Chapter 14: Area

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Exercise 14.1 — Rectangle and Squares

Q1 Identify the missing sidelengths.
(i) A staircase figure made of 4 rectangular strips, each 7 in wide. Heights (top to bottom): 4 in, 3 in, 2 in, and ? in. The two given arrows at the right show the top strip height = 4 in and the total height on the right = 14 in. The '?' at the top labels the unknown width of the top strip, and the '? in' at the bottom labels the unknown height of the bottom strip.
(ii) A large rectangle (total Area = 50 m²) split into two sub-rectangles in the top portion (areas 29 m² and 11 m²), each with a shared height of 4 m. The '?' labels at the top mark the unknown widths of the two sub-rectangles, and the '?' at the bottom-right marks the unknown height of the bottom strip.

SOLUTION

Part (i): The figure is a staircase of 4 rectangular strips, all with the same width.

Strip 1 (top): area = 28 in², height = 4 in => width = 28 ÷ 4 = 7 in
Strip 2: area = 21 in², height = 3 in => width = 21 ÷ 3 = 7 in
Strip 3: area = 14 in², height = 2 in => width = 14 ÷ 2 = 7 in
Strip 4 (bottom): area = 35 in², width = 7 in => height = 35 ÷ 7 = 5 in

Check total height: 4 + 3 + 2 + 5 = 14 in ✓ (matches the '14 in' label on the right)
Missing sidelengths: top width (?) = 7 in; bottom strip height (? in) = 5 in

Part (ii): The outer rectangle has total area = 50 m².
Two sub-rectangles in the top strip share the same height = 4 m.
Area of left sub-rectangle = 29 m² => width = 29 ÷ 4 = 29/4 = 7.25 m
Area of right sub-rectangle = 11 m² => width = 11 ÷ 4 = 11/4 = 2.75 m

Total width of outer rectangle = 7.25 + 2.75 = 10 m

Area in top strip = 29 + 11 = 40 m²
Remaining area (bottom strip) = 50 - 40 = 10 m²
Height of bottom strip = 10 ÷ 10 = 1 m

Missing sidelengths: left width (?) = 7.25 m; right width (?) = 2.75 m; bottom height (?) = 1 m

Answer: (i) Top width = 7 in; bottom strip height = 5 in. (ii) Left width = 29/4 m = 7.25 m; right width = 11/4 m = 2.75 m; bottom height = 1 m.

Q2 Find the area of the spiral tube shown in the figure. The tube has the same width throughout. The rectangular spiral has outer dimensions 20 × 20 (units), with inner concentric rectangles at 15, 10, and 5 (units). The tube width is 1 unit throughout. What should be the length of the straight tube if it is to have the same area as the bent (spiral) tube?

SOLUTION

The spiral tube consists of 8 straight rectangular segments (going from outer to inner):
Segment 1 (top, outer): length = 20 units
Segment 2 (right, outer): length = 20 units
Segment 3 (bottom, second level): length = 15 units
Segment 4 (left, second level): length = 15 units
Segment 5 (top, third level): length = 10 units
Segment 6 (right, third level): length = 10 units
Segment 7 (bottom, inner): length = 5 units
Segment 8 (left, inner): length = 5 units

Total length of the spiral path = 20 + 20 + 15 + 15 + 10 + 10 + 5 + 5 = 100 units

Tube width = 1 unit throughout.
Area of spiral tube = total path length × width = 100 × 1 = 100 sq units

For the straight tube of the same width (1 unit) and same area (100 sq units):
length × 1 = 100 => length = 100 units

Answer: Area of spiral tube = 100 sq units. Length of equivalent straight tube (width = 1 unit) = 100 units.

Q3 In this figure, if the sidelength of the square is doubled, what is the increase in the areas of the regions 1, 2 and 3? Give reasons.

SOLUTION

Let the original sidelength of the square be s.
The two perpendicular lines inside the square divide it into regions 1, 2, 3 (and a fourth region).
Let the original areas be A_1, A_2, A_3 for regions 1, 2, 3 respectively.

When the sidelength is doubled (new side = 2s):
All linear dimensions scale by factor 2, so all areas scale by factor $2^2 = 4$.

New area of region 1 = $4A_1$, increase = $4A_1 - A_1 = 3A_1$
New area of region 2 = $4A_2$, increase = $4A_2 - A_2 = 3A_2$
New area of region 3 = $4A_3$, increase = $4A_3 - A_3 = 3A_3$

Reason: Area is proportional to (sidelength)^2. When sidelength is doubled, area becomes $(2s)^2 = 4s^2$ — four times the original. So the area of EACH region becomes 4 times its original value.
The increase in each region = 3 times its original area.

Answer: The area of each region increases by 3 times its original area (since area scales as sidelength^2, and $(2s)^2 = 4s^2$, giving an increase of 3× the original area for each region).

Exercise 14.2 — Triangles

Q1 Find the areas of the following triangles:
(i) Triangle with base = 4 cm and corresponding altitude = 3 cm (the perpendicular height drawn to the base).
(ii) Triangle DEF with base EF = 5 cm and the perpendicular DN = 3.2 cm drawn from vertex D to the base EF (N lies on EF).
(iii) Triangle NAT: right angle at A, leg AT = 3 cm, leg NA = 4 cm.

SOLUTION

Part (i):
Base = 4 cm, height = 3 cm (perpendicular to the base).
Area = (1/2) × base × height = (1/2) × 4 × 3 = 6 cm²

Part (ii): Area of triangle DEF
Base = EF = 5 cm, and the perpendicular height from D to EF is DN = 3.2 cm.
Area = (1/2) × base × height = (1/2) × 5 × 3.2 = 8 cm²

Part (iii): Area of triangle NAT
Right angle at A, so NA and AT are the two legs (perpendicular to each other).
Base = AT = 3 cm, height = NA = 4 cm.
Area = (1/2) × 3 × 4 = 6 cm²

Answer: (i) 6 cm² (ii) 8 cm² (iii) 6 cm²

Q2 Find the length of the altitude BY. Triangle ABC has AX = 4 units (altitude from A to BC, meeting BC at X), BC = 6 units, and AC = 8 units. BY is the altitude from B to AC.

SOLUTION

First, find the area of triangle ABC using the known altitude AX and base BC.
Area(△ABC) = (1/2) × base × height = (1/2) × BC × AX = (1/2) × 6 × 4 = 12 sq units

Now express the same area using altitude BY and base AC.
Area(△ABC) = (1/2) × AC × BY = (1/2) × 8 × BY = 4 × BY

Setting the two expressions equal:
4 × BY = 12
BY = 12 ÷ 4 = 3 units

Answer: BY = 3 units

Q3 Find the area of △SUB, given that it is isosceles, SE is perpendicular to UB, and the area of △SEB is 24 sq. units.

SOLUTION

Since △SUB is isosceles with SE ⊥ UB, the altitude SE from S to the base UB is also the perpendicular bisector of UB.
Therefore E is the midpoint of UB, so UE = EB.

The altitude SE divides △SUB into two triangles: △SEU and △SEB.
△SEU and △SEB share the same height SE and have equal bases (UE = EB).
Therefore, Area(△SEU) = Area(△SEB) = 24 sq. units

Area(△SUB) = Area(△SEU) + Area(△SEB) = 24 + 24 = 48 sq. units

Answer: Area of △SUB = 48 sq. units

Q4 ABCD, BCEF, and BFGH are identical squares.
(i) If the area of the red region is 49 sq. units, then what is the area of the blue region?
(ii) In another version of this figure, if the total area enclosed by the blue and red regions is 180 sq. units, then what is the area of each square?

SOLUTION

The three identical squares ABCD, BCEF, and BFGH are arranged so that they share common vertices (vertex B is shared by all three; the squares form an L-shape or step pattern).

Part (i):
The red region is a triangle occupying the same fractional area of its square as the blue region does of its (identical) square.
Since all three squares are identical and the colored regions are corresponding triangles in their respective squares, the red region and blue region have equal areas.
Area of blue region = Area of red region = 49 sq. units

Part (ii):
Total area enclosed by blue and red regions = 180 sq. units.
Since blue area = red area (by the same symmetry argument), each = 180 ÷ 2 =

90 sq. units.

The area of each colored region equals the area of one square (the region fills one complete square).

Therefore, area of each square = 90 sq. units.

Answer: (i) Area of blue region = 49 sq. units (ii) Area of each square = 90 sq. units

Exercise 14.3 — Area of any Polygon

Q1 Find the area of the quadrilateral ABCD given that AC = 22 cm, BM = 3 cm, DN = 3 cm, BM is perpendicular to AC, and DN is perpendicular to AC.

SOLUTION

Diagonal AC divides quadrilateral ABCD into two triangles: $\triangle ABC$ and $\triangle ACD$.

BM $\perp$ AC with BM = 3 cm (altitude of $\triangle ABC$ to base AC)
DN $\perp$ AC with DN = 3 cm (altitude of $\triangle ACD$ to base AC)

Area($\triangle ABC$) = (1/2) × AC × BM = (1/2) × 22 × 3 = 33 cm²
Area($\triangle ACD$) = (1/2) × AC × DN = (1/2) × 22 × 3 = 33 cm²

Area(ABCD) = Area($\triangle ABC$) + Area($\triangle ACD$) = 33 + 33 = 66 cm²

Answer: Area of quadrilateral ABCD = 66 cm²

Q2 Find the area of the shaded region given that ABCD is a rectangle. In the figure: AE = 10 cm and EB = 8 cm (so AB = 18 cm); AF = 6 cm and FD = 4 cm (so AD = 10 cm); DC = 18 cm; BC = 10 cm. E is on AB and F is on AD. The segments FE and EC are drawn, and the shaded region is the quadrilateral DFEC.

SOLUTION

Set up coordinates: D = (0, 0), C = (18, 0), B = (18, 10), A = (0, 10).
E is on AB with AE = 10 cm from A: E = (10, 10).
F is on AD with AF = 6 cm from A: F = (0, 4) (6 cm down from A, so FD = 4 cm).

The shaded region is the quadrilateral DFEC with vertices D(0,0), F(0,4), E(10,10), C(18,0).

Method — subtract the two unshaded triangles from the whole rectangle:
Area of rectangle ABCD = AB × BC = 18 × 10 = 180 cm²

Unshaded triangle AFE (top-left): base AE = 10 cm, height AF = 6 cm
Area($\triangle AFE$) = (1/2) × 10 × 6 = 30 cm²

Unshaded triangle EBC (top-right): base EB = 8 cm, height BC = 10 cm
Area($\triangle EBC$) = (1/2) × 8 × 10 = 40 cm²

Shaded area = 180 − 30 − 40 = 110 cm²

Check with the shoelace formula on D(0,0), F(0,4), E(10,10), C(18,0):

Area = (1/2)	0·4 − 0·0 + 0·10 − 10·4 + 10·0 − 18·10 + 18·0 − 0·0	
= (1/2)	0 + (−40) + (−180) + 0	= (1/2)(220) = 110 cm² ✓

Answer: Area of shaded region = 110 cm²

Exercise 14.4 — Parallelogram

Q1 Observe the parallelograms in the figure below (parallelograms a through g, all drawn on a grid with the same base and the same height between the same pair of parallel lines).
(i) What can we say about the areas of all these parallelograms?
(ii) What can we say about their perimeters? Which figure appears to have the maximum perimeter, and which has the minimum perimeter?

SOLUTION

Part (i): All the parallelograms (a) through (g) lie between the same pair of parallel lines and share the same base length.
Area of a parallelogram = base × height.
Since all have the same base and the same perpendicular height (the distance between the parallel lines), all their areas are equal.

Part (ii): The perimeter of a parallelogram = 2 × (base + slant side).
The base is the same for all, but the slant side changes: a more tilted (oblique) parallelogram has a longer slant side.
Parallelogram (a) (or the rectangle-like shape) has the shortest slant side and thus the minimum perimeter.
The most tilted parallelogram (g) has the longest slant side and thus the maximum perimeter.
Minimum perimeter: parallelogram (a); Maximum perimeter: parallelogram (g).

Answer: (i) All parallelograms have equal areas (same base, same height between the parallel lines). (ii) Parallelogram (a) has the minimum perimeter and (g) has the maximum perimeter.

Q2 Find the areas of the following parallelograms:
(i) base = 7 cm, height = 4 cm
(ii) base = 5 cm, height = 3 cm
(iii) base = 4.8 cm, height = 5 cm
(iv) base = 4.4 cm, height = 2 cm

SOLUTION

Area of a parallelogram = base × height

(i) Area = 7 × 4 = 28 cm²
(ii) Area = 5 × 3 = 15 cm²

(iii) Area = 4.8 × 5 = 24 cm²
(iv) Area = 4.4 × 2 = 8.8 cm²

Answer: (i) 28 cm² (ii) 15 cm² (iii) 24 cm² (iv) 8.8 cm²

Q3 Find QN. In parallelogram PQSR: SR = 12 cm (base), QM = 6 cm (perpendicular height from Q to base SR, meeting SR at M), and PS = 7.6 cm (the slant side). QN is the perpendicular from Q to side PS.

SOLUTION

First, find the area of parallelogram PQSR using the known base and height.
Area = base × height = SR × QM = 12 × 6 = 72 cm²

The area can also be expressed using PS as the base and QN as the corresponding height:
Area = PS × QN
72 = 7.6 × QN
QN = 72 ÷ 7.6 = 720/76 = 180/19 ≈ 9.47 cm

Answer: QN = 180/19 cm ≈ 9.47 cm

Q4 Consider a rectangle and a parallelogram of the same sidelengths: 5 cm and 4 cm. Which has the greater area? [Hint: Imagine constructing them on the same base.]

SOLUTION

Rectangle: sidelengths 5 cm and 4 cm.
Area of rectangle = length × width = 5 × 4 = 20 cm²

Parallelogram: same sidelengths 5 cm and 4 cm.
Area of parallelogram = base × height.
If the base = 5 cm, then the height h ≤ 4 cm (since h is the perpendicular distance between the parallel sides, and the slant side = 4 cm; h = 4 only when the slant side is perpendicular, i.e., when it IS a rectangle).
For an oblique parallelogram, h < 4 cm, so area < 5 × 4 = 20 cm².

The rectangle is actually the special case where h = 4 cm exactly.
For any non-rectangular parallelogram with the same sidelengths: area < 20 cm².
Therefore, the rectangle has the greater area.

Answer: The rectangle has the greater area (20 cm²). The parallelogram's height is less than 4 cm unless it is a rectangle, so its area is less than 20 cm².

Q5 Which has greater area — an equilateral triangle or a square of the same sidelength as the triangle? Which has greater area — two identical equilateral triangles together or a square of the same sidelength as the triangle? Give reasons.

SOLUTION

Let the common sidelength be a.

Area of equilateral triangle = $(\sqrt{3}/4) \times a^2 \approx 0.433\ a^2$
Area of square = a^2

Since 0.433 < 1, the SQUARE has the greater area.

For two identical equilateral triangles:
Combined area = $2 \times (\sqrt{3}/4) \times a^2 = (\sqrt{3}/2) \times a^2 \approx 0.866\ a^2$

Since 0.866 < 1, the SQUARE still has the greater area, even compared to two equilateral triangles of the same sidelength.

Reason: $(\sqrt{3}/2) \approx 0.866 < 1$, so two equilateral triangles together still have less area than the square.

Answer: In both cases, the square has the greater area. One equilateral triangle ≈ $0.433a^2 < a^2$ (square); two equilateral triangles ≈ $0.866a^2 < a^2$ (square).

Exercise 14.5 — Rhombus and Trapezium

Q1 Find the area of a rhombus whose diagonals are 20 cm and 15 cm.

SOLUTION

Area of a rhombus = (1/2) × product of diagonals
= (1/2) × d_1 × d_2
= (1/2) × 20 × 15
= (1/2) × 300
= 150 cm^2

Answer: Area of rhombus = 150 cm^2

Q2 Find the areas of the following figures:
(i) A rhombus with side = 10 ft, and one diagonal = 16 ft.
(ii) A trapezium with parallel sides 24 m and 36 m, and height = 14 m.
(iii) A right trapezoid with the two parallel sides (vertical) = 14 in and 6 in, and the horizontal distance between them = 10 in.
(iv) A trapezium with parallel sides 12 ft and 18 ft, and height = 8 ft.

SOLUTION

Part (i): Rhombus with side = 10 ft, diagonal d_1 = 16 ft.
Half of d_1 = 8 ft. Using the Pythagorean theorem to find half of d_2:
half_d_2 = sqrt(side² − (d_1/2)²) = sqrt(10² − 8²) = sqrt(100 − 64) = sqrt(36) = 6 ft
So d_2 = 12 ft.
Area = (1/2) × d_1 × d_2 = (1/2) × 16 × 12 = 96 sq ft

Part (ii): Trapezium with parallel sides a = 24 m and b = 36 m, height h = 14 m.
Area = (1/2) × h × (a + b) = (1/2) × 14 × (24 + 36) = (1/2) × 14 × 60 = 420 m^2

Part (iii): Right trapezoid with parallel sides (vertical) = 14 in and 6 in, horizontal distance = 10 in.
The two parallel sides are the vertical sides of lengths 14 in and 6 in.
The height (perpendicular distance between them) = 10 in.
Area = (1/2) × h × (a + b) = (1/2) × 10 × (14 + 6) = (1/2) × 10 × 20 = 100 sq in

Part (iv): Trapezium with parallel sides 12 ft and 18 ft, height = 8 ft.
Area = (1/2) × 8 × (12 + 18) = (1/2) × 8 × 30 = 120 sq ft

Answer: (i) 96 sq ft (ii) 420 m^2 (iii) 100 sq in (iv) 120 sq ft

Q3 A regular hexagon is divided into a trapezium, an equilateral triangle, and a rhombus, as shown. Find the ratio of their areas.

SOLUTION

Let the side of the regular hexagon be a.
A regular hexagon can be divided into 6 equilateral triangles, each with side a.
Area of one equilateral triangle = $(\sqrt{3}/4)a^2$
Total hexagon area = $6 \times (\sqrt{3}/4)a^2 = (3\sqrt{3}/2)a^2$

From the figure, the hexagon is divided into 3 regions:
Equilateral triangle (1 small equilateral triangle of the 6): area = $(\sqrt{3}/4)a^2$
Rhombus (2 small equilateral triangles): area = $2 \times (\sqrt{3}/4)a^2 = (\sqrt{3}/2)a^2$
Trapezium (3 small equilateral triangles): area = $3 \times (\sqrt{3}/4)a^2 = (3\sqrt{3}/4)a^2$

Verification: 1 + 2 + 3 = 6 small triangles = total hexagon ✓

Ratio of areas = equilateral triangle : rhombus : trapezium
$= 1 \times (\sqrt{3}/4)a^2 : 2 \times (\sqrt{3}/4)a^2 : 3 \times (\sqrt{3}/4)a^2$
= 1 : 2 : 3

Answer: Ratio of areas (equilateral triangle : rhombus : trapezium) = 1 : 2 : 3

Q4 ZYXW is a trapezium with ZY ∥ WX. A is the midpoint of XY. Show that the area of the trapezium ZYXW is equal to the area of △ZWB, where B is the point where line ZA (extended) meets line WX.

SOLUTION

Let ZY = a (shorter parallel side), WX = b (longer parallel side), and h = perpendicular height between ZY and WX.

Set up coordinates: W = (0, 0), X = (b, 0), Y = (b, h) (right side vertical), Z = (b − a, h).
(This is one valid configuration; the argument holds in general.)

Area of trapezium ZYXW = (1/2)(ZY + WX) × h = (1/2)(a + b)h

A is the midpoint of XY:
X = (b, 0), Y = (b, h), so A = midpoint = (b, h/2)

Find where line ZA meets the x-axis (line WX extended):
Z = (b − a, h), A = (b, h/2)
Direction ZA: (b − (b−a), h/2 − h) = (a, −h/2)
Parametric: (b − a + ta, h − t(h/2))

At y = 0: h − t(h/2) = 0 => t = 2
x at t = 2: b − a + 2a = b + a
So B = (b + a, 0), meaning WB = b + a = WX + ZY

Area of △ZWB:
Z = (b − a, h), W = (0, 0), B = (a + b, 0)
Base WB = a + b (along the x-axis), Height from Z to x-axis = h
Area = (1/2) × WB × h = (1/2)(a + b)h

Therefore, Area(△ZWB) = (1/2)(a + b)h = Area(trapezium ZYXW) ✓

Answer: Area of △ZWB = (1/2)(a + b)h = Area of trapezium ZYXW (shown by locating B on line WX extended at distance WB = ZY + WX from W, using A as midpoint of XY).