Chapter 2: Introduction to Linear Polynomials

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Exercise 2.1 — Polynomials — Degree, Coefficients, and Constant Term

Q1 Find the degrees of the following polynomials: (i) $2x^2 - 5x + 3$ (ii) $y^3 + 2y - 1$ (iii) -9 (iv) $4z - 3$

SOLUTION

The degree of a polynomial is the highest power of the variable in it.
(i) $2x^2 - 5x + 3$: the highest power of x is 2. Degree = 2.
(ii) $y^3 + 2y - 1$: the highest power of y is 3. Degree = 3.
(iii) -9: this is a non-zero constant, written as $-9x^0$. Degree = 0.
(iv) $4z - 3$: the highest power of z is 1. Degree = 1.
(i) 2, (ii) 3, (iii) 0, (iv) 1

Q2 Write polynomials of degrees 1, 2 and 3.

SOLUTION

We need one example for each degree.
Degree 1 (linear polynomial): $x + 1$.
(The highest power of x is 1.)
Degree 2 (quadratic polynomial): $x^2 + x + 1$.
(The highest power of x is 2.)
Degree 3 (cubic polynomial): $x^3 + x^2 + x + 1$.
(The highest power of x is 3.)
Note: many other answers are equally valid.
Degree 1: $x + 1$; Degree 2: $x^2 + x + 1$; Degree 3: $x^3 + x^2 + x + 1$ (examples)

Q3 What are the coefficients of x^2 and x^3 in the polynomial $x^4 - 3x^3 + 6x^2 - 2x + 7$?

SOLUTION

Write out the polynomial term by term:

$x^4 - 3x^3 + 6x^2 - 2x + 7$

The term containing x^3 is $-3x^3$, so the coefficient of x^3 is -3.

The term containing x^2 is $6x^2$, so the coefficient of x^2 is 6.

Coefficient of $x^3 = -3$; Coefficient of $x^2 = 6$

Q4 What is the coefficient of z in the polynomial $4z^3 + 5z^2 - 11$?

SOLUTION

The polynomial $4z^3 + 5z^2 - 11$ contains the terms: $4z^3$, $5z^2$, and -11.

There is no term with z^1 (i.e., no z term appears explicitly).

The missing z term is $0 \cdot z$, so the coefficient of z is 0.

Q5 What is the constant term of the polynomial $9x^3 + 5x^2 - 8x - 10$?

SOLUTION

The constant term is the term that does not contain the variable x.

In $9x^3 + 5x^2 - 8x - 10$, the constant term is -10.

Exercise 2.2 — Linear Polynomials — Values and Word Problems

Q1 Find the value of the linear polynomial $5x - 3$ if: (i) $x = 0$ (ii) $x = -1$ (iii) $x = 2$

SOLUTION

Substitute each value of x into $p(x) = 5x - 3$.

(i) $x = 0$: $p(0) = 5(0) - 3 = 0 - 3 = -3$

(ii) $x = -1$: $p(-1) = 5(-1) - 3 = -5 - 3 = -8$

(iii) $x = 2$: $p(2) = 5(2) - 3 = 10 - 3 = 7$

(i) –3, (ii) –8, (iii) 7

Q2 Find the value of the quadratic polynomial $7s^2 - 4s + 6$ if: (i) $s = 0$ (ii) $s = -3$ (iii) $s = 4$

SOLUTION

Substitute each value of s into $p(s) = 7s^2 - 4s + 6$.

(i) $s = 0$: $p(0) = 7(0)^2 - 4(0) + 6 = 0 - 0 + 6 = 6$

(ii) $s = -3$: $p(-3) = 7(-3)^2 - 4(-3) + 6 = 7(9) + 12 + 6 = 63 + 12 + 6 = 81$

(iii) $s = 4$: $p(4) = 7(4)^2 - 4(4) + 6 = 7(16) - 16 + 6 = 112 - 16 + 6 = 102$

(i) 6, (ii) 81, (iii) 102

Q3 The present age of Salil's mother is three times Salil's present age. After 5 years, their ages will add up to 70 years. Find their present ages.

SOLUTION

Let Salil's present age = x years.

Then his mother's present age = 3x years.

After 5 years, Salil's age = x + 5 and his mother's age = 3x + 5.

Given: $(x + 5) + (3x + 5) = 70$

$4x + 10 = 70$

$4x = 60$

$x = 15$

So Salil's present age = 15 years and his mother's present age = $3 \times 15 = 45$ years.

Check: After 5 years, $(15 + 5) + (45 + 5) = 20 + 50 = 70$. Correct.

Salil's age = 15 years; Mother's age = 45 years

Q4 The difference between two positive integers is 63. The ratio of the two integers is 2:5. Find the two integers.

SOLUTION

Let the two integers be 2k and 5k (in ratio 2:5).

The larger integer is 5k and the smaller is 2k.

Given: $5k - 2k = 63$

3k = 63
k = 21
The two integers are 2k = 2 × 21 = 42 and 5k = 5 × 21 = 105.
Check: 105 – 42 = 63, and 42:105 = 2:5. Correct.
The two integers are 42 and 105

Q5 Ruby has 3 times as many two-rupee coins as she has five rupee-coins. If she has a total 88, how many coins does she have of each type?

SOLUTION

Let the number of five-rupee coins = y.
Then the number of two-rupee coins = 3y.
Total amount: 5y + 2(3y) = 88
5y + 6y = 88
11y = 88
y = 8
Five-rupee coins = 8; Two-rupee coins = 3 × 8 = 24.
Check: 8 × 5 + 24 × 2 = 40 + 48 = 88. Correct.
Five-rupee coins = 8; Two-rupee coins = 24

Q6 A farmer cuts a 300 feet fence into two pieces of different sizes. The longer piece is four times as long as the shorter piece. How long are the two pieces?

SOLUTION

Let the length of the shorter piece = s feet.
Then the longer piece = 4s feet.
Total: s + 4s = 300
5s = 300
s = 60
Shorter piece = 60 feet; Longer piece = 4 × 60 = 240 feet.
Check: 60 + 240 = 300 feet. Correct.
Shorter piece = 60 feet; Longer piece = 240 feet

Q7 If the length of a rectangle is three more than twice its width and its perimeter is 24 cm, what are the dimensions of the rectangle?

SOLUTION

Let width = w cm.
Then length = 2w + 3 cm.
Perimeter = 2(length + width) = 24
2((2w + 3) + w) = 24
2(3w + 3) = 24

3w + 3 = 12
3w = 9
w = 3
Length = 2(3) + 3 = 9 cm.
Check: Perimeter = 2(9 + 3) = 2(12) = 24 cm. Correct.
Width = 3 cm; Length = 9 cm

Exercise 2.3 — Exploring Linear Patterns

Q1 A student has 500 in her savings bank account. She gets 150 every month as pocket money. How much money will she have at the end of every month from the second month onwards? Find a linear expression to represent the amount she will have in the nth month.

SOLUTION

Initial savings = 500.
Each month she adds 150.
At end of month n, total = 500 + 150n.
End of month 1: 500 + 150(1) = 650
End of month 2: 500 + 150(2) = 800
End of month 3: 500 + 150(3) = 950
End of month 4: 500 + 150(4) = 1100
The amounts form the pattern: 650, 800, 950, 1100, … with a common difference of 150.
This is a linear pattern because the amount increases by a constant 150 each month.
Linear expression for the nth month: Amount = 500 + 150n
Amount in nth month = 500 + 150n (). Pattern: 650, 800, 950, …

Q2 A rally starts with 120 members. Each hour, 9 members drop out of the group. How many members will remain after 1, 2, 3, … hours? Find a linear expression to represent the number of members at the end of the nth hour.

SOLUTION

Initial members = 120.
Each hour, 9 members drop out.
After n hours, members remaining = 120 – 9n.
After 1 hour: 120 – 9(1) = 111
After 2 hours: 120 – 9(2) = 102
After 3 hours: 120 – 9(3) = 93
The sequence 111, 102, 93, … has a common difference of –9.
This is a linear pattern with constant decrease of 9 members per hour.
Linear expression: Members after nth hour = 120 – 9n
Members after nth hour = 120 – 9n. After 1 h: 111; after 2 h: 102; after 3 h: 93

Q3 Suppose the length of a rectangle is 13 cm. Find the area if the breadth is (i) 12 cm, (ii) 10 cm, (iii) 8 cm. Find the linear pattern representing the area of the rectangle.

SOLUTION

Area = length × breadth = 13 × breadth.
(i) Breadth = 12 cm: Area = 13 × 12 = 156 cm²
(ii) Breadth = 10 cm: Area = 13 × 10 = 130 cm²
(iii) Breadth = 8 cm: Area = 13 × 8 = 104 cm²

The sequence of areas is 156, 130, 104, …
As breadth decreases by 2 cm each step, the area decreases by 13 × 2 = 26 cm².
The linear pattern for breadth b: Area = 13b.
This is linear in b because it increases by a constant amount (13) for every unit increase in breadth.
(i) 156 cm², (ii) 130 cm², (iii) 104 cm². Linear pattern: Area = 13b

Q4 Suppose the length of a rectangular box is 7 cm and breadth is 11 cm. Find the volume if the height is (i) 5 cm, (ii) 9 cm, (iii) 13 cm. Find the linear pattern representing the volume of the rectangular box.

SOLUTION

Volume = length × breadth × height = 7 × 11 × height = 77 × height.
(i) Height = 5 cm: Volume = 77 × 5 = 385 cm³
(ii) Height = 9 cm: Volume = 77 × 9 = 693 cm³
(iii) Height = 13 cm: Volume = 77 × 13 = 1001 cm³
The sequence of volumes is 385, 693, 1001, …
As height increases by 4 cm each step, volume increases by 77 × 4 = 308 cm³.
The linear pattern for height h: Volume = 77h.
This is linear in h because it increases by the constant amount 77 for each unit increase in height.
(i) 385 cm³, (ii) 693 cm³, (iii) 1001 cm³. Linear pattern: Volume = 77h

Q5 Sarita is reading a book of 500 pages. She reads 20 pages every day. How many pages will be left after 15 days? Express this as a linear pattern.

SOLUTION

Pages left after n days = 500 – 20n.
After 15 days: Pages left = 500 – 20(15) = 500 – 300 = 200 pages.
The sequence is 480, 460, 440, … (decreasing by 20 each day).
Linear pattern: Pages left after n days = 500 – 20n.
The difference between consecutive terms is –20 (constant), confirming a linear pattern.
200 pages remain after 15 days. Linear pattern: Pages left = 500 – 20n

Exercise 2.4 — Linear Growth and Linear Decay

Q1 Suppose a plant has height 1.75 feet and it grows by 0.5 feet each month. (i) Find the height after 7 months. (ii) Make a table of values for t varying from 0 to 10 months and show how the height, h, increases every month. (iii) Find an expression that relates h and t, and explain why it represents linear growth.

SOLUTION

(i) Initial height = 1.75 ft; growth rate = 0.5 ft/month.
Height after t months: h(t) = 1.75 + 0.5t
After 7 months: h(7) = 1.75 + 0.5(7) = 1.75 + 3.5 = 5.25 feet.

(ii) Table of values (t in months, h in feet):
t = 0: h = 1.75
t = 1: h = 2.25
t = 2: h = 2.75
t = 3: h = 3.25
t = 4: h = 3.75
t = 5: h = 4.25
t = 6: h = 4.75
t = 7: h = 5.25
t = 8: h = 5.75
t = 9: h = 6.25
t = 10: h = 6.75
The height increases by exactly 0.5 feet every month.

(iii) Expression: h = 1.75 + 0.5t.
This represents linear growth because: the coefficient of t is positive (0.5 > 0), and the height increases by a constant amount (0.5 feet) for every unit increase in t. The expression is a linear polynomial in t.
(i) 5.25 feet. (iii) h = 1.75 + 0.5t; linear growth because height increases by a constant 0.5 ft per month.

Q2 A mobile phone is bought for 10,000. Its value decreases by 800 every year. (i) Find the value of the phone after 3 years. (ii) Make a table of values for t varying from 0 to 8 years and show how the value of the phone, v, depreciates with time. (iii) Find an expression that relates v and t, and explain why it represents linear decay.

SOLUTION

(i) Initial value = 10,000; depreciation = 800/year.
Value after t years: v(t) = 10000 – 800t
After 3 years: v(3) = 10000 – 800(3) = 10000 – 2400 = 7,600.

(ii) Table of values (t in years, v in):
t = 0: v = 10,000

t = 1: v = 9,200
t = 2: v = 8,400
t = 3: v = 7,600
t = 4: v = 6,800
t = 5: v = 6,000
t = 6: v = 5,200
t = 7: v = 4,400
t = 8: v = 3,600
The value decreases by exactly 800 every year.

(iii) Expression: v = 10000 – 800t.
This represents linear decay because: the coefficient of t is negative (–800 < 0), and the value decreases by a constant amount (800) for every unit increase in t. The expression is a linear polynomial in t with a negative slope.
(i) 7,600. (iii) v = 10000 – 800t; linear decay because value decreases by a constant 800 per year.

Q3 The initial population of a village is 750. Every year, 50 people move from a nearby city to the village. (i) Find the population of the village after 6 years. (ii) Make a table of values for t varying from 0 to 10 years and show how the population, P, increases every year. (iii) Find an expression that relates P and t, and explain why it represents linear growth.

SOLUTION

(i) Initial population = 750; growth rate = 50 people/year.
Population after t years: P(t) = 750 + 50t
After 6 years: P(6) = 750 + 50(6) = 750 + 300 = 1050.

(ii) Table of values (t in years, P = population):
t = 0: P = 750
t = 1: P = 800
t = 2: P = 850
t = 3: P = 900
t = 4: P = 950
t = 5: P = 1000
t = 6: P = 1050
t = 7: P = 1100
t = 8: P = 1150
t = 9: P = 1200
t = 10: P = 1250
The population increases by exactly 50 every year.

(iii) Expression: P = 750 + 50t.
This represents linear growth because: the coefficient of t is positive (50 > 0), and the population increases by a constant amount (50 people) for every unit increase in t. The

expression is a linear polynomial in t.

(i) Population after 6 years = 1050. (iii) P = 750 + 50t; linear growth because population increases by a constant 50 per year.

Q4 A telecom company charges 600 for a certain recharge scheme. This prepaid balance is reduced by 15 each day after the recharge. (i) Write an equation that models the remaining balance b(x) after using the scheme for x days. Explain why it represents linear decay. (ii) After how many days will the balance run out? (iii) Make a table of values for x varying from 1 to 10 days and show how the balance b(x) reduces with time.

SOLUTION

(i) Initial balance = 600; daily deduction = 15.

Balance after x days: b(x) = 600 – 15x.

This represents linear decay because the coefficient of x is negative (–15 < 0) and the balance decreases by a constant amount (15) for each additional day. It is a linear polynomial in x.

(ii) Balance runs out when b(x) = 0:

600 – 15x = 0

15x = 600

x = 40 days.

The balance will run out after 40 days.

(iii) Table of values (x in days, b(x) in):

x = 1: b = 585

x = 2: b = 570

x = 3: b = 555

x = 4: b = 540

x = 5: b = 525

x = 6: b = 510

x = 7: b = 495

x = 8: b = 480

x = 9: b = 465

x = 10: b = 450

The balance decreases by 15 each day.

(i) b(x) = 600 – 15x; linear decay. (ii) Balance runs out after 40 days.

Exercise 2.5 — Linear Relationships

Q1 A learning platform charges a fixed monthly fee and an additional cost per digital learning module accessed. A student observes that when she accessed 10 modules, her bill was 400. When she accessed 14 modules, her bill was 500. If the monthly bill y depends on the number of modules accessed, x, according to the relation y = ax + b, find the values of a and b.

SOLUTION

Using y = ax + b with the two given data points:
When x = 10, y = 400: 10a + b = 400 ...(1)
When x = 14, y = 500: 14a + b = 500 ...(2)
Subtract (1) from (2):
(14a + b) – (10a + b) = 500 – 400
4a = 100
a = 25
Substitute a = 25 into (1):
10(25) + b = 400
250 + b = 400
b = 150
Check with (2): 14(25) + 150 = 350 + 150 = 500. Correct.
So a = 25 (cost per module = 25) and b = 150 (fixed monthly fee = 150).
a = 25, b = 150. The relation is y = 25x + 150.

Q2 A gym charges a fixed monthly fee and an additional cost per hour for using the badminton court. A student using the gym observed that when she used the badminton court for 10 hours, her bill was 800. When she used it for 15 hours, her bill was 1100. If the monthly bill y depends on the hours of the use of the badminton court, x, according to the relation y = ax + b, find the values of a and b.

SOLUTION

Using y = ax + b with the two given data points:
When x = 10, y = 800: 10a + b = 800 ...(1)
When x = 15, y = 1100: 15a + b = 1100 ...(2)
Subtract (1) from (2):
(15a + b) – (10a + b) = 1100 – 800
5a = 300
a = 60
Substitute a = 60 into (1):
10(60) + b = 800
600 + b = 800
b = 200
Check with (2): 15(60) + 200 = 900 + 200 = 1100. Correct.
So a = 60 (cost per hour = 60) and b = 200 (fixed monthly fee = 200).

a = 60, b = 200. The relation is y = 60x + 200.

Q3 Consider the relationship between temperature measured in degrees Celsius (°C) and degrees Fahrenheit (°F), which is given by °C = a °F + b. Find a and b, given that ice melts at 0 degrees Celsius and 32 degrees Fahrenheit, and water boils at 100 degrees Celsius and 212 degrees Fahrenheit.

SOLUTION

Using °C = a °F + b with the two given conditions:
When °C = 0, °F = 32: 0 = 32a + b ...(1)
When °C = 100, °F = 212: 100 = 212a + b ...(2)
Subtract (1) from (2):
100 – 0 = (212a + b) – (32a + b)
100 = 180a
a = 100/180 = 5/9
Substitute into (1):
0 = 32(5/9) + b
b = –160/9
Check: °C at 212°F = (5/9)(212) – 160/9 = 1060/9 – 160/9 = 900/9 = 100. Correct.
Therefore: °C = (5/9)°F – 160/9 = (5/9)(°F – 32).
So a = 5/9 and b = –160/9.
a = 5/9, b = –160/9. The relation is °C = (5/9)(°F – 32).

Exercise 2.6 — Visualising Linear Relationships

Q1 Draw the graphs of the following sets of lines. In each case, reflect on the role of 'a' and 'b'. (i) y = 4x, y = 2x, y = x (ii) y = –6x, y = –3x, y = –x (iii) y = 5x, y = –5x (iv) y = 3x – 1, y = 3x, y = 3x + 1 (v) y = –2x – 3, y = –2x, y = 2x + 3

SOLUTION

For each set, plot at least two points per line and draw the straight line through them. The key observations follow.

(i) y = 4x, y = 2x, y = x (all have b = 0, positive slopes):
Sample points — y=4x: (0,0), (1,4), (–1,–4); y=2x: (0,0), (1,2), (–1,–2); y=x: (0,0), (1,1), (–1,–1).
Role of a: all three lines pass through the origin (b = 0). As a increases (1 < 2 < 4), the line becomes steeper. A larger positive a means a steeper upward slope.

(ii) y = –6x, y = –3x, y = –x (all have b = 0, negative slopes):
Sample points — y=–6x: (0,0), (1,–6), (–1,6); y=–3x: (0,0), (1,–3), (–1,3); y=–x: (0,0), (1,–1), (–1,1).
Role of a: all lines pass through the origin (b = 0). Negative a makes the line slope downward from left to right. As |a| increases (1 < 3 < 6), the line becomes steeper in the downward direction.

(iii) y = 5x, y = –5x (same |a|, opposite signs, b = 0):
Sample points — y=5x: (0,0), (1,5), (–1,–5); y=–5x: (0,0), (1,–5), (–1,5).
Role of a: both lines pass through the origin. y = 5x rises steeply to the right; y = –5x falls steeply to the right. They are mirror images of each other across the x-axis (reflections). Same steepness, opposite directions.

(iv) y = 3x – 1, y = 3x, y = 3x + 1 (same a = 3, different b):
Sample points — y=3x–1: (0,–1), (1,2); y=3x: (0,0), (1,3); y=3x+1: (0,1), (1,4).
Role of b: all three lines have the same slope (a = 3), so they are parallel to each other. Changing b shifts the line up or down without changing its slope. The y-intercepts are –1, 0, and +1 respectively.

(v) y = –2x – 3, y = –2x, y = 2x + 3 (mixed):
Sample points — y=–2x–3: (0,–3), (1,–5), (–1,–1); y=–2x: (0,0), (1,–2), (–1,2); y=2x+3: (0,3), (1,5), (–1,1).
y = –2x – 3 and y = –2x have the same slope (a = –2), so they are parallel; y = –2x – 3 is the line y = –2x shifted down by 3 units.
y = 2x + 3 has slope a = +2 (opposite sign), so it is not parallel to the other two and will intersect them.
Role of a and b: a determines the slope and direction; b determines the vertical position (y-intercept). Lines with equal a are parallel; changing b only shifts them.
(i) All pass through origin; steeper as a increases. (ii) All pass through origin; slope negative and steeper as |a| increases. (iii) Both through origin, reflections of each other across x-axis. (iv) Three parallel lines (same slope a=3); b shifts the line up/down. (v) y=–2x–3 and y=–2x are parallel (a=–2); y=2x+3 has opposite slope and intersects the other two.

Exercises — Introduction to Linear Polynomials

Q1 Write a polynomial of degree 3 in the variable x, in which the coefficient of the x^2 term is –7.

SOLUTION

A polynomial of degree 3 (a cubic) has the general form $ax^3 + bx^2 + cx + d$, where $a \neq 0$.
The degree is 3, so the x^3 term must be present (its coefficient $a \neq 0$).
We are told the coefficient of the x^2 term is –7, so $b = -7$.
Choosing any convenient values for the remaining coefficients (say $a = 1$, $c = 4$, $d = 2$) gives one valid answer:
$p(x) = x^3 - 7x^2 + 4x + 2$.

This has degree 3 and the coefficient of x^2 is –7, as required. (Many other answers are possible, e.g. $2x^3 - 7x^2 + 1$.)
One example: $p(x) = x^3 - 7x^2 + 4x + 2$ (any cubic in x with x^2-coefficient –7 is acceptable).

Q2 Find the values of the following polynomials at the indicated values of the variables. (i) $5x^2 - 3x + 7$ if $x = 1$ (ii) $4t^3 - t^2 + 6$ if $t = a$

SOLUTION

(i) Substitute $x = 1$ into $5x^2 - 3x + 7$:
$= 5(1)^2 - 3(1) + 7$
$= 5 - 3 + 7$
$= 9$.

(ii) Substitute $t = a$ into $4t^3 - t^2 + 6$:
$= 4a^3 - a^2 + 6$.
Since a is a variable (not a specific number), the value stays in terms of a: $4a^3 - a^2 + 6$.
(i) 9 (ii) $4a^3 - a^2 + 6$

Q3 If we multiply a number by 5/2 and add 2/3 to the product, we get –7/12. Find the number.

SOLUTION

Let the required number be x.
"Multiply the number by 5/2 and add 2/3" gives the expression $(5/2)x + 2/3$.
This equals –7/12, so the linear equation is:
$(5/2)x + 2/3 = -7/12$.

Subtract 2/3 from both sides. Writing 2/3 as 8/12:
$(5/2)x = -7/12 - 8/12 = -15/12 = -5/4$.

Multiply both sides by 2/5:
$x = (-5/4) \times (2/5) = -10/20 = -1/2$.

Check: $(5/2)(-1/2) + 2/3 = -5/4 + 2/3 = -15/12 + 8/12 = -7/12$. ✓
The number is –1/2.

Q4 A positive number is 5 times another number. If 21 is added to both the numbers, then one of the new numbers becomes twice the other new number. What are the numbers?

SOLUTION

Let the smaller number be x. Then the larger number is 5x.
Add 21 to both: the new numbers are (x + 21) and (5x + 21).
The larger new number (5x + 21) becomes twice the smaller new number (x + 21):
5x + 21 = 2(x + 21).

Expand and solve:
5x + 21 = 2x + 42
5x – 2x = 42 – 21
3x = 21
x = 7.

So the smaller number is 7 and the larger number is 5x = 35.
Check: new numbers are 7 + 21 = 28 and 35 + 21 = 56, and 56 = 2 × 28. ✓
(The other case, x + 21 = 2(5x + 21), gives a negative x, which is rejected since the numbers are positive.)
The two numbers are 7 and 35.

Q5 If you have 800 and you save 250 every month, find the amount you have after (i) 6 months (ii) 2 years. Express this as a linear pattern.

SOLUTION

You start with 800 and add 250 each month. After n months the amount is:
Amount = 800 + 250n (a linear expression in n — the amount increases by the constant 250 each month, so it is a linear pattern).

(i) After 6 months, n = 6:
Amount = 800 + 250 × 6 = 800 + 1500 = 2300.

(ii) 2 years = 24 months, so n = 24:
Amount = 800 + 250 × 24 = 800 + 6000 = 6800.
Linear pattern: 800 + 250n. (i) 2300 (ii) 6800

Q6 The digits of a two-digit number differ by 3. If the digits are interchanged, and the resulting number is added to the original number, we get 143. Find both the numbers.

SOLUTION

Let the tens digit be a and the units digit be b.
Original number = 10a + b. Number with digits interchanged = 10b + a.

Their sum is 143:
(10a + b) + (10b + a) = 143
11a + 11b = 143

11(a + b) = 143
a + b = 143 ÷ 11 = 13. … (1)

The digits differ by 3, so a – b = 3 (or b – a = 3). … (2)

Solving a + b = 13 and a – b = 3: adding gives 2a = 16, so a = 8 and b = 5.
This gives the original number 85 and the interchanged number 58.
(Taking b – a = 3 instead gives a = 5, b = 8, i.e. the pair 58 and 85 — the same two numbers.)

Check: 85 + 58 = 143, and the digits 8 and 5 differ by 3. ✓
The two numbers are 85 and 58.

Q7 Draw the graph of the following equations, and identify their slopes and y-intercepts. Also, find the coordinates of the points where these lines cut the y-axis. (i) y = –3x + 4 (ii) 2y = 4x + 7 (iii) 5y = 6x – 10 (iv) 3y = 6x – 11. Are any of the lines parallel?

SOLUTION

Write each line in the form y = ax + b, where a is the slope and b is the y-intercept; the line cuts the y-axis at (0, b).

(i) y = –3x + 4. Slope a = –3, y-intercept b = 4, cuts y-axis at (0, 4). To draw: plot (0, 4) and, e.g., (1, 1), and join.

(ii) 2y = 4x + 7 => y = 2x + 7/2. Slope a = 2, y-intercept b = 7/2 = 3.5, cuts y-axis at (0, 7/2). To draw: plot (0, 3.5) and (1, 5.5).

(iii) 5y = 6x – 10 => y = (6/5)x – 2. Slope a = 6/5, y-intercept b = –2, cuts y-axis at (0, –2). To draw: plot (0, –2) and (5, 4).

(iv) 3y = 6x – 11 => y = 2x – 11/3. Slope a = 2, y-intercept b = –11/3 ≈ –3.67, cuts y-axis at (0, –11/3). To draw: plot (0, –11/3) and (2, 1/3).

Parallel lines have equal slopes but different y-intercepts. Lines (ii) and (iv) both have slope 2 (with different y-intercepts 7/2 and –11/3), so lines (ii) and (iv) are parallel.
(i) slope –3, y-int 4, cuts (0, 4); (ii) slope 2, y-int 7/2, cuts (0, 7/2); (iii) slope 6/5, y-int –2, cuts (0, –2); (iv) slope 2, y-int –11/3, cuts (0, –11/3). Lines (ii) and (iv) are parallel (equal slope 2).

Q8 If the temperature of a liquid can be measured in Kelvin units as x K and in Fahrenheit units as y °F, the relation between the two systems of measurement of temperature is given by the linear equation y = (9/5)(x – 273) + 32. (i) Find the temperature of the liquid in Fahrenheit if the temperature of the liquid is 313 K. (ii) If the temperature is 158 °F, then find the temperature in Kelvin.

SOLUTION

The relation is y = (9/5)(x – 273) + 32, where x is in Kelvin and y is in Fahrenheit.

(i) Put x = 313 K:
y = (9/5)(313 – 273) + 32 = (9/5)(40) + 32 = 72 + 32 = 104.

So the temperature is 104 °F.

(ii) Put y = 158 °F and solve for x:
158 = (9/5)(x – 273) + 32
158 – 32 = (9/5)(x – 273)
126 = (9/5)(x – 273)
x – 273 = 126 × (5/9) = 70
x = 70 + 273 = 343.
So the temperature is 343 K.
(i) 104 °F (ii) 343 K

Q9 The work done by a body on the application of a constant force is the product of the constant force and the distance travelled by the body in the direction of the force. Express this in the form of a linear equation in two variables (work w and distance d), and draw its graph by taking the constant force as 3 units. What is the work done when the distance travelled is 2 units? Verify it by plotting it on the graph.

SOLUTION

Work = force × distance. With work w and distance d and a constant force F, the linear equation is:
w = F·d.

Taking the constant force as 3 units, the equation becomes:
w = 3d.

This is a linear equation in the two variables w and d, passing through the origin (0, 0) with slope 3. To draw it, plot two points such as (0, 0) and (1, 3), then join and extend the straight line.

Work done when the distance travelled is 2 units (d = 2):
w = 3 × 2 = 6 units.

On the graph, the point (2, 6) lies on the line w = 3d, verifying the answer.
w = 3d; the work done for d = 2 units is 6 units (point (2, 6) on the line).

Q10 The graph of a linear polynomial p(x) passes through the points (1, 5) and (3, 11). (i) Find the polynomial p(x). (ii) Find the coordinates where the graph of p(x) cuts the axes. (iii) Draw the graph of p(x) and verify your answers.

SOLUTION

(i) A linear polynomial has the form p(x) = ax + b. Its graph passes through (1, 5) and (3, 11):
a(1) + b = 5 … (1)
a(3) + b = 11 … (2)
Subtract (1) from (2): 3a + b – (a + b) = 11 – 5, so 2a = 6, giving a = 3.
From (1): b = 5 – a = 5 – 3 = 2.

Therefore p(x) = 3x + 2.

(ii) The graph cuts the x-axis where p(x) = 0:
3x + 2 = 0 => x = –2/3, so the point is (–2/3, 0).
It cuts the y-axis where x = 0: p(0) = 2, so the point is (0, 2).

(iii) To draw the graph, plot (0, 2) and (–2/3, 0) (or the given points (1, 5) and (3, 11)) and join them with a straight line. Checking: p(1) = 3(1) + 2 = 5 and p(3) = 3(3) + 2 = 11, which match the given points. ✓
(i) p(x) = 3x + 2 (ii) x-axis at (–2/3, 0) and y-axis at (0, 2)

Q11 Let p(x) = ax + b and q(x) = cx + d be two linear polynomials such that: (i) p(0) = 5. (ii) The polynomial p(x) – q(x) cuts the x-axis at (3, 0). (iii) The sum p(x) + q(x) is equal to 6x + 4 for all real x. Find the polynomials p(x) and q(x).

SOLUTION

From condition (i): p(0) = a(0) + b = b = 5, so b = 5.

From condition (iii): p(x) + q(x) = (a + c)x + (b + d) = 6x + 4 for all x. Matching coefficients:
a + c = 6 … (1)
b + d = 4 => 5 + d = 4 => d = –1.

From condition (ii): p(x) – q(x) = (a – c)x + (b – d). It cuts the x-axis at (3, 0), so its value is 0 at x = 3:
(a – c)(3) + (b – d) = 0.
Here b – d = 5 – (–1) = 6, so:
3(a – c) + 6 = 0 => a – c = –2 … (2)

Solve (1) and (2): adding, 2a = 4, so a = 2, and c = 6 – 2 = 4.

Therefore p(x) = 2x + 5 and q(x) = 4x – 1.
Check: p + q = 6x + 4 ✓; p – q = –2x + 6, which is 0 at x = 3 ✓; p(0) = 5 ✓.
p(x) = 2x + 5 and q(x) = 4x – 1

Q12 Look at the first three stages of a growing pattern of hexagons made using matchsticks. A new hexagon gets added at every stage which shares a side with the last hexagon of the previous stage. (i) Draw the next two stages of the pattern. How many matchsticks will be required at these stages? (ii) Complete the table (Stage 1,2,3,4,5,…,n vs Number of matchsticks). (iii) Find a rule to determine the number of matchsticks required for the nth stage. (iv) How many matchsticks will be required for the 15th stage of the pattern? (v) Can 200 matchsticks form a stage in this pattern? Justify your answer.

SOLUTION

Stage 1 is a single hexagon, which needs 6 matchsticks. Each new hexagon shares one side (one matchstick) with the previous hexagon, so it adds only 5 new matchsticks.
So the counts are: Stage 1 = 6, Stage 2 = 6 + 5 = 11, Stage 3 = 11 + 5 = 16.

(i) Next two stages: Stage 4 = 16 + 5 = 21 matchsticks; Stage 5 = 21 + 5 = 26 matchsticks. (Draw a chain of 4 hexagons, then 5 hexagons, each sharing one side with the next.)

(ii) Completed table:

Stage	1	2	3	4	5	…	n
Matchsticks	6	11	16	21	26	…	5n + 1

(iii) The number of matchsticks starts at 6 and increases by the constant 5 each stage (a linear pattern). At Stage n:
matchsticks = 6 + 5(n – 1) = 5n + 1.

(iv) 15th stage: 5 × 15 + 1 = 75 + 1 = 76 matchsticks.

(v) Set 5n + 1 = 200: 5n = 199, so n = 199/5 = 39.8, which is not a whole number. Since n must be a positive integer, 200 matchsticks cannot form a stage in this pattern.
(i) Stage 4 = 21, Stage 5 = 26 (ii)/(iii) rule 5n + 1 (iv) 76 matchsticks (v) No — 5n + 1 = 200 gives n = 39.8, not a whole number.

Q13 Let p(x) = ax + b and q(x) = cx + d be two linear polynomials such that: (i) The graph of p(x) passes through the points (2, 3) and (6, 11). (ii) The graph of q(x) passes through the point (4, –1). (iii) The graph of q(x) is parallel to the graph of p(x). Find the polynomials p(x) and q(x). Also, find the coordinates of the point where these lines meet the x-axis.

SOLUTION

Finding p(x) = ax + b through (2, 3) and (6, 11):
2a + b = 3 … (1)
6a + b = 11 … (2)
Subtract (1) from (2): 4a = 8, so a = 2. From (1): b = 3 – 2(2) = –1.
Therefore p(x) = 2x – 1.

Finding q(x) = cx + d: q is parallel to p, so it has the same slope, c = 2. Its graph passes through (4, –1):
2(4) + d = –1 => 8 + d = –1 => d = –9.
Therefore q(x) = 2x – 9.

Point where each line meets the x-axis (y = 0):
For p(x): 2x – 1 = 0 => x = 1/2, so (1/2, 0).
For q(x): 2x – 9 = 0 => x = 9/2, so (9/2, 0).

(Note: p and q are parallel, so they never intersect each other; each meets the x-axis at its own point — p at (1/2, 0) and q at (9/2, 0).)
p(x) = 2x – 1 and q(x) = 2x – 9. They meet the x-axis at (1/2, 0) and (9/2, 0) respectively (being parallel, they do not meet each other).

Q14 What do all linear functions of the form f(x) = ax + a, a > 0, have in common?

SOLUTION

Take out the common factor a from f(x) = ax + a:
f(x) = a(x + 1).

Find where the graph cuts the x-axis, i.e. where f(x) = 0:
a(x + 1) = 0. Since a > 0 (so a ≠ 0), we need x + 1 = 0, giving x = –1.

This value x = –1 does not depend on a. So for every value of a > 0:
f(–1) = a(–1) + a = –a + a = 0.

Therefore every line of the form y = ax + a passes through the same point (–1, 0). That is what they all have in common — they all cut the x-axis at (–1, 0).
They all pass through the same point (–1, 0), since f(x) = a(x + 1) is 0 at x = –1 for every a > 0.