Chapter 1: Orienting Yourself: The Use of Coordinates

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Exercise 1.1 — Orienting Yourself: The Use of Coordinates (Reiaan's room — Fig 1.3)

Q1 Referring to Fig. 1.3, if D1R1 represents the door to Reiaan's room, how far is the door from the left wall (the y-axis) of the room? How far is the door from the x-axis?

SOLUTION

From Fig. 1.3, the door D1R1 lies entirely on the x-axis (the bottom wall of the room), because both endpoints D1 and R1 have y-coordinate = 0.
The door's left endpoint is D1 = (8, 0).
Distance from the left wall (y-axis) = x-coordinate of D1 = 8 ft.
Distance from the x-axis = y-coordinate of any point on the door = 0 ft.
(The door is part of the bottom wall, so it sits directly on the x-axis.)
The door is 8 ft from the left wall (y-axis) and 0 ft from the x-axis (it lies on the x-axis).

Q2 What are the coordinates of D1?

SOLUTION

From Fig. 1.3, D1 is the left endpoint of the door D1R1 on the x-axis.
Reading the grid, D1 is 8 units to the right of the origin along the x-axis.
Since D1 is on the x-axis, its y-coordinate = 0.
D1 = (8, 0)

Q3 If R1 is the point (11.5, 0), how wide is the door? Do you think this is a comfortable width for the room door? If a person in a wheelchair wants to enter the room, will he/she be able to do so easily?

SOLUTION

D1 = (8, 0) and R1 = (11.5, 0) are both on the x-axis.
Width of door = x-coordinate of R1 − x-coordinate of D1 = 11.5 − 8 = 3.5 ft.
In inches: 3.5 × 12 = 42 inches.
A standard comfortable room door is 2.5–3 ft (30–36 inches). At 3.5 ft (42 inches), this door is wider than standard and very comfortable.
The ADA (accessibility) minimum clear width for a wheelchair is 32 inches (≈ 2.67 ft). At 42 inches, this door exceeds that standard.

Yes, a person in a wheelchair can enter easily — the door width of 42 inches gives ample clearance.

Door width = 3.5 ft (42 inches). This is a comfortable, wider-than-standard door. A wheelchair user can enter easily since 42 inches exceeds the 32-inch accessibility minimum.

Q4 If B1 (0, 1.5) and B2 (0, 4) represent the ends of the bathroom door, is the bathroom door narrower or wider than the room door?

SOLUTION

The bathroom door runs along the y-axis from B1(0, 1.5) to B2(0, 4).

Bathroom door width = y-coordinate of B2 − y-coordinate of B1 = 4 − 1.5 = 2.5 ft.

Room door width = 3.5 ft (from Q3 above).

2.5 ft < 3.5 ft, so the bathroom door is narrower than the room door.

Difference = 3.5 − 2.5 = 1.0 ft.

Bathroom door width = 2.5 ft. Room door width = 3.5 ft. The bathroom door is narrower than the room door by 1 ft.

Exercise 1.2 — Orienting Yourself: The Use of Coordinates (Reiaan's room + bathroom — Fig 1.5)

Q1 Place Reiaan's rectangular study table with three of its feet at the points (8, 9), (11, 9) and (11, 7). (i) Where will the fourth foot of the table be? (ii) Is this a good spot for the table? (iii) What is the width of the table? The length? Can you make out the height of the table?

SOLUTION

The three given feet are A = (8, 9), B = (11, 9), C = (11, 7).
AB is horizontal (y = 9, from x = 8 to x = 11), length = 3 ft.
BC is vertical (x = 11, from y = 9 to y = 7), length = 2 ft.
For a rectangle ABCD, the fourth vertex D satisfies: D = A + C − B = (8 + 11 − 11, 9 + 7 − 9) = (8, 7).

(i) Fourth foot = (8, 7).

Verification: AB = CD = 3 ft (horizontal); BC = DA = 2 ft (vertical). All four right angles confirmed.

(ii) The table occupies $x \in [8, 11]$, $y \in [7, 9]$, which is in the right half of the bedroom away from the bed (bed is roughly $x \in [0, 6.5]$) and clear of the wardrobe ($x \in [3, 7]$, $y \in [0, 2]$). It is a reasonable, accessible spot with space to move around the table.

(iii) Horizontal span = 11 − 8 = 3 ft (length). Vertical span = 9 − 7 = 2 ft (width).
The height of the table cannot be determined from a 2D floor plan — the coordinate system only shows the x- and y-dimensions (length and width on the floor).
(i) Fourth foot at (8, 7). (ii) Good spot — right portion of room, away from bed and wardrobe. (iii) Length = 3 ft, Width = 2 ft. Height cannot be found from a 2D plan.

Q2 If the bathroom door has a hinge at B1 and opens into the bedroom, will it hit the wardrobe? Are there any changes you would suggest if the door is made wider?

SOLUTION

The bathroom door has B1 = (0, 1.5) (hinge) and B2 = (0, 4) (free end), door length = 4 − 1.5 = 2.5 ft.
When the door swings 90° into the bedroom (positive x-direction), the free end moves along an arc of radius 2.5 ft centred at B1(0, 1.5).
The free end travels from (0, 4) [closed] to (2.5, 1.5) [fully open].
The swept arc covers the region: $x \in [0, 2.5]$, $y \in [1.5, 4]$.

From Fig. 1.5, the wardrobe occupies $x \in [3, 7]$, $y \in [0, 2]$.
The door's maximum reach is x = 2.5, while the wardrobe begins at x = 3.
2.5 < 3, so the door does NOT hit the wardrobe.

If the door is made wider (longer), its arc radius increases. For example, a 3 ft door would reach x = 3, just touching the wardrobe corner. A door wider than 3 ft would hit the wardrobe.
Suggested changes if door is wider: (a) Make the door swing into the bathroom instead of the bedroom. (b) Move the wardrobe further from the y-axis (e.g., start it at x = 4 or more). (c) Use

a sliding door instead of a hinged door.
The door does NOT hit the wardrobe — its arc reaches x = 2.5 ft, and the wardrobe starts at x = 3 ft. If the door is made wider than 3 ft, it would hit the wardrobe; the fix is to make the door swing into the bathroom or use a sliding door.

Q3 Look at Reiaan's bathroom. (i) What are the coordinates of the four corners O, F, R, and P of the bathroom? (ii) What is the shape of the showering area SHWR in Reiaan's bathroom? Write the coordinates of the four corners. (iii) Mark off a 3 ft × 2 ft space for the washbasin and a 2 ft × 3 ft space for the toilet. Write the coordinates of the corners of these spaces.

SOLUTION

From Fig. 1.5, the bathroom is attached to the left (negative x side) of the bedroom.

(i) Reading the four labeled red-dot corners from the grid:
O = (0, 0) [origin, shared corner with the bedroom]
F = (0, 9) [top of the bathroom on the y-axis]
R = (−6, 9) [top-left corner of the bathroom]
P = (−6, 0) [bottom-left corner of the bathroom]
The bathroom is a rectangle 6 ft wide (in the x-direction) and 9 ft tall (in the y-direction).

(ii) The showering area SHWR has four corners, read from the grid:
S = (−6, 6) [bottom-left]
H = (−3, 6) [bottom-right — NOT on the y-axis]
W = (−2, 9) [top-right]
R = (−6, 9) [top-left]
Top side RW: from (−6, 9) to (−2, 9) — horizontal, 4 ft long.
Bottom side SH: from (−6, 6) to (−3, 6) — horizontal, 3 ft long.
Left side RS: from (−6, 9) to (−6, 6) — vertical, 3 ft long.
Right side WH: from (−2, 9) to (−3, 6) — the slanted wall.
RW || SH (both horizontal) but RW (4 ft) ≠ SH (3 ft), so SHWR is a TRAPEZOID.

(iii) The remaining bathroom space (below the shower, $x \in [-6, 0]$, $y \in [0, 6]$) is used for the washbasin and toilet.
Example placement (student may choose different valid positions):
Washbasin (3 ft × 2 ft): corners at (−2, 0), (0, 0), (0, 2), (−2, 2).
Toilet (2 ft × 3 ft): corners at (−6, 0), (−4, 0), (−4, 3), (−6, 3).
Both fit within the bathroom floor area and do not overlap.
(i) O = (0,0), F = (0,9), R = (−6,9), P = (−6,0). (ii) Shape is a TRAPEZOID; S = (−6,6), H = (−3,6), W = (−2,9), R = (−6,9) (top RW = 4 ft || bottom SH = 3 ft). (iii) One valid placement: washbasin at (−2,0),(0,0),(0,2),(−2,2); toilet at (−6,0),(−4,0),(−4,3),(−6,3).

Q4 Other rooms in the house: (i) Reiaan's room door leads from the dining room which has the length 18 ft and width 15 ft. The length of the dining room extends from point P to point A. Sketch the dining room and mark the coordinates of its corners. (ii) Place a rectangular 5 ft × 3 ft dining table precisely in the centre of the dining room. Write down the coordinates of the feet of the table.

SOLUTION

(i) P = (−6, 0) is the bottom-left corner of the bathroom (from Q3 above).
A = (12, 0) is the bottom-right corner of the bedroom.
Length from P to A = 12 − (−6) = 18 ft. ✓ This matches the given length.
The dining room extends 15 ft downward (in the negative y-direction) from the bottom wall of the house:

Corner 1: P = (−6, 0)
Corner 2: A = (12, 0)
Corner 3: (12, −15)
Corner 4: (−6, −15)

(ii) The dining room spans $x \in [-6, 12]$ and $y \in [-15, 0]$.
Centre of dining room: $x_c = (-6 + 12)/2 = 3$, $y_c = (-15 + 0)/2 = -7.5$.
Centre = (3, −7.5).
Place the 5 ft dimension along the x-axis and the 3 ft dimension along the y-axis.
Table extends from x = 3 − 2.5 = 0.5 to x = 3 + 2.5 = 5.5 (5 ft span).
Table extends from y = −7.5 − 1.5 = −9 to y = −7.5 + 1.5 = −6 (3 ft span).
The four feet of the table are at:

(0.5, −9), (5.5, −9), (5.5, −6), (0.5, −6).

All four feet lie within the dining room ($x \in [-6,12]$, $y \in [-15,0]$). ✓
(i) Dining room corners: P(−6, 0), A(12, 0), (12, −15), (−6, −15). (ii) Table centred at (3, −7.5); feet at (0.5, −9), (5.5, −9), (5.5, −6), (0.5, −6).

Exercises — Orienting Yourself: The Use of Coordinates

Q1 What are the x-coordinate and y-coordinate of the point of intersection of the two axes?

SOLUTION

The point of intersection of the x-axis and the y-axis is called the origin O.
The origin lies on both axes at the same time.
A point on the x-axis has y-coordinate 0, and a point on the y-axis has x-coordinate 0. Since the origin is on both, both its coordinates are 0.
So the coordinates of the origin are (0, 0): x-coordinate = 0 and y-coordinate = 0.
x-coordinate = 0 and y-coordinate = 0 (the origin is (0, 0)).

Q2 Point W has x-coordinate equal to -5. Can you predict the coordinates of point H which is on the line through W parallel to the y-axis? Which quadrants can H lie in?

SOLUTION

A line drawn through W parallel to the y-axis is a vertical line. Every point on a vertical line has the SAME x-coordinate as W.
Since W has x-coordinate -5, every point on this line (including H) also has x-coordinate -5.
So the coordinates of H are of the form (-5, y), where y can be any real number.
We cannot predict the exact value of y — it depends on where H is on the line.
Now, x-coordinate of H is -5 (negative). Points with a negative x-coordinate lie either in Quadrant II (x negative, y positive) or in Quadrant III (x negative, y negative).
If y = 0, H would lie on the x-axis (on no quadrant).
H = (-5, y) for some value of y; H can lie in Quadrant II (if y > 0) or Quadrant III (if y < 0).

Q3 Consider the points R (3, 0), A (0, -2), M (-5, -2) and P (-5, 2). If they are joined in the same order, predict: (i) Two sides of RAMP that are perpendicular to each other. (ii) One side of RAMP that is parallel to one of the axes. (iii) Two points that are mirror images of each other in one axis. Which axis will this be? Now plot the points and verify your predictions.

SOLUTION

Joining R -> A -> M -> P -> R gives sides RA, AM, MP and PR.
Look at side AM: A (0, -2) to M (-5, -2). Both points have the same y-coordinate (-2), so AM is a horizontal segment — it is parallel to the x-axis.
Look at side MP: M (-5, -2) to P (-5, 2). Both points have the same x-coordinate (-5), so MP is a vertical segment — it is parallel to the y-axis.
A horizontal segment (AM) and a vertical segment (MP) meet at right angles.
(i) So sides AM and MP are perpendicular to each other (they meet at M).
(ii) AM is parallel to the x-axis, and MP is parallel to the y-axis — either can be given. For example, AM is parallel to the x-axis.
(iii) Consider M (-5, -2) and P (-5, 2). They have the same x-coordinate (-5) and their y-coordinates are opposites (-2 and +2). Reflecting a point in the x-axis keeps x the same and

changes the sign of y. So M and P are mirror images of each other in the x-axis.
On plotting R (3, 0), A (0, -2), M (-5, -2), P (-5, 2) and joining them in order, all three predictions are confirmed.
(i) AM and MP are perpendicular. (ii) AM is parallel to the x-axis (and MP is parallel to the y-axis). (iii) M (-5, -2) and P (-5, 2) are mirror images in the x-axis.

Q4 Plot point Z (5, -6) on the Cartesian plane. Construct a right-angled triangle IZN and find the lengths of the three sides. (Comment: Answers may differ from person to person.)

SOLUTION

First plot Z (5, -6): move 5 units right of the origin along the x-axis and 6 units down, in Quadrant IV.
Answers differ because you may choose the other two vertices I and N freely, as long as the triangle IZN has a right angle. Here is one convenient choice with the right angle at Z.
Take I = (5, 0) — directly above Z on the x-axis — and N = (0, -6) — directly to the left of Z on the y-axis.

Side ZI: from Z (5, -6) to I (5, 0). Same x-coordinate, so it is vertical. Length =	0 - (-6)	= 6 units.
Side ZN: from Z (5, -6) to N (0, -6). Same y-coordinate, so it is horizontal. Length =	5 - 0	= 5 units.

ZI is vertical and ZN is horizontal, so the angle at Z is a right angle — triangle IZN is right-angled at Z.
Side IN (the hypotenuse): from I (5, 0) to N (0, -6). By the Baudhayana-Pythagoras Theorem, IN = sqrt((5-0)² + (0-(-6))²) = sqrt(25 + 36) = sqrt(61) ~ 7.81 units.
Check: ZI² + ZN² = 6² + 5² = 36 + 25 = 61 = IN². Confirmed.
One valid triangle: I (5, 0), N (0, -6), right angle at Z; sides ZI = 6, ZN = 5, IN = sqrt(61) ~ 7.81 units. (Other correct answers are possible.)

Q5 What would a system of coordinates be like if we did not have negative numbers? Would this system allow us to locate all the points on a 2-D plane?

SOLUTION

Negative numbers are what let us measure distances to the left of the origin (along the x-axis) and below the origin (along the y-axis).
Without negative numbers, we could only use x >= 0 and y >= 0. So we could mark points only where both coordinates are zero or positive.
That means we would be restricted to the first quadrant (together with the positive x-axis, the positive y-axis, and the origin).
The axes divide the plane into four quadrants. Points in Quadrants II, III and IV all need at least one negative coordinate. These points could not be represented at all.
So no — a coordinate system without negative numbers would only cover one quarter of the plane and could NOT locate all the points of a 2-D plane.

It would cover only the first quadrant (x >= 0, y >= 0); it could NOT locate all points of the 2-D plane, since Quadrants II, III, IV need negative coordinates.

Q6 Are the points M (-3, -4), A (0, 0) and G (6, 8) on the same straight line? Suggest a method to check this without plotting and joining the points.

SOLUTION

Method: three points lie on the same straight line if the 'steepness' (slope) between any two of them is the same. Slope between two points = (change in y) / (change in x). If the slopes match for both pairs, the points are collinear.
Slope of MA, from M (-3, -4) to A (0, 0): (0 - (-4)) / (0 - (-3)) = 4/3.
Slope of AG, from A (0, 0) to G (6, 8): (8 - 0) / (6 - 0) = 8/6 = 4/3.
Both slopes equal 4/3, and the point A (0, 0) is common to both segments.
Since the slopes are equal, MA and AG lie along the same line direction; therefore M, A and G are collinear.
(Alternative method: check whether MA + AG = MG using distances, or verify all three satisfy the line y = (4/3)x — indeed -4 = (4/3)(-3), 0 = (4/3)(0), 8 = (4/3)(6).)
Yes — the slope of MA and of AG are both 4/3, so M, A, G are collinear (they all lie on y = (4/3)x).

Q7 Use your method (from Problem 6) to check if the points R (-5, -1), B (-2, -5) and C (4, -12) are on the same straight line. Now plot both sets of points and check your answers.

SOLUTION

Use the slope method: compare the slope of RB with the slope of BC.
Slope of RB, from R (-5, -1) to B (-2, -5): (-5 - (-1)) / (-2 - (-5)) = (-4) / (3) = -4/3.
Slope of BC, from B (-2, -5) to C (4, -12): (-12 - (-5)) / (4 - (-2)) = (-7) / (6) = -7/6.
The two slopes are -4/3 and -7/6, which are NOT equal (-4/3 = -8/6, but the second is -7/6).
Since the slopes are different, the points do not lie in the same direction; therefore R, B and C are NOT collinear.
On plotting, R (-5, -1), B (-2, -5), C (4, -12) form a bent path (a triangle), confirming they are not on one straight line.
No — slope RB = -4/3 but slope BC = -7/6; the slopes differ, so R, B, C are not collinear.

Q8 Using the origin as one vertex, plot the vertices of: (i) A right-angled isosceles triangle. (ii) An isosceles triangle with one vertex in Quadrant III and the other in Quadrant IV.

SOLUTION

(i) Right-angled isosceles triangle with a vertex at the origin O (0, 0). Put the right angle at O and make the two legs along the axes and equal in length.
Choose the other two vertices as P (4, 0) on the x-axis and Q (0, 4) on the y-axis.
Then OP = 4 units (along x-axis) and OQ = 4 units (along y-axis), so the two legs are equal

(isosceles), and OP is perpendicular to OQ, so the angle at O is 90 degrees (right-angled). Hypotenuse PQ = sqrt(4² + 4²) = sqrt(32) = 4*sqrt(2) units.

So O (0, 0), P (4, 0), Q (0, 4) is a right-angled isosceles triangle. (Other equal-leg choices also work.)

(ii) Isosceles triangle with O (0, 0) as the third vertex, one vertex in Quadrant III and one in Quadrant IV.

Choose A (-3, -5) in Quadrant III (x < 0, y < 0) and B (3, -5) in Quadrant IV (x > 0, y < 0).

Check the two sides from O: OA = sqrt((-3)² + (-5)²) = sqrt(9 + 25) = sqrt(34), and OB = sqrt(3² + (-5)²) = sqrt(9 + 25) = sqrt(34).

OA = OB = sqrt(34), so the triangle OAB is isosceles, with A in Quadrant III and B in Quadrant IV. (Any A (-a, -b) and B (a, -b) with a, b > 0 works, since they are reflections in the y-axis and hence equidistant from O.)

(i) e.g. O (0,0), (4,0), (0,4) — legs 4 and 4, right angle at O. (ii) e.g. O (0,0), A (-3,-5) [Q III], B (3,-5) [Q IV] — OA = OB = sqrt(34). (Answers may vary.)

Q9 The following table shows the coordinates of points S, M and T. In each case, state whether M is the midpoint of segment ST. Justify your answer. Rows: (a) S(-3,0), M(0,0), T(3,0); (b) S(2,3), M(3,4), T(4,5); (c) S(0,0), M(0,5), T(0,-10); (d) S(-8,7), M(0,-2), T(6,-3). When M is the mid-point of ST, can you find any connection between the coordinates of M, S and T?

SOLUTION

The midpoint of a segment joining S (x1, y1) and T (x2, y2) is the point ((x1 + x2)/2, (y1 + y2)/2) — the average of the x-coordinates and the average of the y-coordinates. M is the midpoint only if it equals this point.

Row (a): S (-3, 0), T (3, 0). Midpoint = ((-3+3)/2, (0+0)/2) = (0, 0) = M. YES, M is the midpoint.

Row (b): S (2, 3), T (4, 5). Midpoint = ((2+4)/2, (3+5)/2) = (3, 4) = M. YES, M is the midpoint.

Row (c): S (0, 0), T (0, -10). Midpoint = ((0+0)/2, (0 + (-10))/2) = (0, -5). This is NOT M (0, 5). NO, M is not the midpoint.

Row (d): S (-8, 7), T (6, -3). Midpoint = ((-8+6)/2, (7 + (-3))/2) = (-1, 2). This is NOT M (0, -2). NO, M is not the midpoint.

Connection: whenever M is the midpoint of ST, the coordinates of M are the averages of the coordinates of S and T. That is, x-coordinate of M = (x of S + x of T)/2 and y-coordinate of M = (y of S + y of T)/2. Equivalently, S and T are equal shifts on opposite sides of M.

(a) Yes; (b) Yes; (c) No (midpoint is (0,-5)); (d) No (midpoint is (-1,2)). Connection: M is the midpoint iff its coordinates are the averages: M = ((x_S+x_T)/2, (y_S+y_T)/2).

Q10 Use the connection you found to find the coordinates of B given that M (-7, 1) is the midpoint of A (3, -4) and B (x, y).

SOLUTION

The midpoint formula: M = ((x_A + x_B)/2, (y_A + y_B)/2). So x-coordinate of M = (x_A + x_B)/2 and y-coordinate of M = (y_A + y_B)/2.
For the x-coordinates: -7 = (3 + x)/2. Multiply both sides by 2: -14 = 3 + x, so x = -14 - 3 = -17.
For the y-coordinates: 1 = (-4 + y)/2. Multiply both sides by 2: 2 = -4 + y, so y = 2 + 4 = 6.
Therefore B = (-17, 6).
Check: midpoint of A (3, -4) and B (-17, 6) = ((3 + (-17))/2, (-4 + 6)/2) = (-14/2, 2/2) = (-7, 1) = M. Correct.
B = (-17, 6).

Q11 Let P, Q be points of trisection of AB, with P closer to A, and Q closer to B. Using your knowledge of how to find the coordinates of the midpoint of a segment, how would you find the coordinates of P and Q? Do this for the case when the points are A (4, 7) and B (16, -2).

SOLUTION

The points of trisection P and Q divide AB into three equal parts, so AP = PQ = QB, with P nearer A and Q nearer B.
Idea using midpoints: P is the point one-third of the way from A to B, and Q is two-thirds of the way from A to B. A neat midpoint-based route: Q is the midpoint of P and B, and P is the midpoint of A and Q. So the coordinates are: P = A + (1/3)(B - A) and Q = A + (2/3)(B - A) — i.e. add one-third (and two-thirds) of the total shift from A to B.
Total shift from A (4, 7) to B (16, -2): in x it is 16 - 4 = 12; in y it is -2 - 7 = -9.
P (one-third of the shift from A): x = 4 + (1/3)(12) = 4 + 4 = 8; y = 7 + (1/3)(-9) = 7 - 3 = 4. So P = (8, 4).
Q (two-thirds of the shift from A): x = 4 + (2/3)(12) = 4 + 8 = 12; y = 7 + (2/3)(-9) = 7 - 6 = 1. So Q = (12, 1).
Check with midpoints: midpoint of A (4, 7) and Q (12, 1) = (8, 4) = P. Midpoint of P (8, 4) and B (16, -2) = (12, 1) = Q. Both confirm equal thirds.
P = (8, 4) and Q = (12, 1). (P is A plus one-third of the A->B shift; Q is A plus two-thirds.)

Q12 (i) Given the points A (1, -8), B (-4, 7) and C (-7, -4), show that they lie on a circle K whose center is the origin O (0, 0). What is the radius of circle K? (ii) Given the points D (-5, 6) and E (0, 9), check whether D and E lie within the circle, on the circle, or outside the circle K.

SOLUTION

A point lies on a circle centred at O (0, 0) if its distance from O equals the radius. Distance of a point (x, y) from O is sqrt(x² + y²), so we compare x² + y² with the radius squared.
(i) A (1, -8): $OA^2 = 1^2 + (-8)^2 = 1 + 64 = 65$.
B (-4, 7): $OB^2 = (-4)^2 + 7^2 = 16 + 49 = 65$.

C (-7, -4): $OC^2 = (-7)^2 + (-4)^2 = 49 + 16 = 65$.
$OA^2 = OB^2 = OC^2 = 65$, so OA = OB = OC = sqrt(65). All three points are the same distance from O, so they lie on one circle centred at O, of radius sqrt(65) ~ 8.06 units.
(ii) Compare each point's distance-squared with $r^2 = 65$.
D (-5, 6): $OD^2 = (-5)^2 + 6^2 = 25 + 36 = 61$. Since 61 < 65, OD < radius, so D lies INSIDE (within) the circle.
E (0, 9): $OE^2 = 0^2 + 9^2 = 81$. Since 81 > 65, OE > radius, so E lies OUTSIDE the circle.
(i) OA = OB = OC = sqrt(65), so all three lie on circle K centred at O; radius = sqrt(65) ~ 8.06 units. (ii) D (-5, 6) is inside (61 < 65); E (0, 9) is outside (81 > 65).

Q13 The midpoints of the sides of triangle ABC are the points D, E, and F. Given that the coordinates of D, E, and F are (5, 1), (6, 5), and (0, 3), respectively, find the coordinates of A, B and C.

SOLUTION

Let D (5, 1) be the midpoint of AB, E (6, 5) the midpoint of BC, and F (0, 3) the midpoint of CA (D, E, F taken in order around the triangle).
By the midpoint formula, if we add the coordinates: D + F - E gives A, D + E - F gives B, and E + F - D gives C. (This comes from D = (A+B)/2, E = (B+C)/2, F = (C+A)/2; adding/subtracting isolates each vertex.)
A = D + F - E = (5 + 0 - 6, 1 + 3 - 5) = (-1, -1).
B = D + E - F = (5 + 6 - 0, 1 + 5 - 3) = (11, 3).
C = E + F - D = (6 + 0 - 5, 5 + 3 - 1) = (1, 7).
Check: midpoint of A (-1, -1) and B (11, 3) = ((-1+11)/2, (-1+3)/2) = (5, 1) = D. Midpoint of B (11, 3) and C (1, 7) = ((11+1)/2, (3+7)/2) = (6, 5) = E. Midpoint of C (1, 7) and A (-1, -1) = ((1-1)/2, (7-1)/2) = (0, 3) = F. All correct.
A = (-1, -1), B = (11, 3), C = (1, 7).

Q14 A city has two main roads which cross each other at the centre of the city. These two roads are along the North-South (N-S) direction and East-West (E-W) direction. All the other streets of the city run parallel to these roads and are 200 m apart. There are 10 streets in each direction. (i) Using 1 cm = 200 m, draw a model of the city in your notebook. Represent the roads/streets by single lines. (ii) There are street intersections in the model. A street intersection is labelled (a, b) if the a-th street in the N-S direction and the b-th street in the E-W direction meet there. Using this convention, find: (a) how many street intersections can be referred to as (4, 3); (b) how many street intersections can be referred to as (3, 4).

SOLUTION

(i) Model: draw the two main roads as a horizontal line (E-W) and a vertical line (N-S) crossing at the city centre. Then draw the other streets parallel to these, each 200 m apart. At the scale 1 cm = 200 m, adjacent parallel streets are drawn 1 cm apart. With 10 streets in each direction you get a grid of horizontal and vertical lines.

(ii) The label (a, b) means the crossing of a UNIQUE street running N-S (the a-th one) with a UNIQUE street running E-W (the b-th one).
Two specific lines — one particular N-S street and one particular E-W street — cross at exactly one point.
(a) (4, 3) means the 4th N-S street meets the 3rd E-W street. There is exactly ONE such intersection.
(b) (3, 4) means the 3rd N-S street meets the 4th E-W street. There is exactly ONE such intersection.
Note that (4, 3) and (3, 4) are two DIFFERENT single points — order matters, just as with coordinates.
(a) Exactly 1 intersection; (b) Exactly 1 intersection. (They are two distinct points; the ordered label pins down a unique crossing.)

Q15 A computer graphics program displays images on a rectangular screen whose coordinate system has the origin at the bottom-left corner. The screen is 800 pixels wide and 600 pixels high. A circular icon of radius 80 pixels is drawn with its centre at the point A (100, 150). Another circular icon of radius 100 pixels is drawn with its centre at the point B (250, 230). Determine: (i) whether any part of either circle lies outside the screen; (ii) whether the two circles intersect each other.

SOLUTION

The screen covers x from 0 to 800 and y from 0 to 600 (origin at bottom-left). A circle of centre (h, k) and radius r stays fully inside if h - r >= 0, h + r <= 800, k - r >= 0 and k + r <= 600.
(i) Circle A: centre (100, 150), r = 80. Left edge: 100 - 80 = 20 >= 0. Right edge: 100 + 80 = 180 <= 800. Bottom edge: 150 - 80 = 70 >= 0. Top edge: 150 + 80 = 230 <= 600. All four are within bounds, so circle A lies fully inside the screen.
Circle B: centre (250, 230), r = 100. Left: 250 - 100 = 150 >= 0. Right: 250 + 100 = 350 <= 800. Bottom: 230 - 100 = 130 >= 0. Top: 230 + 100 = 330 <= 600. All within bounds, so circle B also lies fully inside the screen.
So NO part of either circle lies outside the screen.
(ii) Two circles intersect (cross at two points) when the distance d between their centres satisfies |r1 - r2| < d < r1 + r2.
Distance between centres A (100, 150) and B (250, 230): d = sqrt((250 - 100)² + (230 - 150)²) = sqrt(150² + 80²) = sqrt(22500 + 6400) = sqrt(28900) = 170 pixels.

Here r1 + r2 = 80 + 100 = 180 and	r1 - r2	=	80 - 100	= 20.
Since 20 < 170 < 180, we have	r1 - r2	< d < r1 + r2, so the two circles DO intersect (at two points).		

(i) No — both circles lie fully inside the 800 x 600 screen. (ii) Yes — centres are 170 px apart and 20 < 170 < 180, so the circles intersect at two points.

Q16 Plot the points A (2, 1), B (-1, 2), C (-2, -1), and D (1, -2) in the coordinate plane. Is ABCD a square? Can you explain why? What is the area of this square?

SOLUTION

Plot A (2, 1) in Quadrant I, B (-1, 2) in Quadrant II, C (-2, -1) in Quadrant III, D (1, -2) in Quadrant IV, and join them in order A -> B -> C -> D -> A.

Find the four side lengths using the distance formula sqrt((x2-x1)² + (y2-y1)²).

AB = sqrt((-1-2)² + (2-1)²) = sqrt(9 + 1) = sqrt(10).

BC = sqrt((-2-(-1))² + (-1-2)²) = sqrt(1 + 9) = sqrt(10).

CD = sqrt((1-(-2))² + (-2-(-1))²) = sqrt(9 + 1) = sqrt(10).

DA = sqrt((2-1)² + (1-(-2))²) = sqrt(1 + 9) = sqrt(10).

All four sides are equal: AB = BC = CD = DA = sqrt(10).

Now check the diagonals: AC = sqrt((-2-2)² + (-1-1)²) = sqrt(16 + 4) = sqrt(20), and BD = sqrt((1-(-1))² + (-2-2)²) = sqrt(4 + 16) = sqrt(20). The diagonals are equal (both sqrt(20)).

A quadrilateral with all four sides equal AND equal diagonals is a square. (Equal sides -> rhombus; equal diagonals then force right angles.) You can also check a corner directly: the side-vector AB = (-3, 1) and BC = (-1, -3) have dot product (-3)(-1) + (1)(-3) = 3 - 3 = 0, so the angle at B is 90 degrees.

So ABCD is a square, with side sqrt(10).

Area of a square = side² = (sqrt(10))² = 10 square units.

Yes, ABCD is a square — all sides = sqrt(10), diagonals = sqrt(20) equal, and adjacent sides are perpendicular. Area = (sqrt(10))² = 10 square units.