Chapter 7: The Mathematics of Maybe: Introduction to Probability

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Exercise 7.1 — The Probability Scale

Q1 Rank the following events on a scale from 0 (Impossible) to 1 (Certain). Label each event: Impossible, less likely, equally likely (even chance), more likely, certain. Give reasons why you gave each event its ranking.
(i) The next Monday will come after Sunday.
(ii) It will snow in Mumbai in July.
(iii) An elephant will walk through your classroom today.
(iv) You will greet at least one friend at school tomorrow.

SOLUTION

(i) The next Monday will come after Sunday.
Label: Certain. Probability = 1.
Reason: Days of the week follow a fixed, unchangeable order. Monday always follows Sunday — this is guaranteed by definition. There is no possibility of it not happening.

(ii) It will snow in Mumbai in July.
Label: Impossible. Probability = 0 (or extremely close to 0).
Reason: Mumbai has a tropical climate. July is the peak of the monsoon season with very high temperatures. Snowfall requires sub-zero temperatures, which have never been recorded in Mumbai. This event is practically impossible.

(iii) An elephant will walk through your classroom today.
Label: Impossible (or less likely, effectively 0).
Reason: Elephants are wild animals not found in urban schools. There is no reasonable mechanism by which this could happen. The probability is essentially 0.

(iv) You will greet at least one friend at school tomorrow.
Label: More likely (probability close to 1, but not certain).
Reason: At school you regularly interact with classmates. It is highly probable that you will greet at least one friend, but it is not absolutely certain (e.g., you might be absent, or school might be closed). The event is more likely but not guaranteed.
(i) Certain (probability = 1). (ii) Impossible (probability = 0). (iii) Impossible (probability ≈ 0). (iv) More likely (probability close to 1, but not certain).

Exercise 7.2 — Experimental Probability

Q1 A teacher mixes a large bag of sweets of different colours and randomly selects a sample of 30 sweets. She counts the number of sweets of each colour: 10 red sweets | 8 green sweets | 7 yellow sweets | 5 blue sweets.
(i) Calculate the probability that a randomly picked sweet from the sample is green.
(ii) If there are 600 sweets in total in the large bag, estimate how many are likely to be yellow, based on the sample results.

SOLUTION

(i) Total sweets in sample = 30. Number of green sweets = 8.
P(green) = Number of green sweets / Total sweets in sample
P(green) = 8/30 = 4/15

(ii) Experimental probability of picking a yellow sweet from the sample:
P(yellow) = 7/30
Estimated number of yellow sweets in 600 = P(yellow) × Total sweets
= (7/30) × 600
= 7 × 20
= 140

Q2 A survey is conducted at a school where a random sample of 40 students is asked about their favourite club. The responses are: 14 students: Science Club | 11 students: Arts Club | 9 students: Sports Club | 6 students: Debate Club. Assume there are 800 students in the whole school.
(i) What is the probability that a randomly chosen student from the sample prefers the Arts Club?
(ii) Using the sample results, estimate how many students in the whole school are likely to prefer the Sports Club.

SOLUTION

(i) Total students in sample = 40. Students preferring Arts Club = 11.
P(Arts Club) = 11/40

(ii) Experimental probability of preferring Sports Club = 9/40.
Estimated number in whole school = P(Sports Club) × Total students
= (9/40) × 800
= 9 × 20
= 180

Q3 Toss a coin 20 times and record the result each time (heads or tails).
(i) How many times did you get heads?
(ii) How many times did you get tails?
(iii) Calculate the experimental probability of getting heads.
(iv) If you toss the coin once more, what is the probability of getting tails?

SOLUTION

This is a hands-on activity. Let the number of heads obtained in 20 tosses be h, and the number of tails be t, where h + t = 20.

(i) The number of heads = h (recorded from the actual experiment).

(ii) The number of tails = t = 20 - h.

(iii) Experimental probability of getting heads:
P(heads) = (Number of heads) / (Total tosses) = h / 20
For example, if h = 11, then P(heads) = 11/20.
As the number of tosses increases, experimental probability approaches the theoretical value of 1/2.

(iv) The probability of getting tails on the next toss is based on theoretical probability (since the coin is fair):
P(tails) = 1/2
Note: Each toss is an independent event. Previous results do not affect the next toss.
(i)–(ii) Recorded from experiment (h heads, t = 20 - h tails). (iii) Experimental P(heads) = h/20.
(iv) Theoretical P(tails) = 1/2.

Q4 Toss a paper cup into the air 100 times. After each toss record whether the cup lands on its bottom, upside down on its top, or on its side. Assign probabilities to the outcomes by using experimental probability.

SOLUTION

This is a hands-on activity. Toss a paper cup 100 times and record the outcome each time.
Let: n(bottom) = number of times it lands on the bottom,
n(top) = number of times it lands upside down on top,
n(side) = number of times it lands on its side.
We have: n(bottom) + n(top) + n(side) = 100.

Assign experimental probabilities:
P(bottom) = n(bottom) / 100
P(top) = n(top) / 100
P(side) = n(side) / 100

Note: P(bottom) + P(top) + P(side) = 1.
Unlike a fair coin or die, the cup is not symmetric, so the three outcomes are NOT equally likely. The side outcome is typically the most common since the cup's side presents the largest area.

P(bottom) = n(bottom)/100, P(top) = n(top)/100, P(side) = n(side)/100, where the three counts sum to 100. Outcomes are not equally likely.

Q5 What is the probability of getting an even number when rolling a fair 6-sided die?

SOLUTION

Sample space when rolling a 6-sided die: S = {1, 2, 3, 4, 5, 6}, so n(S) = 6.
Even numbers in the sample space: {2, 4, 6}, so number of favourable outcomes = 3.
P(even number) = Number of favourable outcomes / Total number of outcomes
= 3/6
= 1/2

Q6 Suppose you roll a 6-sided die 12 times and get a '3' three times.
(i) What is the experimental probability of rolling a '3'?
(ii) What is the theoretical probability of rolling a '3'?
(iii) Why might these probabilities be different? What would you expect to happen if you roll the die 60, 600, or 6000 times?

SOLUTION

(i) Experimental probability of rolling a '3':
Number of times '3' appeared = 3. Total rolls = 12.
Experimental P('3') = 3/12 = 1/4

(ii) Theoretical probability of rolling a '3':
A fair die has 6 equally likely outcomes. Favourable outcome = {3}, count = 1.
Theoretical P('3') = 1/6

(iii) Experimental and theoretical probabilities may differ because:
With only 12 rolls, the sample size is small. Random variation means actual results can deviate noticeably from expected results.
By the Law of Large Numbers, as the number of trials increases, the experimental probability gets closer to the theoretical probability.

Expected number of '3's in repeated trials:
60 rolls: expected '3's = 60 × (1/6) = 10
600 rolls: expected '3's = 600 × (1/6) = 100
6000 rolls: expected '3's = 6000 × (1/6) = 1000
So the experimental probability (approx. 1/6) would get closer and closer to the theoretical probability of 1/6 as the number of rolls increases.
(i) Experimental P('3') = 1/4. (ii) Theoretical P('3') = 1/6. (iii) Small sample causes variation; with more rolls the experimental probability converges to 1/6.

Exercise 7.3 — Elements of Probability: Sample Spaces and Events

Q1 When a single 6-sided die is rolled, what is the total number of possible outcomes in the sample space?

SOLUTION

When a 6-sided die is rolled, the possible outcomes are the numbers 1, 2, 3, 4, 5, and 6.
Sample space: S = {1, 2, 3, 4, 5, 6}.
Total number of possible outcomes = n(S) = 6.

Q2 For the following experiments write down the sample space S.
(i) Rolling a die and tossing a coin together.
(ii) Choosing a random integer between -5 and +5.
(iii) A box containing 5 green and 7 red balls. One ball is drawn at random.

SOLUTION

(i) Rolling a die and tossing a coin together:
Die outcomes: {1, 2, 3, 4, 5, 6}. Coin outcomes: {H, T}.
Each element of S is a pair (die result, coin result).
S = {(1,H), (1,T), (2,H), (2,T), (3,H), (3,T), (4,H), (4,T), (5,H), (5,T), (6,H), (6,T)}
n(S) = 12.

(ii) Choosing a random integer between -5 and +5 (strictly between, i.e., excluding -5 and +5):
S = {-4, -3, -2, -1, 0, 1, 2, 3, 4}
n(S) = 9.

(iii) A box with 5 green balls and 7 red balls; one ball drawn at random:
The ball drawn is either green or red.
S = {Green ball, Red ball}
n(S) = 2 (by type), or equivalently there are 12 equally likely individual balls,
which can be labelled S = {G1, G2, G3, G4, G5, R1, R2, R3, R4, R5, R6, R7}, n(S) = 12.
(i) S = {(1,H),(1,T),(2,H),(2,T),(3,H),(3,T),(4,H),(4,T),(5,H),(5,T),(6,H),(6,T)}, n(S) = 12. (ii) S = {-4,-3,-2,-1,0,1,2,3,4}, n(S) = 9. (iii) S = {G1,G2,G3,G4,G5, R1,R2,R3,R4,R5,R6,R7}, n(S) = 12 (the 12 individual balls). By colour type only, {Green, Red} gives a coarser n = 2.

Q3 In a village fair, there are 3 popular snacks available: Samosa, Pakora, and Bhaji. For drinks, villagers can choose either Chai or Lassi.
(i) List the sample space of all possible snack and drink combinations a person could choose at the fair.
(ii) List the event 'Selecting Samosa as a snack.'

SOLUTION

(i) Snacks = {Samosa, Pakora, Bhaji}. Drinks = {Chai, Lassi}.
Each combination is a (snack, drink) pair.

S = {(Samosa, Chai), (Samosa, Lassi), (Pakora, Chai), (Pakora, Lassi), (Bhaji, Chai), (Bhaji, Lassi)}
n(S) = 3 × 2 = 6.

(ii) Event 'Selecting Samosa as a snack' = all outcomes in S where the snack is Samosa:
E = {(Samosa, Chai), (Samosa, Lassi)}

Exercise 7.4 — Tree Diagrams

Q1 There are two fruit baskets A and B. Basket A has one apple and two oranges. Basket B has one banana and one mango. You randomly pick one fruit from each basket.
(i) Draw a tree diagram showing all possible pairs of fruits.
(ii) List the sample space.
(iii) What is the probability of picking one apple and one banana?

SOLUTION

(i) Tree diagram:
Basket A has 3 fruits: Apple (A), Orange-1 (O1), Orange-2 (O2).
Basket B has 2 fruits: Banana (Ba), Mango (M).

Start -> Apple -> Banana : (Apple, Banana)
-> Mango : (Apple, Mango)
-> Orange1 -> Banana: (Orange, Banana)
-> Mango : (Orange, Mango)
-> Orange2 -> Banana: (Orange, Banana)
-> Mango : (Orange, Mango)

(ii) Sample space (listing all equally likely pairs treating the two oranges as distinct):
S = {(Apple, Banana), (Apple, Mango), (Orange1, Banana), (Orange1, Mango), (Orange2, Banana), (Orange2, Mango)}
n(S) = 3 × 2 = 6.

(iii) Favourable outcome for (Apple AND Banana) = {(Apple, Banana)}, which is 1 outcome.
P(Apple and Banana) = 1/6

Q2 Let us say that you have a box containing 3 red pens, 4 black pens and 2 green pens. You pick a pen (without looking) from the box and put it back. Then your friend does the same.
(i) What are the possible outcomes of the pen colours? Can you draw a tree diagram representing the possible outcomes?
(ii) Can you use the tree diagram to guess the probability that both you and your friend pick pens of the same colour?

SOLUTION

(i) Total pens = 3 + 4 + 2 = 9. Colours: Red (R), Black (B), Green (G).
Since you replace the pen before your friend picks, the two picks are independent.
Tree diagram (each first-level branch is your pick; each second-level branch is your friend's pick):
Your pick R (3/9) -> Friend picks R (3/9): (R,R)
-> Friend picks B (4/9): (R,B)
-> Friend picks G (2/9): (R,G)
Your pick B (4/9) -> Friend picks R (3/9): (B,R)
-> Friend picks B (4/9): (B,B)

-> Friend picks G (2/9): (B,G)

Your pick G (2/9) -> Friend picks R (3/9): (G,R)

-> Friend picks B (4/9): (G,B)

-> Friend picks G (2/9): (G,G)

Total possible colour-pair outcomes: 9 (listed above).

(ii) 'Both pick same colour' means outcome is (R,R), (B,B), or (G,G).

Using theoretical probability with replacement:

P(both Red) = (3/9) × (3/9) = 9/81 = 1/9

P(both Black) = (4/9) × (4/9) = 16/81

P(both Green) = (2/9) × (2/9) = 4/81

P(same colour) = 1/9 + 16/81 + 4/81

= 9/81 + 16/81 + 4/81

= 29/81

Exercises — The Mathematics of Maybe: Introduction to Probability

Q1 Fill in the blanks. (i) The probability of an impossible event is _____. (ii) The set of all possible outcomes of a random experiment is called the _____. (iii) The probability of an event that is certain to happen is _____. (iv) Tossing a fair coin has a probability of _____ for getting heads.

SOLUTION

(i) An impossible event can never occur, so it has no favourable outcomes. Its probability is 0.
(ii) The list of all possible outcomes of a random experiment is called the sample space.
(iii) An event that is certain to happen occurs in every outcome, so its probability is 1.
(iv) A fair coin has 2 equally likely outcomes (Head, Tail). Favourable = 1 (Head).
P(Head) = 1/2.
(i) 0 (ii) sample space (iii) 1 (iv) 1/2

Q2 In a survey of 50 students, 15 students said they liked football. The number of students who like football is 15, and the _____ (frequency/relative frequency) is _____ (fill in the fraction or decimal).

SOLUTION

The count 15 is the frequency of students who like football.
The relative frequency is the fraction of the total that this represents.
Relative frequency = 15/50 = 3/10 = 0.3.
So the blank asks for the relative frequency, which is 3/10 (0.3).
relative frequency; 3/10 = 0.3

Q3 Which of the following experiments have equally likely outcomes? Explain. (i) A driver attempts to start a car. The car starts or does not start. (ii) Tossing a fair coin once. (iii) Rolling a fair 6-sided die. (iv) Choosing a marble randomly from a bag that contains 3 red marbles and 7 blue marbles. (v) A baby is born. It is a boy or a girl.

SOLUTION

Equally likely means each outcome has the same chance of occurring.
(i) Not equally likely. Whether a car starts depends on its condition, fuel, battery, etc.; 'starts' and 'does not start' are generally not equally likely.
(ii) Equally likely. A fair coin gives Head and Tail with equal probability 1/2 each.
(iii) Equally likely. A fair die gives each of 1, 2, 3, 4, 5, 6 with equal probability 1/6.
(iv) Not equally likely (for colour). There are 3 red and 7 blue marbles, so drawing blue is more likely than red; P(red)=3/10, P(blue)=7/10. (Each individual marble is equally likely, but red and blue as outcomes are not.)
(v) Approximately equally likely. A baby being a boy or a girl is usually taken as two equally likely outcomes, each with probability about 1/2.
(ii) and (iii) have equally likely outcomes (fair coin, fair die); (v) is treated as equally likely; (i)

and (iv) are not.

Q4 Write the sample space and calculate the probability based on the given information. (i) Two coins are tossed at the same time. What is the probability of getting at least one head? (ii) Ten identical cards numbered 1 to 10 are placed in a box. One card is drawn at random. What is the probability of drawing a card with an even number? (iii) A die is rolled once. What is the probability of getting a number greater than 4? (iv) A bag contains 3 red balls, 2 blue balls, and 1 green ball. One ball is picked at random. What is the probability that it is not red? (v) Three coins are tossed simultaneously. What is the probability of getting exactly two heads?

SOLUTION

(i) Sample space S = {HH, HT, TH, TT}, so 4 outcomes.
At least one head = {HH, HT, TH} => 3 favourable outcomes.
P(at least one head) = 3/4.

(ii) S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, 10 outcomes.
Even numbers = {2, 4, 6, 8, 10} => 5 favourable.
P(even) = 5/10 = 1/2.

(iii) S = {1, 2, 3, 4, 5, 6}, 6 outcomes.
Greater than 4 = {5, 6} => 2 favourable.
P(>4) = 2/6 = 1/3.

(iv) Total balls = 3 + 2 + 1 = 6. Not red = blue or green = 2 + 1 = 3 favourable.
P(not red) = 3/6 = 1/2.

(v) S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}, 8 outcomes.
Exactly two heads = {HHT, HTH, THH} => 3 favourable.
P(exactly two heads) = 3/8.
(i) 3/4 (ii) 1/2 (iii) 1/3 (iv) 1/2 (v) 3/8

Q5 A bag has 3 candies: strawberry, lemon, and mint. One is picked at random. What is the probability of picking a strawberry candy?

SOLUTION

There are 3 equally likely candies: strawberry, lemon, mint.
Favourable outcome (strawberry) = 1.
P(strawberry) = 1/3.

Q6 A child has 2 shirts (one red and one blue) and 3 types of pants (jeans, khakis, and shorts). List all the possible combinations of outfits consisting of one shirt and one pair of pants. Display your answer in a table format.

SOLUTION

Each outfit is one shirt paired with one pair of pants.
Number of outfits = 2 shirts × 3 pants = 6.
The combinations, in table form (rows = shirts, columns = pants):

Shirt	Jeans	Khakis	Shorts
Red	Red, Jeans	Red, Khakis	Red, Shorts
Blue	Blue, Jeans	Blue, Khakis	Blue, Shorts

So the 6 outfits are: (Red, Jeans), (Red, Khakis), (Red, Shorts), (Blue, Jeans), (Blue, Khakis), (Blue, Shorts).
6 outfits: Red/Blue shirt each paired with Jeans, Khakis, Shorts.

Q7 A tyre company records distances before replacement in 1000 cases. Distance (km): Less than 4000 -> 20 cases; 4001 to 9000 -> 210 cases; 9001 to 14000 -> 325 cases; More than 14000 -> 445 cases. Find the probability that a randomly chosen tyre lasts: (i) Less than 4000 km. (ii) Between 4000 and 14000 km. (iii) More than 14000 km.

SOLUTION

Total number of cases = 20 + 210 + 325 + 445 = 1000.

(i) Less than 4000 km: 20 cases.
P = 20/1000 = 1/50 = 0.02.

(ii) Between 4000 and 14000 km covers the 4001–9000 and 9001–14000 groups: 210 + 325 = 535 cases.
P = 535/1000 = 107/200 = 0.535.

(iii) More than 14000 km: 445 cases.
P = 445/1000 = 89/200 = 0.445.
(i) 1/50 (0.02) (ii) 107/200 (0.535) (iii) 89/200 (0.445)

Q8 The letters of the word 'PEACE' are placed on cards (P, E, A, C, E). Leela draws a card without looking. (i) What is the probability that it is a P, E or C? (ii) What is the probability that it is not an E?

SOLUTION

The word PEACE has 5 cards: P, E, A, C, E. Total outcomes = 5.
Letter counts: P = 1, E = 2, A = 1, C = 1.

(i) P, E or C: favourable = 1 (P) + 2 (E) + 1 (C) = 4.

P(P, E or C) = 4/5.

(ii) Not an E: cards that are not E = P, A, C = 3 cards.
P(not E) = 3/5.
(i) 4/5 (ii) 3/5

Q9 A game of chance consists of spinning an arrow (see Fig. 7.7) which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8, and these are equally likely outcomes. What is the probability that it will point at (i) 8? (ii) An odd number? (iii) A number greater than 2? (iv) A number less than 9? (v) A multiple of 3?

SOLUTION

The spinner has 8 equally likely outcomes: {1, 2, 3, 4, 5, 6, 7, 8}. Total = 8.

(i) Pointing at 8: 1 favourable.
P = 1/8.

(ii) Odd numbers = {1, 3, 5, 7} => 4 favourable.
P = 4/8 = 1/2.

(iii) Numbers greater than 2 = {3, 4, 5, 6, 7, 8} => 6 favourable.
P = 6/8 = 3/4.

(iv) Numbers less than 9 = all of {1,...,8} => 8 favourable (a certain event).
P = 8/8 = 1.

(v) Multiples of 3 = {3, 6} => 2 favourable.
P = 2/8 = 1/4.
(i) 1/8 (ii) 1/2 (iii) 3/4 (iv) 1 (v) 1/4

Q10 A basket contains 4 red balls and 5 blue balls. One ball is drawn and laid aside, and a second ball is drawn. Draw a tree diagram to represent the possible outcomes and probabilities. Use the tree diagram to answer: (i) What is the probability of drawing a red ball and then a blue ball? (ii) What is the probability of drawing 2 blue balls?

SOLUTION

Total balls = 4 red + 5 blue = 9. Two balls are drawn without replacement.
Tree diagram (first draw, then second draw with the remaining 8 balls):
First draw Red (P = 4/9): then Red (3/8) or Blue (5/8).
First draw Blue (P = 5/9): then Red (4/8) or Blue (4/8).

(i) Red then Blue = P(Red first) × P(Blue second | Red first).
= (4/9) × (5/8) = 20/72 = 5/18.

(ii) Two blue = P(Blue first) × P(Blue second | Blue first).
= (5/9) × (4/8) = 20/72 = 5/18.
(i) 5/18 (ii) 5/18

Q11 I throw a pair of 6-sided dice. Write down an event that has a probability of 0 and an outcome that has a probability of 1.

SOLUTION

A pair of 6-sided dice gives sums from 2 to 12; total outcomes = 6 × 6 = 36.

Event with probability 0 (impossible event): 'the sum of the two dice is 1' — impossible, since the smallest sum is 1 + 1 = 2. So P = 0.
(Another valid example: 'the sum is greater than 12'.)

Outcome/event with probability 1 (certain event): 'the sum of the two dice is between 2 and 12 (inclusive)' — this always happens, so P = 1.
(Another valid example: 'each die shows a number from 1 to 6'.)
Probability 0: e.g. 'the sum equals 1' (impossible). Probability 1: e.g. 'the sum lies between 2 and 12' (certain).

Q12 Write the sample space and calculate the probability based on the given information. (i) Two dice are rolled. What is the probability that the sum is a prime number greater than 5? (ii) A bag contains 4 red, 3 green, and 2 blue balls. Two balls are drawn without replacement. What is the probability that both are of different colours? (iii) Three coins are tossed. What is the probability that the first coin shows heads and exactly two heads occur in total? (iv) A four-digit number is formed using the digits 1, 2, 3, and 4 with no repetition. What is the probability that the number is even? (v) A student takes a multiple-choice test with 3 questions, each having 4 options (A, B, C, D), with only one correct answer. What is the probability that the student guesses and gets exactly 2 answers correct?

SOLUTION

(i) Two dice: total outcomes = 6 × 6 = 36. Possible sums are 2 to 12; primes greater than 5 in this range are 7 and 11.
Ways to get 7: (1,6),(2,5),(3,4),(4,3),(5,2),(6,1) = 6. Ways to get 11: (5,6),(6,5) = 2. Total favourable = 6 + 2 = 8.
P = 8/36 = 2/9.

(ii) Bag has 4 + 3 + 2 = 9 balls. Draw 2 without replacement: total ways = C(9,2) = 36.
Same-colour pairs = C(4,2) + C(3,2) + C(2,2) = 6 + 3 + 1 = 10.
Different-colour pairs = 36 − 10 = 26.
P(different colours) = 26/36 = 13/18.

(iii) Three coins: sample space has 8 equally likely outcomes {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}.
First coin heads AND exactly two heads total: {HHT, HTH} = 2 favourable.
P = 2/8 = 1/4.

(iv) Four-digit numbers from 1,2,3,4 with no repetition: total = 4! = 24.
Number is even when the last digit is 2 or 4. For each choice of last digit (2 ways), the first three positions can be arranged in 3! = 6 ways: 2 × 6 = 12 favourable.

P = 12/24 = 1/2.

(v) Each question guessed independently among 4 options: total outcomes = 4 × 4 × 4 = 64. Exactly 2 correct: choose which 2 of the 3 questions are correct = C(3,2) = 3 ways; the correct ones have 1 way each and the remaining wrong one has 3 wrong options => 3 × (1 × 1 × 3) = 9 favourable.
P = 9/64.
(i) 2/9 (ii) 13/18 (iii) 1/4 (iv) 1/2 (v) 9/64

Q13 A box contains 4 balls numbered 1 to 4. Record a sample space using a tree diagram for the following experiments: (i) A ball is drawn, and the number is recorded. Then the ball is returned, and a second ball is drawn and recorded. (ii) A ball is drawn and recorded. Without replacing the first ball, the experimenter draws and records a second ball. (iii) What are the sizes of these two sample spaces?

SOLUTION

Balls are numbered 1, 2, 3, 4.

(i) With replacement: the first draw can be any of 4, and after returning it the second draw can again be any of 4. Each outcome is an ordered pair (a, b).
Sample space = {(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(4,3),(4,4)}.
Size = 4 × 4 = 16.

(ii) Without replacement: first draw = 4 choices, second draw = 3 remaining choices (the second number differs from the first).
Sample space = all ordered pairs (a, b) with a ≠ b: {(1,2),(1,3),(1,4),(2,1),(2,3),(2,4),(3,1),(3,2),(3,4),(4,1),(4,2),(4,3)}.
Size = 4 × 3 = 12.

(iii) The sizes are 16 (with replacement) and 12 (without replacement).
(i) 16 ordered pairs (with replacement) (ii) 12 ordered pairs a≠b (without replacement) (iii) sizes 16 and 12

Q14 List the elements of a sample space for the simultaneous tossing of a coin and drawing of a card from a set of 6 cards numbered 1 through 6.

SOLUTION

A coin has 2 outcomes: Head (H), Tail (T). The cards give 6 outcomes: 1, 2, 3, 4, 5, 6.
Each outcome pairs the coin result with a card number.
Sample space S = {(H,1),(H,2),(H,3),(H,4),(H,5),(H,6),(T,1),(T,2),(T,3),(T,4),(T,5),(T,6)}.
Size = 2 × 6 = 12.
S = {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}; 12 outcomes.

Q15 Three coins are tossed, and the number of heads is recorded. Which of the following lists is a sample space for this experiment? Why do the other lists fail to qualify as a sample space? (i) {1, 2, 3} (ii) {0, 1, 2} (iii) {0, 1, 2, 3, 4} (iv) {0, 1, 2, 3}

SOLUTION

When three coins are tossed, the number of heads can be 0, 1, 2, or 3.
A sample space must list every possible outcome exactly once — no possible value missing and no impossible value included.

(iv) {0, 1, 2, 3} is the correct sample space: it contains all and only the possible counts of heads.

Why the others fail:
(i) {1, 2, 3} omits 0 (getting no heads, i.e. TTT, is possible), so it is incomplete.
(ii) {0, 1, 2} omits 3 (getting three heads, HHH, is possible), so it is incomplete.
(iii) {0, 1, 2, 3, 4} includes 4, but you cannot get 4 heads from only 3 coins — it contains an impossible outcome.
(iv) {0, 1, 2, 3} is the sample space; (i) omits 0, (ii) omits 3, (iii) includes the impossible value 4.

Q16 Suppose you drop a dye at random on the rectangular region shown in Fig. 7.8 (a rectangle 3 m by 2 m). What is the probability that it will land inside the circle with a diameter of 1 m?

SOLUTION

This is a geometric probability: P = (area of favourable region) / (area of whole region).
Rectangle: length 3 m and width 2 m, so area = 3 × 2 = 6 m².
Circle: diameter 1 m => radius r = 1/2 m. Area = πr² = π × (1/2)² = π/4 m².
P(inside circle) = (π/4) / 6 = π/24.
Taking π = 22/7: P = (22/7)/24 = 22/168 = 11/84 ≈ 0.131.
Numerically, π/24 ≈ 0.131.
P = π/24 ≈ 0.131 (≈ 11/84 using π = 22/7)