Chapter 10: How Quantities Combine: Understanding Data

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Exercise 10.1 — Exercise Set 10.1

Q1 The average score of students on a test in Section A is 72 and that of students in Section B is 76. What is the combined average of both the sections given that Section A has 30 students and Section B has 25 students?

SOLUTION

The combined average is the weighted mean of the two section averages, with weights equal to the number of students.

Combined average = (72 × 30 + 76 × 25) / (30 + 25)
= (2160 + 1900) / 55
= 4060 / 55
= 812/11 ≈ 73.82

The combined average score of both sections is 812/11 ≈ 73.82.

Q2 A farmer mixes three equal quantities of fertilisers. The first one contains 1/10 nitrogen, the second contains 9/50 nitrogen, and the third contains 3/60 nitrogen. What is the fraction of nitrogen in the mixture?

SOLUTION

Since the three quantities mixed are equal, the resulting nitrogen fraction is the simple arithmetic mean of the three fractions.

Note: 3/60 = 1/20.

Fraction of nitrogen = (1/10 + 9/50 + 1/20) / 3

Finding a common denominator of 100:
1/10 = 10/100, 9/50 = 18/100, 1/20 = 5/100

Sum = (10 + 18 + 5)/100 = 33/100

Fraction of nitrogen = (33/100) / 3 = 33/300 = 11/100

The fraction of nitrogen in the mixture is 11/100.

Q3 (Śrīdharācārya, Pāṭīgaṇita, c. 750 CE) A goldsmith melts together three pieces of gold: 9 units at 12 varṇa, 5 units at 10 varṇa, and 17 units at 11 varṇa. Find the purity in varṇa of the combined gold.

SOLUTION

The purity of the combined gold is the weighted mean, where the weights are the quantities of each piece.

Purity = (9 × 12 + 5 × 10 + 17 × 11) / (9 + 5 + 17)
= (108 + 50 + 187) / 31
= 345 / 31
≈ 11.13 varṇa

The purity of the combined gold is 345/31 ≈ 11.13 varṇa.

Q4 The average rainfall per day in the months of May, June, and July in a certain location are 3.5 mm, 10 mm and 8.7 mm respectively. Write an expression that gives their combined average.

SOLUTION

Let n_May, n_June, and n_July be the number of days in May, June, and July respectively.

The combined average daily rainfall is the weighted mean:
Combined average = (3.5 × n_May + 10 × n_June + 8.7 × n_July) / (n_May + n_June + n_July)

If the months are taken to have equal numbers of days (or we treat each month equally), then n_May = n_June = n_July, and the combined average simplifies to:
= (3.5 + 10 + 8.7) / 3 = 22.2 / 3 = 7.4 mm per day

The general expression giving the combined average daily rainfall is:
(3.5 × n_May + 10 × n_June + 8.7 × n_July) / (n_May + n_June + n_July)

Q5 Calculate the concentration of spice mix in these two scenarios. (i) A 100 mL kashayam/kadha with 5% spice mix, a 200 mL one with 10% spice mix, and a 300 mL one with 15% spice mix are combined. (ii) A 300 mL kashayam/kadha with 5% spice mix, a 200 mL one with 10% spice mix, and a 100 mL one with 15% spice mix are mixed.

SOLUTION

(i) Total volume = 100 + 200 + 300 = 600 mL
Total spice = 100 × 5% + 200 × 10% + 300 × 15%
= 5 + 20 + 45 = 70 mL
Concentration = 70/600 × 100 = 7000/600 = 35/3 ≈ 11.67%

(ii) Total volume = 300 + 200 + 100 = 600 mL
Total spice = 300 × 5% + 200 × 10% + 100 × 15%
= 15 + 20 + 15 = 50 mL
Concentration = 50/600 × 100 = 5000/600 = 25/3 ≈ 8.33%

(i) The concentration of spice mix is 35/3 ≈ 11.67%.
(ii) The concentration of spice mix is 25/3 ≈ 8.33%.

Note: In (i) the larger volume has the higher concentration, so the combined concentration is closer to 15%. In (ii) the larger volume has the lower concentration, so the combined concentration is closer to 5%.

Exercise 10.2 — Exercise Set 10.2

Q1 Savitri's marks in Kashmiri are as follows: 35 out of 50 in internal tests, 44 out of 60 in the project, and 80 out of 100 in the final exam. The annual percentage score is calculated by combining the internals, project, and final exam in the ratio 3 : 4 : 5. Which of the following expression(s) gives her annual score (as a percentage) in Kashmiri? (i) (35×3 + 44×4 + 80×5)/(3+4+5) (ii) ((35/100)×3 + (44/100)×4 + (80/100)×5)/(3+4+5) (iii) ((35/50)×3 + (44/60)×4 + (80/100)×5)/(3+4+5) × 100 (iv) ((35/50×100)×3 + (44/60×100)×4 + (80/100×100)×5)/(3+4+5)

SOLUTION

The annual score combines the percentage marks in each component with weights 3, 4, 5. First convert each score to a percentage out of 100:

Internals: 35/50 × 100 = 70%

Project: 44/60 × 100 ≈ 73.33%

Final: 80/100 × 100 = 80%

Weighted annual score = (70×3 + 73.33×4 + 80×5) / (3+4+5)

Now check each expression:

(i) (35×3 + 44×4 + 80×5)/(3+4+5): This treats raw marks as percentages (35 instead of 70%), so it gives a wrong value. INCORRECT.

(ii) ((35/100)×3 + (44/100)×4 + (80/100)×5)/(3+4+5): Uses 35/100 instead of 35/50 for internals, so it treats 35 as a percentage out of 100 instead of 50. INCORRECT.

(iii) ((35/50)×3 + (44/60)×4 + (80/100)×5)/(3+4+5) × 100: Converts each score to a fraction of its maximum, then multiplies by 100 to express as percentage. This is correct.
= (0.7×3 + 0.7333×4 + 0.8×5)/12 × 100 = (2.1 + 2.933 + 4)/12 × 100 ≈ 75.28%. CORRECT.

(iv) ((35/50×100)×3 + (44/60×100)×4 + (80/100×100)×5)/(3+4+5): This converts each score to percentage first, then applies weights. This equals expression (iii). CORRECT.

Expressions (iii) and (iv) are correct. Savitri's annual score is approximately 75.28%.

Exercise 10.3 — Exercise Set 10.3

Q1 A stationery shop owner made 8000 selling books, of which 30% is the profit amount, and 1000 selling book covers, of which 50% is the profit amount. What is the percentage of profit on the total sales?

SOLUTION

Profit from books = 30% of 8000 = 2400
Profit from covers = 50% of 1000 = 500
Total profit = 2400 + 500 = 2900
Total sales = 8000 + 1000 = 9000

Percentage profit = (Total profit / Total sales) × 100
= (2900 / 9000) × 100
= 290/9 ≈ 32.22%

The percentage of profit on total sales is 290/9 ≈ 32.22%.

Q2 A white stork's migration is tracked. The average daily distance travelled, calculated over 20 days, is 44.5 km. On the 21st day, it flew 55 km. What is the average daily distance travelled over these 21 days? Make a guess before you calculate.

SOLUTION

Guess: Since 55 km > 44.5 km, adding a higher-than-average value will pull the average up slightly.

Total distance over 20 days = 44.5 × 20 = 890 km
Distance on day 21 = 55 km
Total distance over 21 days = 890 + 55 = 945 km

New average = 945 / 21 = 45 km per day

The average daily distance over 21 days is 45 km.

Q3 A 600 mL solution with 5% salt is mixed with a 300 mL solution with 8% sugar. What are the concentrations of salt and sugar in the mixture?

SOLUTION

Since salt and sugar are independent solutes, their concentrations in the mixture are calculated separately.

Total mixture volume = 600 + 300 = 900 mL

Salt concentration: Salt comes only from the 600 mL solution.
Amount of salt = 600 × 5% = 30 mL
Salt concentration in mixture = 30/900 × 100 = 10/3 ≈ 3.33%

Sugar concentration: Sugar comes only from the 300 mL solution.

Amount of sugar = 300 × 8% = 24 mL
Sugar concentration in mixture = 24/900 × 100 = 8/3 ≈ 2.67%

The correct answer is option (v): Salt: 3.33%, Sugar: 2.67%.

Note: The concentrations decrease because each solute is now diluted across 900 mL instead of the original smaller volume.

Q4 At a panipuri (golgappa) stall, the concentration of spice in the pani (spiced water) was 8%. Many customers complained that it was too spicy. What quantity of regular water should be mixed into the 10 litres of pani so that the spice level is reduced to (3/4)th of the original concentration?

SOLUTION

Original concentration = 8%
Target concentration = (3/4) × 8% = 6%

Amount of spice in the 10 litres of pani = 10 × 8% = 0.8 litres
Let x litres of water be added.

After adding water, the spice concentration becomes:
0.8 / (10 + x) = 6/100

Solving: 0.8 = 0.06 × (10 + x)
0.8 = 0.6 + 0.06x
0.2 = 0.06x
x = 0.2 / 0.06 = 10/3 ≈ 3.33 litres

A quantity of 10/3 ≈ 3.33 litres of water should be mixed.

Q5 A physical fitness evaluation is being undertaken. The final marks are calculated by combining the marks for strength, flexibility, and agility in the ratio 4 : 5 : 6. Rashi has scored 60, 65, and 70. Keerthi has scored 55, 65, and 75 respectively. (i) Find out whose total is more without calculating. (ii) What are their final marks?

SOLUTION

(i) Without calculating:
Rashi and Keerthi have equal flexibility scores (65). For strength, Rashi > Keerthi (60 > 55), but strength has weight 4. For agility, Keerthi > Rashi (75 > 70), and agility has weight 6.
The difference in favour of Keerthi: (75 − 70) × 6 = 30 extra weighted points from agility.
The difference in favour of Rashi: (60 − 55) × 4 = 20 extra weighted points from strength.
Since 30 > 20, Keerthi's total weighted score is more.

(ii) Final marks:
Rashi = (60 × 4 + 65 × 5 + 70 × 6) / (4 + 5 + 6)
= (240 + 325 + 420) / 15
= 985 / 15
= 197/3 ≈ 65.67

Keerthi = (55 × 4 + 65 × 5 + 75 × 6) / (4 + 5 + 6)
= (220 + 325 + 450) / 15
= 995 / 15
= 199/3 ≈ 66.33

Rashi's final marks ≈ 65.67 and Keerthi's final marks ≈ 66.33. Keerthi's total is more.

Q6 A restaurant collected ratings from 10 customers on a scale of 1 to 5. Food ratings: 5★ from 5 customers, 4★ from 3, 3★ from 2, 2★ from 0, 1★ from 0. Ambience ratings: 5★ from 0, 4★ from 4, 3★ from 5, 2★ from 1, 1★ from 0. Service ratings: 5★ from 1, 4★ from 2, 3★ from 2, 2★ from 4, 1★ from 1. What is the average rating if the metrics are combined with the weights food : ambience : service = 6 : 5 : 4?

SOLUTION

First compute the average rating for each metric across 10 customers:

Food average = (5×5 + 4×3 + 3×2 + 2×0 + 1×0) / 10
= (25 + 12 + 6) / 10 = 43/10 = 4.3

Ambience average = (5×0 + 4×4 + 3×5 + 2×1 + 1×0) / 10
= (0 + 16 + 15 + 2) / 10 = 33/10 = 3.3

Service average = (5×1 + 4×2 + 3×2 + 2×4 + 1×1) / 10
= (5 + 8 + 6 + 8 + 1) / 10 = 28/10 = 2.8

Combined weighted rating = (4.3 × 6 + 3.3 × 5 + 2.8 × 4) / (6 + 5 + 4)
= (25.8 + 16.5 + 11.2) / 15
= 53.5 / 15
= 107/30 ≈ 3.57

The average rating of the restaurant is 107/30 ≈ 3.57.

Q7 The following table shows the weight data of langurs in an animal facility. Male langurs: average weight 16.5 kg. Female langurs: average weight 13.8 kg. All langurs: average weight 14.925 kg, total number 60. Without doing any computations, can you tell whether there are more male langurs or more female langurs? (i) Which expression describes the scenario? (ii) Find how many male langurs are present. (iii) A female langur weighing 15.2 kg is admitted. What is the new average weight of female langurs? (iv) Two male langurs weighing 16.9 kg and 16.1 kg are released. What is the new average weight of male langurs? (v) Suppose one of the male langurs (after iv) lost 1 kg. What is the new average weight of all male langurs?

SOLUTION

Without computing: The overall average (14.925 kg) is closer to the female average (13.8 kg) than to the male average (16.5 kg), so there must be more females than males.

(i) The scenario is described by expression (a): (16.5x + 13.8y)/(x + y) = 14.925

where x = number of males and y = number of females, with x + y = 60.
This is correct because x + y = 60 is the total.

(ii) Let x = number of male langurs, y = 60 − x = number of females.
(16.5x + 13.8(60 − x)) / 60 = 14.925
16.5x + 828 − 13.8x = 895.5
2.7x = 67.5
x = 25
There are 25 male langurs (and 35 female langurs).

(iii) Current female total weight = 13.8 × 35 = 483 kg
New female admitted weighs 15.2 kg.
New total weight = 483 + 15.2 = 498.2 kg, new count = 36
New average = 498.2 / 36 = 2491/180 ≈ 13.84 kg

(iv) Current male total weight = 16.5 × 25 = 412.5 kg
Weights of released males: 16.9 + 16.1 = 33 kg (note: this equals 2 × 16.5 = 33)
Remaining total = 412.5 − 33 = 379.5 kg, remaining count = 23
New average = 379.5 / 23 = 16.5 kg
(The average is unchanged because the two released males had combined weight exactly equal to 2 times the average.)

(v) After (iv), total male weight = 379.5 kg, count = 23
One male loses 1 kg: new total = 379.5 − 1 = 378.5 kg
New average = 378.5 / 23 = 757/46 ≈ 16.46 kg

Q8 Dorjee has collected 1 litre of water from the Dead Sea. Using the given salinity table (Dead Sea ≈ 34%, Other seas/oceans ≈ 3.5%, Ground water ≈ 0.01%, Purified drinking water ≈ 0.001%), answer the following. (i) What is the salinity of the mixture if he mixes 1 litre of Dead Sea water with 2 litres of purified drinking water? (ii) Is it possible to mix Dead Sea water and purified drinking water to get groundwater salinity (≈ 0.01%)? If so, what quantity of purified water is needed? (iii) Is it possible to mix Dead Sea water and groundwater to get purified drinking water salinity (≈ 0.001%)? If so, what quantity of groundwater is needed?

SOLUTION

(i) Mixture: 1 L at 34% + 2 L at 0.001%
Salinity = (1 × 34 + 2 × 0.001) / (1 + 2)
= (34 + 0.002) / 3
= 34.002 / 3 ≈ 11.334%

(ii) Can we achieve 0.01% salinity?
The mixture of 34% and 0.001% always yields a salinity between 0.001% and 34%. Since 0.01% lies in this range, it is possible.

Let x = litres of purified water added to 1 L of Dead Sea water.
(1 × 34 + x × 0.001) / (1 + x) = 0.01

$34 + 0.001x = 0.01 + 0.01x$
$33.99 = 0.009x$
$x = 33.99 / 0.009 = 3777$ litres (approx.)

Yes, it is possible. Approximately 3777 litres of purified drinking water must be mixed with 1 litre of Dead Sea water.

(iii) Can we achieve 0.001% salinity by mixing Dead Sea water (34%) with groundwater (0.01%)?
Any mixture of 34% and 0.01% gives a salinity between 0.01% and 34%. Since 0.001% is less than 0.01% (the lower bound), it is impossible to get a salinity of 0.001% by mixing these two.

No, it is not possible. The salinity of groundwater (0.01%) is already higher than the target (0.001%), so adding any amount of groundwater to Dead Sea water will always produce a mixture with salinity ≥ 0.01% > 0.001%.

Exercise 10.4 — Exercise Set 10.4

Q1 The following stacked column chart shows the number of animal species in the IUCN Red List, by class, over time. Total counts at the top of each column: 2007 = 7,851; 2010 = 9,618; 2013 = 11,212; 2016 = 12,630; 2019 = 14,234. Classes shown: Mammals, Birds, Reptiles, Fish, Amphibians, Molluscs, Insects, Others. (i) What does the number 14,234 on the top of the 2019 column represent? (ii) Approximately how many reptile species were in the list in 2016? (iii) Which class(es) of species have seen a relatively small increase in count between 2007 and 2019?

SOLUTION

(i) The number 14,234 at the top of the 2019 column represents the total number of animal species of all classes listed in the IUCN Red List in 2019.

(ii) In the 2016 column (total 12,630), the width of the reptile band is read off the chart. Approximately 1,700 reptile species were in the list in 2016 (approximate reading from the figure).

(iii) From the stacked chart, the bands for Mammals and Birds occupy the bottom of each column. Their widths appear relatively stable across all years (2007–2019), indicating a relatively small increase compared to classes like Reptiles, Fish, Insects, and Others whose bands widen noticeably.
Mammals and Birds have seen a relatively small increase in count between 2007 and 2019.

Q2 The wickets taken by a bowler in International Cricket matches till 2025 are shown in the table (Home — Test: 62, ODI: 100, T20: 32; Overseas — Test: 172, ODI: 49, T20: 71). Complete the given stacked bar charts: (i) comparing the total wickets taken at home vs. overseas (bars show Test, ODI, T20 stacked), (ii) the total wickets taken in each format (bars show Home, Overseas stacked).

SOLUTION

From the table: Home — Test: 62, ODI: 100, T20: 32; Overseas — Test: 172, ODI: 49, T20: 71.

(i) Stacked bar for Home vs Overseas (stacking format-wise within each venue):
Home total = 62 + 100 + 32 = 194 wickets
Test segment: 62, ODI segment: 100, T20 segment: 32
Overseas total = 172 + 49 + 71 = 292 wickets
Test segment: 172, ODI segment: 49, T20 segment: 71

The Home bar extends to 194 (Test 0–62, ODI 62–162, T20 162–194).
The Overseas bar extends to 292 (Test 0–172, ODI 172–221, T20 221–292).

(ii) Stacked bar for each format (stacking Home and Overseas):
Test total = 62 + 172 = 234 wickets (Home: 62, Overseas: 172)
ODI total = 100 + 49 = 149 wickets (Home: 100, Overseas: 49)
T20 total = 32 + 71 = 103 wickets (Home: 32, Overseas: 71)

Test bar: 0–62 (Home), 62–234 (Overseas).

ODI bar: 0–100 (Home), 100–149 (Overseas).
T20 bar: 0–32 (Home), 32–103 (Overseas).

The bowler has taken more wickets overseas overall (292 vs 194) and dominates in overseas Tests (172 wickets).

Exercise 10.5 — Exercise Set 10.5

Q1 The smart watch data of 5 people was tracked from Monday to Friday. Table 10.3 shows average hours per day in each body-state (Lying down / Sitting / Standing or Moving). Pavani (Patient): 20, 3, 1. Raghu (Nurse): 7, 4, 13. Zakir (Teacher): 8, 6, 10. Sahana (Student): ?, ?, ?. Julie (__): 8, 10, 6. (i) How much time does Sahana spend per day in each body-state? Make a reasonable guess. (ii) Guess what activity Julie could be engaged in. (iii) Complete the 100% stacked bar chart based on the tabular data.

SOLUTION

(i) The total must equal 24 hours per day.
A typical student attends school/classes (mostly sitting) for several hours, sleeps about 8 hours (lying down), and has some time standing or moving.
Reasonable guess for Sahana (Student): Lying down ≈ 8 h, Sitting ≈ 10 h, Standing/Moving ≈ 6 h. (Total = 24 h.)

(ii) Julie's data: Lying down = 8 h, Sitting = 10 h, Standing/Moving = 6 h.
A large proportion of time sitting (10 h) and a moderate amount lying down (8 h) — this pattern suits someone doing desk-based work such as an office worker or someone working from home (e.g., data entry operator, programmer, writer, or driver who spends extended hours seated).

(iii) 100% stacked bar chart percentages (each row sums to 100%):
Pavani (Patient): Lying = 20/24 ≈ 83.3%, Sitting = 3/24 ≈ 12.5%, Standing = 1/24 ≈ 4.2%
Raghu (Nurse): Lying = 7/24 ≈ 29.2%, Sitting = 4/24 ≈ 16.7%, Standing = 13/24 ≈ 54.2%
Zakir (Teacher): Lying = 8/24 ≈ 33.3%, Sitting = 6/24 = 25%, Standing = 10/24 ≈ 41.7%
Sahana (Student, using guess 8/10/6): Lying ≈ 33.3%, Sitting ≈ 41.7%, Standing ≈ 25%
Julie: Lying = 8/24 ≈ 33.3%, Sitting = 10/24 ≈ 41.7%, Standing = 6/24 = 25%

Draw each person's bar from 0% to 100%, divided into the three segments in proportion to the percentages above, in the order: Lying down (grey), Sitting (red/orange), Standing/Moving (blue).

Exercises — End-of-Chapter Exercises

Q1 In cricket, the run rate is the average number of runs scored per over. In a T20 match, a team scored 6 runs in the first over making the run rate 6. (i) In the second over they scored 12 runs. What is the run rate now? (ii) After Over 19, their run rate was 6. What is the run rate after 20 overs given the team made 12 runs in the last over?

SOLUTION

(i) After 2 overs: Total runs = 6 + 12 = 18
Run rate = Total runs / Total overs = 18 / 2 = 9
The run rate after 2 overs is 9.

(ii) Total runs after 19 overs = 6 × 19 = 114
Runs in over 20 = 12
Total runs after 20 overs = 114 + 12 = 126
Run rate = 126 / 20 = 6.3
The run rate after 20 overs is 6.3.

Q2 Five friends collected prices of shuttlecocks (N = Nylon, F = Feather, in): Yusuf: 40(N), 105(N), 383(F), 108(N), 165(F), 116(F); Srikanth: 194(N), 85(N), 93(N), 121(N); Kashvi: 49(N), 297(F), 105(N), 275(F), 40(N); Prasanna: 124(N), 333(F), 182(N), 258(F); Gracy: 220(F), 458(F), 129(F), 183(N). (i) Each calculated the average of prices they gathered (a_y, a_s, a_k, a_p, a_g respectively). Write an expression for the combined average. (ii) Suppose a_n and a_f are the average prices of nylon and feather shuttlecocks. Write an expression for the average price of a shuttlecock. Will this equal the answer from part (i)?

SOLUTION

(i) The combined average must weight each person's average by the number of prices they gathered:
Yusuf gathered 6 prices, Srikanth 4, Kashvi 5, Prasanna 4, Gracy 4.

Combined average = (a_y × 6 + a_s × 4 + a_k × 5 + a_p × 4 + a_g × 4) / (6 + 4 + 5 + 4 + 4)
= (6a_y + 4a_s + 5a_k + 4a_p + 4a_g) / 23

(ii) Count nylon and feather shuttlecocks across all five friends:
Nylon (N): Yusuf 3, Srikanth 4, Kashvi 3, Prasanna 2, Gracy 1 → total n_N = 13
Feather (F): Yusuf 3, Kashvi 2, Prasanna 2, Gracy 3 → total n_F = 10
Total = 23.

Combined average = (a_n × 13 + a_f × 10) / (13 + 10) = (13a_n + 10a_f) / 23

This will be equal to the answer in part (i) because both expressions represent the same overall average of all 23 prices — they just group prices differently. The weighted mean is the same regardless of how the groups are formed, as long as the weights reflect the number of prices in each group.

Q3 Shreyas holds 25 shares of a company at an average price of 150 and Vaishnavi holds 5 shares of the same company at an average price of 150. (i) Shreyas buys 10 shares of this company at a price of 30 each. What is the average price per share for him after the purchase? (ii) Vaishnavi buys some shares of this company at a price of 30 each and the average price per share for her after the purchase is 70. How many shares did she buy?

SOLUTION

(i) Shreyas currently owns 25 shares at 150 average.
Total value of current holdings = 25 × 150 = 3750
Value of new purchase = 10 × 30 = 300
Total value after purchase = 3750 + 300 = 4050
Total shares = 25 + 10 = 35
New average price = 4050 / 35 = 810/7 ≈ 115.71

(ii) Vaishnavi currently owns 5 shares at 150 average.
Let n = number of new shares bought at 30 each.
New average price = (5 × 150 + n × 30) / (5 + n) = 70
750 + 30n = 70(5 + n) = 350 + 70n
750 − 350 = 70n − 30n
400 = 40n
n = 10

Vaishnavi bought 10 shares.
Check: (5 × 150 + 10 × 30) / 15 = (750 + 300) / 15 = 1050 / 15 = 70 ✓

Q4 (Pṛthūdakasvāmī, commentary on Brahmagupta's Brāhmasphuṭasiddhānta, c. 864 CE) A pool 30 hastas long is divided into 5 sections having lengths 4, 5, 6, 7, and 8 hastas, and is dug to depths of 9, 7, 7, 3, and 2 hastas, respectively. Find the mean depth of the pool.

SOLUTION

The mean depth is the weighted average of the depths, with the section lengths as weights.

Mean depth = (4 × 9 + 5 × 7 + 6 × 7 + 7 × 3 + 8 × 2) / (4 + 5 + 6 + 7 + 8)
= (36 + 35 + 42 + 21 + 16) / 30
= 150 / 30
= 5 hastas

The mean depth of the pool is 5 hastas.

Q5 Suvarna had purchased 1g gold at 15k. This is shown by point O on a number line (0 to 30). Six scenarios for her next transaction are given. For each, estimate and mark the new average price of gold she will have after the transaction. (i) Purchase 1g at 30k (ii) Purchase 2g at 30k (iii) Purchase 1g at 10k (iv) Purchase 10g at 10k (v) Purchase 0.5g at 30k (vi) Purchase 0.5g at 15k

SOLUTION

Current holding: 1g at 15k. New average = (1×15 + quantity×price) / (1 + quantity).

(i) Buy 1g at 30k:
New avg = (1×15 + 1×30) / 2 = 45 / 2 = 22.5k → mark at 22.5 (midpoint of 15 and 30)

(ii) Buy 2g at 30k:
New avg = (1×15 + 2×30) / 3 = 75 / 3 = 25k → mark at 25 (closer to 30 since more gold bought at 30)

(iii) Buy 1g at 10k:
New avg = (1×15 + 1×10) / 2 = 25 / 2 = 12.5k → mark at 12.5 (midpoint of 15 and 10)

(iv) Buy 10g at 10k:
New avg = (1×15 + 10×10) / 11 = 115 / 11 ≈ 10.45k → mark near 10.45 (very close to 10 since most gold is at 10k)

(v) Buy 0.5g at 30k:
New avg = (1×15 + 0.5×30) / 1.5 = (15 + 15) / 1.5 = 30 / 1.5 = 20k → mark at 20

(vi) Buy 0.5g at 15k:
New avg = (1×15 + 0.5×15) / 1.5 = 22.5 / 1.5 = 15k → mark at 15 (same as O; buying at the same price as the current average leaves the average unchanged)

Q6 Given some data with corresponding weights, how would the weighted average change if all the weights are doubled? If required, experiment with some data. What do you observe? Justify your answer using algebra.

SOLUTION

Observation (from experiment): Doubling all weights does not change the weighted average.

Algebraic justification:
Let the data values be $x_1, x_2, \ldots, x$ with weights $w_1, w_2, \ldots, w$.
Original weighted average = $(w_1x_1 + w_2x_2 + \ldots + wx) / (w_1 + w_2 + \ldots + w)$

When all weights are doubled (replaced by $2w_1, 2w_2, \ldots, 2w$):
New weighted average = $(2w_1x_1 + 2w_2x_2 + \ldots + 2wx) / (2w_1 + 2w_2 + \ldots + 2w)$
$= 2(w_1x_1 + w_2x_2 + \ldots + wx) / 2(w_1 + w_2 + \ldots + w)$
$= (w_1x_1 + w_2x_2 + \ldots + wx) / (w_1 + w_2 + \ldots + w)$
= Original weighted average

The weighted average remains unchanged when all weights are doubled (or multiplied by any non-zero constant), because only the relative weights matter.

Q7 Answer the following questions based on the stacked column chart: 'Cumulative number of objects currently orbiting Earth, by year of launch/separation (as of Apr. 24, 2026)' — showing Payload objects and Other objects from 1960 to 2025+. (i) In 2003, approximately how many total objects were found orbiting Earth? In what year did this number double? (ii) Find the approximate number and share of payload objects and other objects in 2025. (iii) What can you say about the number of payload objects over time, and the number of other objects over time? What can you say about their respective shares over time?

SOLUTION

(i) From the chart, in 2003 the total cumulative count of objects orbiting Earth was approximately 8,000–9,000. Reading from the chart: approximately 8,500 objects.
This number roughly doubled (to about 17,000) around 2014–2015.

(ii) In 2025 (as of April 24, 2026), the chart shows approximately:
Payload objects (green/teal bars): approximately 30,000–32,000
Other objects (debris, rocket stages, etc.): approximately 4,000–5,000
Total: approximately 35,000
Share of payload objects ≈ 30,000/35,000 ≈ 86%
Share of other objects ≈ 5,000/35,000 ≈ 14%
(These are approximate readings from the chart.)

(iii) The number of payload objects has increased dramatically over time, especially after 2017, with a very steep rise in recent years — this reflects the launch of large satellite constellations (e.g., Starlink).
The number of other objects (debris, rocket stages) has also increased but more gradually and at a lower rate compared to payload objects.

Regarding shares: The share of payload objects has been increasing over time (from a lower fraction in earlier decades to the majority now), while the share of other objects has been decreasing as a proportion of the total — even though the absolute number of other objects also grew. The recent surge in satellite launches has caused payload objects to dominate.

Q8 Observe the infographic showing 'What Percentage of Schools have a Playground?' — a map of India with state-wise percentages (e.g., Haryana: 90%, Uttarakhand: 78%, Arunachal Pradesh: 68%, Punjab: 98%, Maharashtra: 93%, Telangana: 74%). (i) Identify the correct inference(s): (a) More schools have a playground in Haryana than Uttarakhand. (b) Approximately every 2 out of 3 schools in Arunachal Pradesh have a playground. (c) Punjab has the highest number of schools with a playground. (d) Given Maharashtra has more schools than Telangana, the number of schools with a playground is more in Maharashtra. (ii) Can we find the nation-wide percentage from this map? What additional information is needed?

SOLUTION

(i) Evaluating each statement:

(a) 'More schools have a playground in Haryana compared to Uttarakhand.'
Haryana: 90% have a playground; Uttarakhand: 78%. A higher percentage means a higher

share, but we do not know the total number of schools in each state. If Uttarakhand has far more schools, its total with playgrounds could exceed Haryana's. We cannot infer actual count from percentages alone. This statement refers to the number of schools (not percentage), so it CANNOT be inferred. INCORRECT.

(b) 'Approximately every 2 out of 3 schools in Arunachal Pradesh have a playground.'
2 out of 3 ≈ 66.7%. The map shows Arunachal Pradesh ≈ 68%, which is approximately 2 out of 3. CORRECT.

(c) 'Punjab has the highest number of schools with a playground.'
Punjab has 98% — the highest percentage on the map. But the number of schools with a playground depends on the total schools, which we don't know. Punjab may have fewer schools than larger states. This CANNOT be inferred. INCORRECT.

(d) 'Suppose Maharashtra has more schools than Telangana. Then the number of schools having a playground is more in Maharashtra.'
Maharashtra ≈ 93%, Telangana ≈ 74%. If Maharashtra has more total schools, 93% of a larger number versus 74% of a smaller number → Maharashtra will have more schools with playgrounds. CORRECT.

Correct inferences: (b) and (d).

(ii) We cannot find the nationwide percentage from this map alone. The map only shows the percentage of schools with a playground in each state, not the actual number of schools. To compute the nationwide percentage, we need the total number of schools in each state, so that we can compute a weighted average of the state percentages (weighted by number of schools).

Q9 Decision Dilemma: (i) Which of the plays would you choose to watch based on the 100% stacked bar chart showing share of ratings (★★★★★, ★★★★, ★★★, ★★, ★) for Play A, Play B, Play C? (ii) The corresponding stacked bar chart shows absolute number of ratings (bars extend to about 100 for A, 750 for B, and 100 for C). Would you change your decision after seeing this? Why/Why not?

SOLUTION

(i) From the 100% stacked bar chart (showing proportions of ratings):
Play A has a large share of 5-star ratings (most of its bar is blue/5★), followed by 4★.
Play B has a moderate share of 5★ and 4★ but also notable 3★ and lower.
Play C appears to have a high proportion of 5★ but the total bar is short in the stacked chart.
Based on proportions alone, Play A appears to have the best rating distribution (highest share of 5★ and 4★). One would likely choose Play A.

(ii) After seeing the stacked bar chart (absolute numbers):
Play B's bar extends far beyond the others (to roughly 800 ratings), while Play A and Play C have much shorter bars (on the order of a few hundred ratings each).
This means Play B has far more total ratings than Play A or Play C.

A larger number of ratings makes the average more reliable and representative.

The decision might change: even if Play A has a higher proportion of 5★ ratings, it is based on very few reviews. Play B's rating is more trustworthy because it is based on many more ratings. One might prefer Play B for this reason, or need more context (venue, story, etc.).

Key lesson: A 100% stacked bar chart only shows proportions, not volumes. The absolute chart reveals that Play B has vastly more data points, which affects reliability of the rating.

Q10 A triathlon consists of swimming 3.8 km, cycling 180 km, and running 42.2 km. Table 10.4 shows finish times: Athlete 1 — Swimming 01:08, Cycling 05:00, Running 03:15; Athlete 2 — 01:05, 05:10, 03:35; Athlete 3 — 01:22, 05:55, 03:50. (i) Is a stacked bar chart or a 100% stacked bar chart more suitable? (ii) If a 100% stacked bar chart is drawn, which questions can be answered: (a) Who finished the race first? (b) Approximately what fraction of their race time did Athlete 1 spend cycling? (c) Which athlete took the longest for running?

SOLUTION

(i) A stacked bar chart is more suitable for this data.
Reason: The total time taken by each athlete is the primary interest (it determines who wins the race). A stacked bar chart shows both the total time for each athlete and the breakdown by segment (swimming, cycling, running). A 100% stacked bar chart would show only proportions, hiding the actual total times, which are needed to determine the winner.

Total times for reference: Athlete 1 = 68+300+195 = 563 min = 9h 23m; Athlete 2 = 65+310+215 = 590 min = 9h 50m; Athlete 3 = 82+355+230 = 667 min = 11h 7m.

(ii) From the 100% stacked bar chart:

(a) 'Who finished the race first?' — CANNOT be answered.
The 100% chart normalises each athlete's bar to 100%, hiding the actual total times. Athlete 1 finished first (smallest total time) but this cannot be read from the 100% chart.

(b) 'Approximately what fraction of their race time did Athlete 1 spend cycling?' — CAN be answered.
Athlete 1 spent 300 minutes cycling out of 563 minutes total = 300/563 ≈ 53.3% of race time.
The 100% stacked bar shows this proportion directly as the length of the cycling segment in Athlete 1's bar.

(c) 'Which athlete took the longest for running?' — CANNOT be answered reliably.
The 100% chart shows the proportion of time spent running within each athlete's total. But different totals mean we cannot compare absolute running times from proportions. Athlete 3 took the longest for running (230 min) but could have a smaller running proportion if their total is much larger — the actual comparison requires the stacked bar.

Q11 Look at the following graph: 'Distribution of disabled persons by age group and type of disability in India (Census 2011)' — a 100% stacked bar chart with age groups 0–19, 20–39, 40–59, 60 & above on the y-axis and percentage (0–100) on the x-axis. Disability types: seeing, hearing, speech, movement, mental retardation, mental illness, any other, multiple disability. What do you notice? What do you wonder? Write your inferences.

SOLUTION

Observations from the chart:

1. 'Seeing' disability (the first segment) has a notably larger share in older age groups (40–59 and 60+) compared to younger groups, suggesting vision-related disabilities become more prevalent with age.

2. 'Mental retardation' and 'mental illness' together appear to have a relatively larger share in the 0–19 age group compared to older groups, consistent with early-life conditions.

3. 'Hearing' disability appears fairly consistently distributed across age groups.

4. The share of 'movement' disability increases with age, likely reflecting age-related mobility issues.

5. 'Multiple disability' appears as a small segment and seems present in all age groups.

Inferences:
- As age increases, disabilities related to sensory and mobility impairments (seeing, movement) become a larger share of the disabled population, consistent with the ageing process.
- The 0–19 group has a larger share of conditions like mental retardation, reflecting congenital/developmental conditions.
- Since this is a 100% stacked bar, it tells us only about the composition of disabilities within each age group, not the total number of disabled persons in each age group.

Wonders:
- How do the total numbers of disabled persons compare across age groups?
- How have these distributions changed since the 2011 census?
- Do the patterns differ significantly between rural and urban areas, or between genders?

Q12 Individual project: Do at least one of the following. (i) Recall the previous day and fill in the approximate time spent lying down, sitting, and standing/moving around, etc. Ask at least 2 family members about their day. Visualise using a stacked bar chart. (ii) Visualise your family's monthly expenditure using a 100% stacked bar chart for at least 3 months.

SOLUTION

This is an individual project/activity. No unique numerical answer is expected — it depends on each student's personal and family data.

(i) Steps: Recall the previous day's approximate hours in each body-state (lying down, sitting, standing/moving). Ensure the three values sum to 24. Collect data from at least 2 family members. Draw a 100% stacked bar chart with one bar per person, divided into three

colour-coded segments proportional to the time in each state. Discuss observations — e.g., who is most active, how the student's pattern compares to a working adult.

(ii) Steps: Identify expense categories (e.g., housing, food, education, transport, healthcare, recreation, others). Collect monthly expense data for at least 3 months. For each month, compute each category's share of total expenditure. Draw a 100% stacked bar chart with one bar per month. Write observations — e.g., which category dominates, whether the distribution changes across months.

Q13 Small-group project: Make a group of 3–4 students. Choose one scenario to design a custom rating scheme: (a) shopping experience at a cloth store, (b) travel experience in a bus, (c) tourism experience of a nearby tourist spot, (d) clinic/hospital experience. (i) Discuss and arrive at 4–6 aspects to rate. (ii) Decide relative weights and justify. (iii) Collect/imagine ratings from at least 10 people on a scale of 1–5. (iv) Compute each individual rating and overall average. (v) Make a 100% stacked bar chart. (vi) Write observations.

SOLUTION

This is a small-group project. The solution depends on the scenario chosen and data collected. A sample for scenario (a) Shopping experience at a cloth store is given below.

(i) Aspects to rate: (1) Variety/range of products, (2) Quality of fabric, (3) Staff behaviour/helpfulness, (4) Pricing/value for money, (5) Store ambience/cleanliness.

(ii) Sample weights: Quality 5, Price 4, Variety 3, Staff 3, Ambience 2. Justification: Quality and price are the primary factors in a purchase decision; variety expands choice; staff and ambience matter for experience but are secondary.

(iii–iv) Collect ratings from at least 10 people on a 1–5 scale for each aspect. Compute each individual's overall rating using the weighted mean. Then average across all individuals for an overall store rating.

(v) In the 100% stacked bar chart, each bar represents one individual. The bar is divided into 5 segments (one per aspect) with lengths proportional to (aspect rating × weight) as a share of that individual's total weighted score.

(vi) Observations: Identify which aspects received consistently high/low ratings. Note if certain aspects drive the overall score. Discuss what the rating scheme captures well (systematic quality aspects) and what it misses (personal taste, seasonality of products, wait time, etc.).

Q14 Whole class project: Each student shares the average age of their family and the number of family members. Discuss among the class and come up with a way to find out the average age of all the families of the class.

SOLUTION

The key insight is that the overall average age cannot be found by simply averaging the family averages — each family average must be weighted by the number of family members.

Method: Each student contributes their family's average age ($\bar{a}_i$) and family size (n_i).

Sum of all ages in a family = $\bar{a}_i \times n_i$.

Overall average age = $(\Sigma\, \bar{a}_i \times n_i) / (\Sigma\, n_i)$

where the sums run over all families in the class.

This is the weighted mean of the family averages, with weights equal to family sizes.

Note: Taking the unweighted average of the family averages would give equal importance to a 2-member family and a 6-member family, which is incorrect — the larger family contributes more ages to the total.

Q15 *(Challenge) Given some data with corresponding weights, what would happen to the weighted average if all the weights are increased by a constant value, say 1? If required, experiment with some data. What do you observe? Justify your answer using algebra.

SOLUTION

Observation (from experiment): Adding 1 to all weights generally changes the weighted average.

Algebraic justification:
Let the data values be $x_1, x_2, \ldots, x$ with original weights $w_1, w_2, \ldots, w$.
Original weighted average = $(\Sigma\, wx) / (\Sigma\, w) = W_{avg}$.

When each weight is increased by 1 (new weights: $w_1+1, w_2+1, \ldots, w+1$):
New weighted average = $(\Sigma\, (w+1)x) / (\Sigma\, (w+1))$
$= (\Sigma\, wx + \Sigma\, x) / (\Sigma\, w + n)$

This equals the original weighted average W_{avg} only if:
$W_{avg} \times (\Sigma\, w + n) = \Sigma\, wx + \Sigma\, x$
$W_{avg} \times \Sigma\, w + n \times W_{avg} = \Sigma\, wx + \Sigma\, x$
$n \times W_{avg} = \Sigma\, x$ (since $W_{avg} \times \Sigma\, w = \Sigma\, wx$)
$W_{avg} = (\Sigma\, x)/n$ = simple arithmetic mean of the data.

Conclusion: Adding 1 to all weights changes the weighted average UNLESS the original weighted average equals the simple arithmetic mean of the data. In general, the new average is a blend of the old weighted average and the simple mean: $(\Sigma\, wx + \Sigma\, x) / (\Sigma\, w + n)$, which lies between the two.

Q16 *(Challenge) A farm has some cows, sheep, and chickens. Last year the cows made up 60%, the sheep 25%, and the chickens 15%. There was a decrease in the number of all three animals' population over the year. Choose the possibilities for the change in their respective shares of the population: (i) % cows ↓, % sheep ↓, % chickens ↓ (ii) % cows ↑, % sheep ↑, % chickens ↑ (iii) % cows same, % sheep same, % chickens same (iv) % cows ↓, % sheep ↑, % chickens same (v) % cows ↑, % sheep ↑, % chickens ↓

SOLUTION

Key constraint: the percentages of cows, sheep, and chickens must always sum to 100%. Therefore, not all three can increase simultaneously, and not all three can decrease simultaneously.

(i) All three percentages decrease: Impossible — if all three decrease, their sum would be less than 100%, which is a contradiction.

(ii) All three percentages increase: Impossible — if all three increase, their sum would exceed 100%.

(iii) All percentages remain the same: Possible — if all three populations decrease by the same proportional factor (e.g., all decrease by 10%), the relative shares remain 60%, 25%, 15%. ✓

(iv) % cows ↓, % sheep ↑, % chickens same: Possible — for example, if cows decrease more steeply than the others, their share falls, and at least one other share (here, sheep) rises to compensate, while chickens' share stays constant. This requires sheep to decrease less steeply than cows, and chickens to decrease at the same rate as the total. ✓

(v) % cows ↑, % sheep ↑, % chickens ↓: Possible — if chickens decrease much faster than cows and sheep, the shares of cows and sheep can both increase while chickens' share falls. The shares still sum to 100%. ✓

Possible scenarios: (iii), (iv), and (v).
Impossible scenarios: (i) and (ii).