, 1, 2, 3, 3, 0, 1, 1, 2, 8, 10, 8.
Connect with line segments. The Mangaluru line peaks sharply in June–July, Port Blair stays elevated from May–November, and Rameswaram peaks in October–November.

(iii) New Delhi values read from the graph (typical values based on the plotted line):
Jan: 1, Feb: 1, Mar: 1, Apr: 1, May: 1, Jun: 4, Jul: 10, Aug: 10, Sep: 4, Oct: 1, Nov: 0, Dec: 1.
(Exact values depend on the scale reading; students should read off their own graph.)

(iv) Annual rainfall days (total the 12 rounded monthly values):
Port Blair: 2+1+1+3+16+19+17+19+17+14+11+5 ≈ 125 days — MOST.
Mangaluru: 0+0+0+2+6+24+28+25+14+9+4+1 ≈ 113 days.
Rameswaram: 3+1+2+3+3+0+1+1+2+8+10+8 ≈ 42 days.
New Delhi (read from the graph): ≈ 35 days — LEAST.
City with most rainy days: Port Blair. City with fewest rainy days: New Delhi.

(v) Rainy season:
New Delhi: July and August have the most rainy days (rainy season =

July–August, i.e., the south-west monsoon).
Rameswaram: October and November have the most rainy days (rainy season = October–December, i.e., the north-east monsoon).

Answer: (i) Multi-year daily records averaged per month. (iv) Most: Port Blair; Least: New Delhi. (v) New Delhi rainy season = July–August; Rameswaram rainy season = October–December.

Q3 The following line graph shows the number of births in every month in India over a time period. (i) What are your observations? (ii) What was the approximate number of births in July 2017? (iii) What time period does the graph capture? (iv) Compare the number of births in the month of January in the years 2018, 2019, and 2020. (v) Estimate the number of births in the year 2019.

SOLUTION

(i) Observations:
There is a clear seasonal pattern — births peak around March–April each year and dip around November–January.
The pattern repeats roughly year-over-year, showing that birth rates in India have a consistent seasonal rhythm.
Overall the monthly birth count hovers between about 1.4 million and 1.9 million throughout the period.

(ii) From the graph, the number of births in July 2017 is approximately 1.8 million (1,800,000).

(iii) The graph captures the period from approximately July 2017 to January 2020 — about 2.5 years.

(iv) January births comparison:
January 2018: approximately 1.5 million.
January 2019: approximately 1.5 million.
January 2020: approximately 1.5 million.
The January figures are roughly similar across all three years, suggesting a consistent low-birth month at the start of each year.

(v) For the year 2019 (12 months, Jan–Dec 2019), the monthly values from the graph average roughly 1.5–1.6 million.
Estimated total births in 2019 ≈ 12 × 1.55 million ≈ 18.6 million births.

Answer: (ii) ≈ 1.8 million. (iii) July 2017 to January 2020. (iv) January figures are similar (~1.5M) across 2018, 2019, 2020. (v) ≈ 18–19 million births in 2019.

Q4 Share your observations on the infographic (Wheat vs Rice preference in Indian states). Based on this infographic, answer the following: (i) The value of Karnataka is hidden. Can you guess what it could be? (ii) Which are the top 5 states where rice is the most popular? (iii) Which are the top 5 states where wheat is the most popular? (iv) List a few states where the preference between rice and wheat is more or less balanced.

SOLUTION

(i) Karnataka is surrounded by states with very high rice preference: Andhra Pradesh (+92), Kerala (+79), Tamil Nadu (+85), Goa (+57), and Telangana (part of Andhra region). Karnataka likely has a value around +80 (strong rice preference), consistent with its southern coastal location.

(ii) Top 5 rice-preferring states (highest positive values from the map):
1. Manipur (+100), 2. Nagaland (+99), 3. Mizoram (+97), 4. Tripura (+96), 5. Meghalaya (+95).
These are all north-eastern states where rice is the staple grain.

(iii) Top 5 wheat-preferring states (most negative values from the map):
1. Rajasthan (−93), 2. Haryana (−81), 3. Punjab (−78), 4. Madhya Pradesh (−60), 5. Gujarat (−37).
These are all northern/north-western states where wheat is the primary crop.

(iv) States with balanced preference (values close to 0):
Chhattisgarh (+3) — nearly balanced.
Uttar Pradesh (−30) — slightly wheat-leaning but relatively moderate.
Odisha (+45) — mild rice preference.
These states consume significant quantities of both grains.

Answer: (i) Karnataka ≈ +80. (ii) Top rice: Manipur, Nagaland, Mizoram, Tripura, Meghalaya. (iii) Top wheat: Rajasthan, Haryana, Punjab, Madhya Pradesh, Gujarat. (iv) Balanced: Chhattisgarh, Uttar Pradesh, Odisha.

Exercise 12.3 — Data Story and Further Practice (Mean, Median, Line Graphs)

Q1 Mean Grids: (i) Fill the grid with 9 distinct numbers such that the average along each row, column, and diagonal is 10. (ii) Can we fill the grid by changing a few numbers and still get 10 as the average in all directions?

SOLUTION

(i) For the average in every row, column, and diagonal to be 10 with 3 numbers, each row/column/diagonal must sum to 30.

This is a magic square centred on 10. One solution (derived from the standard 3×3 magic square by adding 5 to each entry):

	7	12	11	
	14	10	6	
	9	8	13	

Verify rows: 7+12+11=30, 14+10+6=30, 9+8+13=30. ✓
Verify columns: 7+14+9=30, 12+10+8=30, 11+6+13=30. ✓
Verify diagonals: 7+10+13=30, 11+10+9=30. ✓
All 9 numbers (6,7,8,9,10,11,12,13,14) are distinct. ✓

(ii) Yes. We can change some numbers and still get average 10 in all directions, as long as the magic-square property is kept (each row, column, and diagonal must still sum to 30).
For example, take a completely different set of 9 distinct numbers centred on 10, such as 4, 14, 12 / 18, 10, 2 / 8, 6, 16 — each row, column, and diagonal still sums to 30, so the average stays 10 everywhere.
Many arrangements are possible; the centre cell must always equal 10, because both diagonals, the middle row, and the middle column all pass through it.

Answer: One solution: Row 1 = (7, 12, 11), Row 2 = (14, 10, 6), Row 3 = (9, 8, 13). (ii) Yes, other arrangements are possible by keeping row/column/diagonal sums equal to 30.

Q2 Give two examples of data that satisfy each of the following conditions: (i) 3 numbers whose mean is 8. (ii) 4 numbers whose median is 15.5. (iii) 5 numbers whose mean is 13.6. (iv) 6 numbers whose mean = median. (v) 6 numbers whose mean > median.

SOLUTION

(i) 3 numbers with mean 8 (sum = 24):
Example 1: 7, 8, 9 (sum = 24). ✓
Example 2: 6, 8, 10 (sum = 24). ✓

(ii) 4 numbers with median 15.5 (average of 2nd and 3rd sorted values = 15.5):
Example 1: 10, 15, 16, 20 — median = (15+16)/2 = 15.5. ✓
Example 2: 1, 14, 17, 100 — median = (14+17)/2 = 15.5. ✓

(iii) 5 numbers with mean 13.6 (sum = 68):
Example 1: 10, 12, 14, 16, 16 (sum = 68). ✓
Example 2: 8, 10, 12, 18, 20 (sum = 68). ✓

(iv) 6 numbers with mean = median:
Example 1: 1, 2, 3, 5, 6, 7 — sorted, median = (3+5)/2 = 4; mean = 24/6 = 4. ✓
Example 2: 2, 4, 6, 6, 8, 10 — median = (6+6)/2 = 6; mean = 36/6 = 6. ✓

(v) 6 numbers with mean > median:
Example 1: 1, 2, 3, 4, 5, 20 — median = (3+4)/2 = 3.5; mean = 35/6 ≈ 5.83. 5.83 > 3.5. ✓
Example 2: 1, 1, 1, 1, 1, 100 — median = (1+1)/2 = 1; mean = 105/6 = 17.5. 17.5 > 1. ✓

Answer: See solution for two examples of each condition.

Q3 Fill in the blanks such that the median of the collection is 13: 5, 21, 14, _____, _____, _____. How many possibilities exist if only counting numbers are allowed?

SOLUTION

The collection has 6 values (3 given + 3 blanks). Median of 6 values = average of 3rd and 4th values when sorted.
For median = 13: (3rd value + 4th value) / 2 = 13, so 3rd + 4th = 26.

The known values sorted: 5, 14, 21.
We need to choose blanks a, b, c (counting numbers ≥ 1) so that the 3rd and 4th positions in the sorted 6-value list sum to 26.

One way to achieve this: place 12 and 14 at positions 3 and 4.
Since 14 is already in the list, add blank = 12 and one blank ≤ 12 and one blank ≥ 14.
Example: blanks = 5, 12, 100 — sorted list: 5, 5, 12, 14, 21, 100 — median = (12+14)/2 = 13. ✓

Another way: place 13 and 13 at positions 3 and 4.
Example: blanks = 1, 13, 13 — sorted: 1, 5, 13, 13, 14, 21 — median = (13+13)/2 = 13. ✓

Since the third blank can be any counting number that does not disturb the 3rd and 4th positions, and there is no upper bound, there are infinitely many possibilities.
For each valid (3rd, 4th) pair summing to 26 with values p ≤ q (e.g. p=12, q=14 or p=13, q=13 or p=5, q=21 etc.) the remaining blank can be freely chosen from an unbounded set.
Therefore: infinitely many counting-number possibilities.

Answer: Infinitely many possibilities. Example 1: blanks = 5, 12, 100 (sorted list: 5,5,12,14,21,100, median=13). Example 2: blanks = 1, 13, 13 (sorted: 1,5,13,13,14,21, median=13).

Q4 Fill in the blanks such that the mean of the collection is 6.5: 3, 11, ____, _____, 15, 6. How many possibilities exist if only counting numbers are allowed?

SOLUTION

The collection has 6 values (4 given + 2 blanks).
Required sum = mean × count = 6.5 × 6 = 39.
Sum of given values = 3 + 11 + 15 + 6 = 35.
Sum of 2 blanks = 39 − 35 = 4.

We need two counting numbers (≥ 1) that sum to 4:
(1, 3): 1+3 = 4. ✓
(2, 2): 2+2 = 4. ✓
(3, 1): 3+1 = 4. ✓

Since the two blank positions are ordered (3rd and 4th positions in the written list), (1,3) and (3,1) are different ordered solutions.
There are 3 ordered pairs of counting numbers summing to 4.

Verify: 3+11+1+3+15+6 = 39, mean = 39/6 = 6.5. ✓

Verify: 3+11+2+2+15+6 = 39, mean = 6.5. ✓

Answer: The two blanks must sum to 4. Ordered possibilities: (1,3), (2,2), (3,1) — 3 possibilities. If unordered: {1,3} and {2,2} — 2 possibilities.

Q5 Check whether each of the statements below is true. Justify your reasoning. Use algebra, if necessary, to justify. (i) The average of two even numbers is even. (ii) The average of any two multiples of 5 will be a multiple of 5. (iii) The average of any 5 multiples of 5 will also be a multiple of 5.

SOLUTION

(i) NOT always true — SOMETIMES TRUE.
Let the two even numbers be 2a and 2b (where a, b are integers).
Average = (2a + 2b) / 2 = a + b.
a + b is an integer, but it is even only when a and b have the same parity.
Counter-example: average of 2 and 4 = (2+4)/2 = 3, which is ODD.
Example where it is even: average of 2 and 6 = 4 (even).
Conclusion: the average of two even numbers is sometimes even and sometimes odd.

(ii) NOT always true — SOMETIMES TRUE.
Let the two multiples be 5a and 5b.
Average = (5a + 5b) / 2 = 5(a+b) / 2.
This is a multiple of 5 only when (a+b) is even, i.e., when a and b have the same parity.
Counter-example: average of 5 and 10 = 7.5, which is not even an integer (let alone a multiple of 5).
Example where it works: average of 10 and 20 = 15 (a multiple of 5).
Conclusion: sometimes true.

(iii) NOT always true — SOMETIMES TRUE.
Let the 5 multiples be $5a_1$, $5a_2$, $5a_3$, $5a_4$, $5a_5$.
Average = $5(a_1+a_2+a_3+a_4+a_5) / 5 = a_1+a_2+a_3+a_4+a_5$.
This equals the sum of the five 'multipliers', which can be any integer.
Counter-example: average of 5, 5, 5, 5, 10 = 30/5 = 6, which is NOT a multiple of 5.
Example where it works: average of 5, 10, 15, 20, 25 = 75/5 = 15 (a multiple of 5).
Conclusion: sometimes true.

Answer: (i) Sometimes true (e.g., avg(2,4)=3 is odd). (ii) Sometimes true (e.g., avg(5,10)=7.5 not a multiple of 5). (iii) Sometimes true (e.g., avg(5,5,5,5,10)=6 not a multiple of 5).

Q6 There were 2 new admissions to Sudhakar's class just a couple of days after the class average height was found to be 150.2 cm. (i) Which of the following statements are correct? (a) The average height of the class will increase as there are 2 new values. (b) The average height of the class will remain the same. (c) The heights of the new students have to be measured to find out the new average height. (d) The heights of everyone in the class has to be measured again. (ii) The heights of the two new joinees are 149 cm and 152 cm. Which statements about the class' average height are correct? (a) average remains same (b) average increases (c) average decreases (d) information not sufficient. (iii) Which statements about the new class median height are correct? (a) median remains (b) increases (c) decreases (d) information not sufficient.

SOLUTION

(i) Correct statement: (c) — the heights of the new students have to be measured to find the new average.
(a) is wrong: adding 2 values does not automatically increase the average (depends on whether the new heights are above or below the current average).
(b) is wrong: the average will change unless the new heights both equal exactly 150.2 cm.
(d) is wrong: we only need the old sum (= 150.2 × 24 = 3604.8) and the two new heights — no re-measurement of existing students needed.

(ii) New heights: 149 cm and 152 cm.
Old class: n = 24, mean = 150.2, so sum = 24 × 150.2 = 3604.8 cm.
New sum = 3604.8 + 149 + 152 = 3905.8 cm.
New mean = 3905.8 / 26 ≈ 150.22 cm.
Since 149 + 152 = 301 > 2 × 150.2 = 300.4, the two new heights together pull the mean up slightly.
Correct statement: (b) the average will increase (by about 0.02 cm).

(iii) Without knowing the full sorted list of the original 24 heights, we cannot determine where 149 and 152 fall relative to the current median.
Correct statement: (d) the information is not sufficient to make a claim about the median.

Answer: (i) Only (c) is correct. (ii) (b) Average increases (to ≈ 150.22 cm). (iii) (d) Information not sufficient to determine median change.

Q7 Is 17 the average of the data shown in the dot plot below? Share the method you used to answer this question.

SOLUTION

Read the dot plot. Values and their frequencies (from the dot plot image):
14: 2, 15: 2, 16: 3, 17: 5, 18: 4, 19: 4, 20: 3, 21: 1, 23: 1.
Total n = 2+2+3+5+4+4+3+1+1 = 25.

Method 1 — Balance / distance method:
Compute total distance from 17 on each side.
LHS (values < 17): 14 is 3 below (2 dots, total 6), 15 is 2 below (2 dots, total 4), 16 is 1 below (3 dots, total 3). LHS total = 13.
RHS (values > 17): 18 is 1 above (4 dots, total 4), 19 is 2 above (4 dots, total 8), 20 is 3 above (3 dots, total 9), 21 is 4 above (1 dot, total 4), 23 is 6 above (1 dot, total 6). RHS total = 31.
LHS (13) ≠ RHS (31). Since RHS > LHS, the true mean is above 17.

Method 2 — Direct calculation:
Sum = 14×2 + 15×2 + 16×3 + 17×5 + 18×4 + 19×4 + 20×3 + 21×1 + 23×1
= 28 + 30 + 48 + 85 + 72 + 76 + 60 + 21 + 23 = 443.
Mean = 443 / 25 = 17.72.

17.72 ≠ 17. Therefore, 17 is NOT the average of this data.

Answer: No, 17 is not the average. The actual mean = 443/25 = 17.72. The right-side distances from 17 (total 31) exceed the left-side distances (total 13), confirming the mean is above 17.

Q8 The weights of people in a group were measured every month. The average weight for the previous month was 65.3 kg and the median weight was 67 kg. The data for this month showed that one person has lost 2 kg and two have gained 1 kg. What can we say about the change in mean weight and median weight this month?

SOLUTION

Change in mean weight:
The total weight of the group changes by: (−2) + (+1) + (+1) = 0 kg.

Since the total weight is unchanged and the number of people is the same, the mean weight remains 65.3 kg.

Change in median weight:
To determine how the median changes, we need to know the positions of the three people (the one who lost 2 kg and the two who gained 1 kg) relative to the current median of 67 kg.
If the person who lost 2 kg was above the median, their weight moved closer to or possibly crossed the median, potentially shifting it.
Without knowing exactly which individuals changed and where they sit in the sorted data, we cannot determine whether the median increased, decreased, or stayed at 67 kg.
Conclusion: the information is not sufficient to make a definitive claim about the median.

Answer: Mean stays the same at 65.3 kg (net change = −2+1+1 = 0). Median: cannot be determined without knowing which individuals changed and their positions in the distribution.

Q9 The following table shows the retail price (in) of iodised salt in the month of January in a few states over 10 years (2016–2025). (i) Choose data from any 3 states you find interesting and present it through a line graph using an appropriate scale. (ii) What do you find interesting in this data? Share your observations. (iii) Compare the price variation in Gujarat and Uttar Pradesh. (iv) In which state has the price increased the most from 2016 to 2025? (v) What are you curious to explore further?

SOLUTION

(i) Example selection: Mizoram, West Bengal, and Assam (wide variation and contrasting trends).
Plot three lines on a graph with x-axis = years (2016–2025) and y-axis = price in (scale 0–30).
Mizoram line: 20, 20, 22, 22, 20, 22, 25, 27.65, 29.03, 29.8 — steadily rising.
West Bengal line: 9.47, 11.65, 11.63, 11.43, 11.11, 12.79, 16.14, 18.43, 21.66, 23.99 — sharp rise after 2020.
Assam line: 6, 12, 12, 12, 12, 15, 14, 12.02, 13.72, 12.35 — jumped then fluctuated.

(ii) Interesting observations:

Assam had the lowest price (6) in 2016 but it doubled by 2017 and then stabilised.
Mizoram and Uttar Pradesh have consistently had some of the highest prices, likely due to inland transport costs.
West Bengal shows strong price growth after 2020.
Gujarat's price dipped to 13 in 2020 before rising to 19.2 in 2025.

(iii) Gujarat vs Uttar Pradesh:
Gujarat price range: 13 (2020) to 19.2 (2025) — range ≈ 6.20. Relatively stable with moderate increase.
Uttar Pradesh price range: 16.15 (2016) to 26.9 (2024) — range ≈ 10.75. Larger and more consistent upward trend.
UP prices are generally higher than Gujarat throughout the period.

(iv) Price increase from 2016 to 2025:
Andaman: 16 → 20.99 (+4.99, +31%)
Assam: 6 → 12.35 (+6.35, +106%)
Gujarat: 16.5 → 19.2 (+2.70, +16%)
Mizoram: 20 → 29.8 (+9.80, +49%)
Uttar Pradesh: 16.15 → 24.81 (+8.66, +54%)
West Bengal: 9.47 → 23.99 (+14.52, +153%)
West Bengal has the largest absolute increase (14.52) and the largest percentage increase (153%).

(v) Possible further explorations: Why did Assam's price spike sharply in 2017? Does distance from the sea (iodised salt production sites) explain the higher prices in inland states? How does the salt price trend compare with overall food inflation?

Answer: (iii) UP has higher prices and greater variation (range ≈ 10.75) vs Gujarat (range ≈ 6.20). (iv) West Bengal has the largest price increase from 2016 to 2025 (+14.52, +153%).

Q10 Referring to the graph showing primary source of energy for household lighting over time, which of the following statements are valid? (i) In 1983, the majority in rural areas used kerosene while the majority in urban areas used electricity. (ii) The use of kerosene as a primary lighting source has decreased over time in both rural and urban areas. (iii) In the year 2000, 10% of the urban households used electricity as a primary lighting source. (iv) In 2023, there were no power cuts.

SOLUTION

(i) VALID. From the graph: in 1983, the rural kerosene line is above 60% while the rural electricity line is below 30%. In urban areas in 1983, the electricity line is already above 60%. So the rural majority used kerosene and the urban majority used electricity.

(ii) VALID. Both the rural and urban kerosene lines trend downward from 1983 to 2023 — kerosene usage fell in both areas as electricity expanded.

(iii) INVALID. In the year 2000 the urban electricity line is approximately 90–95%, not 10%. The statement appears to confuse the lines or misread the graph.

(iv) INVALID. The graph shows the percentage of households using electricity as their primary lighting source. It tells us nothing about reliability (power cuts). We cannot infer the absence of power cuts from this data.

Answer: Valid: (i) and (ii). Invalid: (iii) — urban electricity in 2000 was ~90%, not 10%; (iv) — graph cannot tell us about power cuts.

Q11 Answer the following questions based on the line graph showing average daily time spent on hobbies and games by urban and rural kids. (i) How long do children aged 10 in urban areas spend each day on hobbies and games? (ii) At what age is the average time spent daily on hobbies and games by rural kids 1.5 hours? (a) 8 years (b) 10 years (c) 12 years (d) 14 years (e) 18 years. (iii) Are the following statements correct? (a) The average time spent daily on hobbies and games by kids aged 15 is twice that of kids aged 10. (b) All rural kids aged 15 spend at least 1 hour on hobbies and games every day.

SOLUTION

(i) From the graph, urban children aged 10 spend approximately 2 hours per day on hobbies and games.

(ii) Reading the rural line at 1.5 hours: the line reaches 1.5 hours at approximately

age 12.
Answer: (c) 12 years.

(iii)(a) NOT necessarily correct. From the graph, urban kids at 15 spend about 1.5 hours and at 10 about 2 hours — so time actually decreases from age 10 to 15; the value at 15 is not twice that at 10.
Rural kids at 15 spend roughly 1 hour and at 10 roughly 2 hours — the 15-year-old value is about half, not double.
Statement (a) is incorrect.

(iii)(b) NOT necessarily correct. The graph shows the AVERAGE daily time for rural kids aged 15 is about 1 hour. An average of 1 hour does not mean every individual child spends at least 1 hour — some may spend less, some more.
Statement (b) is not a valid inference from the graph.

Answer: (i) About 2 hours. (ii) (c) 12 years. (iii)(a) Incorrect — time decreases from age 10 to 15; (iii)(b) Incorrect — average ≠ minimum for every individual.

Q12 The following graphs show the sunrise and sunset times across the year at 4 locations in India (Kibithu, Ghuar Moti, Srinagar, Kanyakumari). (i) At which place does the sun rise the earliest in January? What is the approximate day length at this place in January? (ii) Which place has the longest day length over the year? (iii) Share your observations.

SOLUTION

(i) In January, comparing the sunrise lines (the lower lines near 04:00–08:00 on each graph), Kibithu (the easternmost location, in Arunachal Pradesh) has the earliest sunrise — approximately 06:00. The other locations rise later: Kanyakumari ≈ 06:40, Srinagar ≈ 07:35, Ghuar Moti ≈ 07:40.
At Kibithu in January: sunrise ≈ 06:00, sunset ≈ 16:30.
Approximate day length ≈ 16:30 − 06:00 = 10 hours 30 minutes (about 10–11 hours).

(ii) Day length = sunset time − sunrise time. The location with the greatest variation (and longest summer day) will show the largest gap between its upper (sunset) and lower (sunrise) lines.
Kibithu shows the most variation — its very early summer sunrise (near 04:00) combined with a late sunset pushes its summer day length up to about 14–15 hours.

Kibithu has the longest day length over the year.

(iii) Observations:
Kanyakumari (near the equator) shows very little variation in day length throughout the year — both its sunrise and sunset times stay nearly constant.
Kibithu shows the greatest variation — long summer days and short winter days.
The eastern locations (Kibithu) have sunrise much earlier than western locations (Ghuar Moti), because the sun reaches the east of India first.

Answer: (i) Kibithu has the earliest sunrise in January (≈ 06:00); day length ≈ 10–11 hours. (ii) Kibithu has the longest day over the year (summer day ≈ 14–15 hours).

Q13 The following graph shows the moonrise and moonset time over a month. (i) Find out on what dates amavasya (new moon) and purnima (full moon) were in this month. (ii) What do you notice? What do you wonder?

SOLUTION

(i) Identifying amavasya (new moon) and purnima (full moon) from the moonrise/moonset graph:
On amavasya the moon is very close to the sun — moonrise is near sunrise and moonset is near sunset (the moon is 'invisible' as it rises and sets with the sun). On the graph, this corresponds to dates where the moonrise line (gold/yellow) and the moonset line (blue) are very close together at approximately 06:00 or where one line 'jumps' (the moonrise time wraps around midnight).
On purnima the moon rises at sunset (≈18:00–19:00) and sets at sunrise (≈06:00). On the graph this shows as moonrise at ~18:00 and moonset at ~06:00.

From the graph: the moonrise time crosses midnight (from late evening to early morning) around day 19–20, which marks purnima (full moon) — around the 19th to 20th of the month.
The graph begins (day 1) with moonrise very early (~06:00) and moonset near midnight — suggesting amavasya was just before day 1 (the new moon resets the cycle).
The next amavasya would fall around day 29–30 when the lines converge again.

(ii) Observations:
The moonrise and moonset times shift by approximately 50 minutes later each day.
Unlike the sun (which rises and sets at nearly the same time each day), the

moon's schedule changes dramatically over the month.

The moonrise/moonset lines cross through all 24 hours over the course of one month.

What to wonder: Why does the moon rise about 50 minutes later each day? How does this relate to the moon's orbital period (~29.5 days)?

Answer: (i) Purnima (full moon) ≈ day 19–20 (moonrise at ~18:00); Amavasya (new moon) ≈ day 1 and day 29–30 (moonrise near sunrise time). (ii) The moon rises ~50 minutes later each successive day due to its orbital motion.