Chapter 13: Two Variables, One Line

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Exercise 13.1 — Exercise Set 13.1

Q1 Write a linear equation in two variables in which a = 3, b = 0 and c = −1/5.

SOLUTION

The standard form of a linear equation in two variables is ax + by + c = 0.
Substituting a = 3, b = 0, c = −1/5:
3x + 0·y + (−1/5) = 0
3x − 1/5 = 0 is the required linear equation.

Q2 Complete the following table after expressing the given linear equations in standard form: (i) y − 15 = √2 x (ii) 3y − 2x = 0 (iii) 5x = 3y (iv) x = 8 (v) 3y = 1

SOLUTION

(i) y − 15 = √2 x
Rearranging: −√2 x + y − 15 = 0
Standard form: −√2 x + y − 15 = 0
Coefficient of x: −√2, Coefficient of y: 1, Constant: −15

(ii) 3y − 2x = 0
Rearranging: −2x + 3y + 0 = 0
Coefficient of x: −2, Coefficient of y: 3, Constant: 0

(iii) 5x = 3y
Rearranging: 5x − 3y + 0 = 0
Coefficient of x: 5, Coefficient of y: −3, Constant: 0

(iv) x = 8
Rearranging: x + 0·y − 8 = 0
Coefficient of x: 1, Coefficient of y: 0, Constant: −8

(v) 3y = 1

Rearranging: $0 \cdot x + 3y - 1 = 0$
Coefficient of x: 0, Coefficient of y: 3, Constant: −1

Q3 (i) The cost of a notebook is twice the cost of a pen. Cost of notebook = t, cost of pen = p. Charlie wrote $t = 2p$, whereas Meera wrote $p = 2t$. Which is correct? (ii) In a cricket match, two Indian batsmen together scored 176 runs. Manisha expressed this as $x + y = 176$. Is this correct?

SOLUTION

(i) The statement means notebook costs 2 times the pen.
So $t = 2p$ (Charlie's representation).
Meera wrote $p = 2t$, which would mean the pen costs twice the notebook — this is incorrect.
Charlie's representation $t = 2p$ is correct.

(ii) Let x = runs scored by one batsman, y = runs scored by the other.
Together they scored 176 runs, so $x + y = 176$.
Manisha's representation $x + y = 176$ is correct.

Exercise 13.2 — Exercise Set 13.2

Q1 Verify if the ordered pair (4, 3) is a solution of 5x − 6y = 2. Explain your reasoning.

SOLUTION

Substitute x = 4 and y = 3 into the left-hand side (LHS) of 5x − 6y = 2:
LHS = 5(4) − 6(3) = 20 − 18 = 2
LHS = 2 = RHS
Since substituting (4, 3) satisfies the equation, (4, 3) is a solution of 5x − 6y = 2.

Q2 Find any two solutions for each of the following equations: (i) 7x − 3y = 21 (ii) 2x + 3y = 5

SOLUTION

(i) 7x − 3y = 21
Solution 1: Substitute x = 0: 7(0) − 3y = 21 => −3y = 21 => y = −7. Solution: (0, −7).
Solution 2: Substitute x = 3: 7(3) − 3y = 21 => 21 − 3y = 21 => y = 0. Solution: (3, 0).
Two solutions of 7x − 3y = 21 are (0, −7) and (3, 0).

(ii) 2x + 3y = 5
Solution 1: Substitute x = 1: 2(1) + 3y = 5 => 3y = 3 => y = 1. Solution: (1, 1).
Solution 2: Substitute x = −2: 2(−2) + 3y = 5 => −4 + 3y = 5 => 3y = 9 => y = 3. Solution: (−2, 3).
Two solutions of 2x + 3y = 5 are (1, 1) and (−2, 3).

Q3 In the equations given below, m and n are unknown constants: 2mx + 3y = 7; 4x + ny = −10. If (2, −1) is the solution of both equations, find the values of m and n.

SOLUTION

Since (2, −1) satisfies the first equation 2mx + 3y = 7:
2m(2) + 3(−1) = 7
4m − 3 = 7
4m = 10
m = 5/2

Since (2, −1) satisfies the second equation 4x + ny = −10:
4(2) + n(−1) = −10
8 − n = −10
n = 18

Therefore m = 5/2 and n = 18.

Q4 Find two solutions which lie in different quadrants for each of the following linear equations. Identify the quadrants in which the points lie: (i) 5x + 3y = 7 (ii) 5x − 3y = 7 (iii) −5x + 3y = 7 (iv) −5x − 3y = 7

SOLUTION

(i) 5x + 3y = 7
Substitute x = 2: 10 + 3y = 7 => 3y = −3 => y = −1. Point: (2, −1), which lies in Quadrant IV.
Substitute x = −1: −5 + 3y = 7 => 3y = 12 => y = 4. Point: (−1, 4), which lies in Quadrant II.

(ii) 5x − 3y = 7
Substitute x = 2: 10 − 3y = 7 => 3y = 3 => y = 1. Point: (2, 1), which lies in Quadrant I.
Substitute x = −1: −5 − 3y = 7 => −3y = 12 => y = −4. Point: (−1, −4), which lies in Quadrant III.

(iii) −5x + 3y = 7
Substitute x = 1: −5 + 3y = 7 => 3y = 12 => y = 4. Point: (1, 4), which lies in Quadrant I.
Substitute x = −1: 5 + 3y = 7 => 3y = 2 => y = 2/3. Point: (−1, 2/3), which lies in Quadrant II.

(iv) −5x − 3y = 7
Substitute x = 1: −5 − 3y = 7 => −3y = 12 => y = −4. Point: (1, −4), which lies in Quadrant IV.
Substitute x = −1: 5 − 3y = 7 => −3y = 2 => y = −2/3. Point: (−1, −2/3), which lies in Quadrant III.

Q5 Consider the graph of the equation 3x − 7y = 21 shown, passing through A(7, 0) and B(0, −3). Does the point C(2, 3) lie on the line? Does it satisfy the equation? Can points that do not lie on the line satisfy the equation?

SOLUTION

Substitute x = 2, y = 3 into 3x − 7y = 21:
LHS = 3(2) − 7(3) = 6 − 21 = −15 ≠ 21
Since LHS ≠ RHS, the point C(2, 3) does NOT satisfy the equation.
Therefore C(2, 3) does not lie on the line.

A point lies on the graph of a linear equation if and only if it satisfies the equation.
So points that do not lie on the line cannot satisfy the equation.

Q6 State whether the following sentences are True or False. Justify your answer: (i) A linear equation in two variables has only one solution. (ii) The graph of a linear equation in two variables always passes through the origin. (iii) A linear equation in two variables can never have rational solutions. (iv) x = 3 is a valid linear equation in two variables. (v) The equation 2x + 3y = 7 has infinitely many solutions. (vi) The point (1, 2) is a solution of the equation 2x + 3y = 7.

SOLUTION

(i) False.
A linear equation in two variables has infinitely many solutions — for every value assigned to x, there is a corresponding value of y that satisfies the equation.

(ii) False.
The graph passes through the origin only if the constant term c = 0 (i.e., the equation is ax + by = 0). For example, x + y = 5 does not pass through (0, 0) since 0 + 0 ≠ 5.

(iii) False.
A linear equation in two variables can certainly have rational solutions. For example, (1/2, 2) is a rational solution of x + y = 5/2.

(iv) True.
x = 3 can be written as x + 0·y − 3 = 0, which is of the form ax + by + c = 0 with a = 1, b = 0, c = −3. So it is a valid linear equation in two variables (with the coefficient of y being zero).

(v) True.
A linear equation in two variables has infinitely many solutions. For any value of x, we can find a corresponding y satisfying 2x + 3y = 7.

(vi) False.
Substitute x = 1, y = 2: 2(1) + 3(2) = 2 + 6 = 8 ≠ 7. So (1, 2) is not a solution.

Q7 (i) Compare the solutions of 3x + 4y = 7 and 6x + 8y = 14. Argue that they have the same set of solutions. (ii) Show that the equations ax + by = c and kax + kby = kc, with k ≠ 0, have the same set of solutions.

SOLUTION

(i) Observe that 6x + 8y = 14 is exactly 2 times the equation 3x + 4y = 7:
2(3x + 4y) = 2(7) => 6x + 8y = 14.
If (x_o, y_o) satisfies 3x + 4y = 7, then $3x_o + 4y_o = 7$, so $6x_o + 8y_o = 2(7) = 14$, meaning (x_o, y_o) also satisfies 6x + 8y = 14.
Conversely, if (x_o, y_o) satisfies 6x + 8y = 14, dividing both sides by 2 gives $3x_o + 4y_o = 7$.
Hence every solution of one is also a solution of the other; the two equations have identical solution sets.

(ii) Suppose (x_o, y_o) satisfies ax + by = c, so $ax_o + by_o = c$.
Multiplying both sides by k (k ≠ 0): $k(ax_o + by_o) = kc$ => $kax_o + kby_o = kc$.
So (x_o, y_o) also satisfies kax + kby = kc.
Conversely, if (x_o, y_o) satisfies kax + kby = kc, then $kax_o + kby_o = kc$.
Since k ≠ 0, divide both sides by k: $ax_o + by_o = c$.
So (x_o, y_o) also satisfies ax + by = c.
Therefore the two equations ax + by = c and kax + kby = kc have the same set of solutions.

Exercise 13.3 — Exercise Set 13.3

Q1 The following diagrams represent ski hills A, B and C. Hill A: rise = 60 m, run = 70 m. Hill B: rise = 60 m, run = 110 m. Hill C: rise = 80 m, run = 100 m. Rank the hills in order of their steepness, from least to greatest.

SOLUTION

Slope = Rise ÷ Run (a larger slope means a steeper hill).
Slope of A = 60/70 = 6/7 ≈ 0.857
Slope of B = 60/110 = 6/11 ≈ 0.545
Slope of C = 80/100 = 4/5 = 0.800

Comparing: B (≈ 0.545) < C (0.800) < A (≈ 0.857)
Order from least to greatest steepness: B, C, A.

Q2 The ramp at a loading dock rises 2.5 metres over a run of 4 metres. Find the slope of the ramp.

SOLUTION

Slope = Rise / Run = 2.5 / 4 = 0.625
The slope of the ramp is 5/8 (or 0.625).

Q3 Find the slope of the line l in each of the following diagrams: (i) Line l is horizontal (ii) Line l goes from upper-left to lower-right, reading from the graph it passes through (−1, 1) and (1, −1) (iii) Line l has positive slope, passing through approximately (−2, −1) and (0, 1) (iv) Line l is vertical.

SOLUTION

(i) The line l is horizontal (parallel to the x-axis).
A horizontal line has zero rise for any run.
Slope = 0/run = 0.

(ii) The line l goes from upper-left to lower-right (negative slope).
Reading from the graph, the line passes through (−1, 1) and (1, −1).
Slope = $(y_2 - y_1)/(x_2 - x_1)$ = (−1 − 1)/(1 − (−1)) = −2/2 = −1.

(iii) The line l has a positive slope.
Reading from the graph, the line passes through (−2, −1) and (0, 1).
Slope = (1 − (−1))/(0 − (−2)) = 2/2 = 1.

(iv) The line l is vertical (parallel to the y-axis).
For a vertical line, x is constant so the run = 0 and the slope formula gives division by zero.
Slope is undefined.

Q4 An accessibility ramp for wheelchairs is to be made alongside the staircase. The guidelines given for the slope of the ramp is 1 cm vertical rise to 12 cm horizontal length. What should be the horizontal length of the ramp if the total height of the stairs is 18 cm?

SOLUTION

The prescribed slope = Rise/Run = 1/12.
Total rise (height of stairs) = 18 cm.
Using Slope = Rise/Run:
1/12 = 18/Run
Run = 18 × 12 = 216 cm.
The horizontal length of the ramp should be 216 cm.

Exercise 13.4 — Exercise Set 13.4

Q1 On two different days a family buys movie tickets and snack boxes. Cost of one movie ticket = x, cost of one snack box = y. Frame a pair of linear equations: (i) Day 1: 2 movie tickets and 3 snack boxes for 850. (ii) Day 2: 4 movie tickets and 1 snack box for 1100.

SOLUTION

(i) On the first day, 2 movie tickets and 3 snack boxes cost 850.
Equation: $2x + 3y = 850$

(ii) On the second day, 4 movie tickets and 1 snack box cost 1100.
Equation: $4x + y = 1100$

The required pair of linear equations is: $2x + 3y = 850$ and $4x + y = 1100$.

Q2 Two friends, Sahil and Meena, travelled by taxi. Fixed charge for one taxi trip = x, additional charge per km = y. Frame a pair of linear equations: (i) Sahil travelled 6 km and paid 122. (ii) Meena travelled 8 km and paid 160.

SOLUTION

(i) Sahil's fare = fixed charge + 6 × (charge per km) = $x + 6y = 122$.
Equation: $x + 6y = 122$

(ii) Meena's fare = fixed charge + 8 × (charge per km) = $x + 8y = 160$.
Equation: $x + 8y = 160$

The required pair of linear equations is: $x + 6y = 122$ and $x + 8y = 160$.

Q3 In a sports meet, adult tickets cost 150 each and children's tickets cost 100 each. Total attendance = 200, total ticket revenue = 25,000. Let x = number of adult tickets, y = number of children's tickets. Frame a pair of linear equations.

SOLUTION

Total number of people: $x + y = 200$ ... (1)
Total amount collected: $150x + 100y = 25,000$ ... (2)

The required pair of linear equations is:
$x + y = 200$ and $150x + 100y = 25,000$.

Exercise 13.5 — Exercise Set 13.5

Q1 Form a pair of linear equations for each of the following problems and find their solutions: (i) The sum of two integers is +5 and their difference is −21. (ii) The difference between two numbers is 26 and one number is three times the other. (iii) The coach of a cricket team buys 7 bats and 6 balls for 8880. Later, she buys 3 bats and 5 balls for 4000. Find the cost of each. (iv) Taxi: fixed charge + per-km charge. For 10 km: 155; for 15 km: 220. Find the fixed charge and charge per km. Also find the cost for 25 km. (v) A fraction becomes 9/11 if 2 is added to both numerator and denominator. If 3 is added to both, it becomes 5/6. Find the fraction. (vi) Adding 1 to numerator and subtracting 1 from denominator gives 1. Adding 1 only to denominator gives 1/2. Find the fraction. (vii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. (viii) Sum of digits of a two-digit number is 9. Nine times the number equals twice the number obtained by reversing digits. (ix) Meena withdrew 2000 in 50 and 100 notes, getting 25 notes in all. (x) Lending library: fixed charge for first 3 days, extra per day. Saritha paid 27 for 7 days; Susy paid 21 for 5 days.

SOLUTION

(i) Let the two integers be x and y.
x + y = 5 ...(1) and x − y = −21 ...(2)
Adding (1) and (2): 2x = −16 => x = −8
From (1): −8 + y = 5 => y = 13
The two integers are −8 and 13.

(ii) Let the two numbers be x (larger) and y (smaller).
x − y = 26 ...(1) and x = 3y ...(2)
Substituting (2) in (1): 3y − y = 26 => 2y = 26 => y = 13
x = 3(13) = 39
The two numbers are 39 and 13.

(iii) Let cost of one bat = b and cost of one ball = Ba.
7b + 6Ba = 8880 ...(1) and 3b + 5Ba = 4000 ...(2)
Multiply (1) by 5 and (2) by 6:
35b + 30Ba = 44,400 ...(3)
18b + 30Ba = 24,000 ...(4)
Subtracting (4) from (3): 17b = 20,400 => b = 1200
From (1): 7(1200) + 6Ba = 8880 => 8400 + 6Ba = 8880 => 6Ba = 480 => Ba = 80
Cost of each bat = 1200 and each ball = 80.

(iv) Let fixed charge = f and charge per km = k.
f + 10k = 155 ...(1) and f + 15k = 220 ...(2)
Subtracting (1) from (2): 5k = 65 => k = 13
From (1): f + 130 = 155 => f = 25
Fixed charge = 25, charge per km = 13.
Cost for 25 km = 25 + 25(13) = 25 + 325 = 350.

(v) Let the fraction be n/d.

$(n + 2)/(d + 2) = 9/11 \Rightarrow 11(n + 2) = 9(d + 2) \Rightarrow 11n - 9d = -4$ ...(1)
$(n + 3)/(d + 3) = 5/6 \Rightarrow 6(n + 3) = 5(d + 3) \Rightarrow 6n - 5d = -3$ ...(2)
Multiply (1) by 5 and (2) by 9:
$55n - 45d = -20$ ...(3)
$54n - 45d = -27$ ...(4)
Subtracting (4) from (3): $n = 7$
From (1): $11(7) - 9d = -4 \Rightarrow 77 - 9d = -4 \Rightarrow 9d = 81 \Rightarrow d = 9$
The fraction is 7/9.

(vi) Let the fraction be n/d.
$(n + 1)/(d - 1) = 1 \Rightarrow n + 1 = d - 1 \Rightarrow d = n + 2$ ...(1)
$n/(d + 1) = 1/2 \Rightarrow 2n = d + 1$ ...(2)
Substituting (1) in (2): $2n = (n + 2) + 1 = n + 3 \Rightarrow n = 3$
$d = 3 + 2 = 5$
The fraction is 3/5.

(vii) Let Nuri's current age = N years and Sonu's current age = S years.
Five years ago: $N - 5 = 3(S - 5) \Rightarrow N - 3S = -10$ ...(1)
Ten years later: $N + 10 = 2(S + 10) \Rightarrow N - 2S = 10$ ...(2)
Subtracting (1) from (2): $S = 20$
From (2): $N = 2(20) + 10 = 50$
Nuri is 50 years old and Sonu is 20 years old.

(viii) Let the tens digit = x and units digit = y.
$x + y = 9$ ...(1)
Nine times the number = twice the reversed number:
$9(10x + y) = 2(10y + x) \Rightarrow 90x + 9y = 20y + 2x \Rightarrow 88x = 11y \Rightarrow 8x = y$ ...(2)
Substituting (2) in (1): $x + 8x = 9 \Rightarrow 9x = 9 \Rightarrow x = 1,\ y = 8$
The two-digit number is 18.

(ix) Let the number of 50 notes = a and 100 notes = b.
$a + b = 25$ ...(1) and $50a + 100b = 2000 \Rightarrow a + 2b = 40$ ...(2)
Subtracting (1) from (2): $b = 15$
From (1): $a = 25 - 15 = 10$
Meena received 10 notes of 50 and 15 notes of 100.

(x) Let the fixed charge for first 3 days = f and extra charge per day = e.
Saritha kept the book for 7 days (= 3 + 4 extra days): $f + 4e = 27$ ...(1)
Susy kept the book for 5 days (= 3 + 2 extra days): $f + 2e = 21$ ...(2)
Subtracting (2) from (1): $2e = 6 \Rightarrow e = 3$
From (2): $f + 6 = 21 \Rightarrow f = 15$
Fixed charge = 15, charge for each extra day = 3.

Q2 Form a pair of linear equations and find their common solutions graphically. 10 students of Grade 9 took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls.

SOLUTION

Let b = number of boys and g = number of girls.
Total students: b + g = 10 ...(1)
Girls are 4 more than boys: g = b + 4 => b − g = −4 ...(2)

To solve graphically, find two points on each line:
Line 1 (b + g = 10): (0, 10) and (10, 0).
Line 2 (g = b + 4): (0, 4) and (3, 7).

Plotting these lines on the Cartesian plane, they intersect at one point.
Solving algebraically: Adding (1) and (2): 2b = 6 => b = 3
g = 10 − 3 = 7

The intersection point is (3, 7): 3 boys and 7 girls took part in the quiz.

Q3 Use the ratios a_1/a_2, b_1/b_2, c_1/c_2 to determine whether the lines representing the following pairs of linear equations intersect at a point, are parallel or are coincident: (i) 5x − 4y + 8 = 0; 7x + 6y − 9 = 0 (ii) 9x + 3y + 12 = 0; 18x + 6y + 24 = 0 (iii) 6x − 3y + 10 = 0; 2x − y + 9 = 0

SOLUTION

(i) 5x − 4y + 8 = 0 and 7x + 6y − 9 = 0
$a_1/a_2 = 5/7$, $b_1/b_2 = -4/6 = -2/3$
Since $a_1/a_2 \neq b_1/b_2$ ($5/7 \neq -2/3$), the lines intersect at a unique point.

(ii) 9x + 3y + 12 = 0 and 18x + 6y + 24 = 0
$a_1/a_2 = 9/18 = 1/2$, $b_1/b_2 = 3/6 = 1/2$, $c_1/c_2 = 12/24 = 1/2$
Since $a_1/a_2 = b_1/b_2 = c_1/c_2 = 1/2$, the lines are coincident (infinitely many solutions).

(iii) 6x − 3y + 10 = 0 and 2x − y + 9 = 0
$a_1/a_2 = 6/2 = 3$, $b_1/b_2 = -3/-1 = 3$, $c_1/c_2 = 10/9$
Since $a_1/a_2 = b_1/b_2$ but $c_1/c_2 \neq a_1/a_2$ ($10/9 \neq 3$), the lines are parallel (no solution).

Q4 Which of the following pairs of linear equations have solutions? If they have solutions, find them graphically: (i) x + y = 5, 2x + 2y = 10 (ii) x − y = 8, 3x − 3y = 16 (iii) 2x + y − 6 = 0, 4x − 2y − 4 = 0 (iv) 2x − 2y − 2 = 0, 4x − 4y − 5 = 0

SOLUTION

(i) x + y = 5 and 2x + 2y = 10
$a_1/a_2 = 1/2$, $b_1/b_2 = 1/2$, $c_1/c_2 = 5/10 = 1/2$
All three ratios are equal => the lines are coincident.
This pair has infinitely many solutions (every point on x + y = 5 is a solution).

(ii) x − y = 8 and 3x − 3y = 16

$a_1/a_2 = 1/3$, $b_1/b_2 = -1/-3 = 1/3$, $c_1/c_2 = 8/16 = 1/2$
$a_1/a_2 = b_1/b_2$ but $c_1/c_2 \neq a_1/a_2$ => the lines are parallel.
This pair has no solution.

(iii) $2x + y - 6 = 0$ and $4x - 2y - 4 = 0$
$a_1/a_2 = 2/4 = 1/2$, $b_1/b_2 = 1/-2 = -1/2$
$a_1/a_2 \neq b_1/b_2$ => the lines intersect at a unique point.
Solving: from $2x + y = 6$, we get $y = 6 - 2x$.
Substituting in $4x - 2y = 4$: $4x - 2(6 - 2x) = 4$ => $4x - 12 + 4x = 4$ => $8x = 16$ => $x = 2$
$y = 6 - 2(2) = 2$
Unique solution: (2, 2).

(iv) $2x - 2y - 2 = 0$ and $4x - 4y - 5 = 0$
$a_1/a_2 = 2/4 = 1/2$, $b_1/b_2 = -2/-4 = 1/2$, $c_1/c_2 = -2/-5 = 2/5$
$a_1/a_2 = b_1/b_2$ but $c_1/c_2 \neq a_1/a_2$ => the lines are parallel.
This pair has no solution.

Q5 Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

SOLUTION

Let length = l and width = w (in metres).
Half the perimeter = $(l + w) = 36$ ...(1)
Length is 4 m more than width: $l = w + 4$ ...(2)
Substituting (2) in (1): $(w + 4) + w = 36$ => $2w + 4 = 36$ => $2w = 32$ => $w = 16$
$l = 16 + 4 = 20$
The length of the garden is 20 m and the width is 16 m.

Q6 Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables so that the graphs of the pair formed represent (i) intersecting lines (ii) parallel lines (iii) coincident lines.

SOLUTION

The given equation is $2x + 3y - 8 = 0$ (here $a_1 = 2$, $b_1 = 3$, $c_1 = -8$).

(i) Intersecting lines: need $a_1/a_2 \neq b_1/b_2$.
Choose any equation with a different ratio, e.g., $x + 2y - 5 = 0$.
Here $a_2 = 1$, $b_2 = 2$: $a_1/a_2 = 2$, $b_1/b_2 = 3/2$ — these are unequal ✓
Example: $x + 2y - 5 = 0$.

(ii) Parallel lines: need $a_1/a_2 = b_1/b_2$ but $\neq c_1/c_2$.
Keep the same ratio 2:3 for coefficients but change the constant.
Example: $2x + 3y - 12 = 0$.
$a_1/a_2 = 2/2 = 1$, $b_1/b_2 = 3/3 = 1$, $c_1/c_2 = -8/-12 = 2/3 \neq 1$ ✓

(iii) Coincident lines: need $a_1/a_2 = b_1/b_2 = c_1/c_2$.
Multiply the entire equation by any non-zero constant, e.g., $4x + 6y - 16 = 0$.
$a_1/a_2 = 2/4 = 1/2$, $b_1/b_2 = 3/6 = 1/2$, $c_1/c_2 = -8/-16 = 1/2$ ✓

Q7 Problem from Mahāvīrāchārya's Gaṇita sāra saṅgraha (c. 850 CE): The price of 9 citrons and 7 fragrant wood-apples together is 107; and the price of 7 citrons and 9 fragrant wood-apples together is 101. Find the price of each citron and each fragrant wood-apple.

SOLUTION

Let price of one citron = x and price of one wood-apple = y.
$9x + 7y = 107$ ...(1)
$7x + 9y = 101$ ...(2)

Adding (1) and (2): $16x + 16y = 208 \Rightarrow x + y = 13$ ...(3)
Subtracting (2) from (1): $2x - 2y = 6 \Rightarrow x - y = 3$ ...(4)

Adding (3) and (4): $2x = 16 \Rightarrow x = 8$
From (3): $y = 13 - 8 = 5$

Verification: $9(8) + 7(5) = 72 + 35 = 107$ ✓ and $7(8) + 9(5) = 56 + 45 = 101$ ✓
Price of each citron = 8 (units) and each fragrant wood-apple = 5 (units).

Q8 5 pencils and 7 pens together cost 50, whereas 7 pencils and 5 pens together cost 46. Find the cost of each pencil and pen.

SOLUTION

Let cost of one pencil = p and cost of one pen = q.
$5p + 7q = 50$ ...(1)
$7p + 5q = 46$ ...(2)

Adding (1) and (2): $12p + 12q = 96 \Rightarrow p + q = 8$ ...(3)
Subtracting (2) from (1): $-2p + 2q = 4 \Rightarrow -p + q = 2$ ...(4)

Adding (3) and (4): $2q = 10 \Rightarrow q = 5$
From (3): $p = 8 - 5 = 3$

Verification: $5(3) + 7(5) = 15 + 35 = 50$ ✓ and $7(3) + 5(5) = 21 + 25 = 46$ ✓
Cost of each pencil = 3 and each pen = 5.

Q9 Find the height of the stool. (The figure shows: a cat sitting on the stool reaches a combined height of 85 cm; the stool is 25 cm taller than the cat.)

SOLUTION

Let the height of the stool = s cm and the height of the cat = c cm.

From the figure, the combined height of stool and cat = 85 cm:

$s + c = 85$ ...(1)

The stool is 25 cm taller than the cat:

$s - c = 25$ ...(2)

Adding (1) and (2): $2s = 110 \Rightarrow s = 55$

From (2): $c = s - 25 = 55 - 25 = 30$

The height of the stool is 55 cm.

Exercises — End-of-Chapter Exercises

Q1 The graph of the line y = 3x passes through (0, 0) and (2, 6). (i) Identify the slope from the graph. (ii) What is the y-intercept? (iii) A water tank is filled so that water level y cm after x minutes follows y = 3x. (a) How high after 5 minutes? (b) How many minutes to reach 21 cm? (iv) Without a new graph, does (4, 10) lie on this line? (v) Where does y = 3x intersect the line x = 3?

SOLUTION

(i) Using the two points on the graph, (0, 0) and (2, 6):
Slope = $(y_2 - y_1)/(x_2 - x_1)$ = (6 − 0)/(2 − 0) = 6/2 = 3.
The slope of the line is 3. This means for every 1 unit moved to the right, the line rises 3 units.

(ii) The y-intercept is the value of y when x = 0.
From the equation y = 3(0) = 0. The y-intercept is 0.
This means the line passes through the origin. Significance: the water level is 0 cm at time 0 (tank starts empty).

(iii) Using y = 3x for the water tank:
(a) At x = 5 minutes: y = 3 × 5 = 15 cm.
(b) Set y = 21: 21 = 3x => x = 7 minutes.

(iv) Substitute x = 4 into y = 3x: y = 3 × 4 = 12.
But the point is (4, 10), and 10 ≠ 12.
Therefore (4, 10) does NOT lie on the line y = 3x.

(v) The line x = 3 is a vertical line at x = 3.
Substituting x = 3 in y = 3x: y = 3 × 3 = 9.
The two lines intersect at the point (3, 9).

Q2 Temperature conversion: F = (9/5)C + 32. (i) Draw the graph with Celsius on x-axis and Fahrenheit on y-axis. (ii) Temperature at 30°C in Fahrenheit. (iii) Temperature at 95°F in Celsius. (iv) Temperature at 0°C in Fahrenheit and at 0°F in Celsius. (v) Is there a temperature numerically the same in both scales?

SOLUTION

(i) To draw the graph, find two points:
C = 0: F = (9/5)(0) + 32 = 32. Point: (0, 32).
C = 100: F = (9/5)(100) + 32 = 180 + 32 = 212. Point: (100, 212).
Plot these points and draw the straight line through them.

(ii) C = 30: F = (9/5)(30) + 32 = 54 + 32 = 86.
At 30°C the temperature is 86°F.

(iii) F = 95: 95 = (9/5)C + 32 => (9/5)C = 63 => C = 63 × 5/9 = 35.
At 95°F the temperature is 35°C.

(iv) At C = 0: F = (9/5)(0) + 32 = 32. Temperature is 32°F.
At F = 0: 0 = (9/5)C + 32 => (9/5)C = −32 => C = −32 × 5/9 = −160/9 ≈ −17.8°C.

(v) Set F = C: C = (9/5)C + 32 => C − (9/5)C = 32 => (−4/5)C = 32 => C = −40.
At −40°, both scales give the same numerical reading.
Verification: F = (9/5)(−40) + 32 = −72 + 32 = −40 = C ✓

Q3 Solve the following system of equations graphically: 2x + y = 6, 2x − y − 2 = 0.

SOLUTION

Find two points on each line:
Line 1: 2x + y = 6. When x = 0: y = 6. When x = 3: y = 0. Points: (0, 6) and (3, 0).
Line 2: 2x − y = 2. When x = 0: y = −2. When x = 1: y = 0. Points: (0, −2) and (1, 0).

Plot these points and draw the lines. The lines intersect at one point.

Solving algebraically: From Line 1: y = 6 − 2x.
Substituting in Line 2: 2x − (6 − 2x) = 2 => 2x − 6 + 2x = 2 => 4x = 8 => x = 2.
y = 6 − 2(2) = 2.

The solution is x = 2, y = 2, i.e., the point (2, 2).

Q4 Find the point of intersection of the lines shown on the cover page.

SOLUTION

The cover page shows two lines intersecting. The point of intersection is the ordered pair (x, y) that satisfies both linear equations represented on the cover.
Read the equations of the two lines from the cover page, then solve them simultaneously.
The intersection point is the unique solution of the pair of linear equations shown on the cover.

Q5 Give a formula to find the x-intercept of the line y = mx + c.

SOLUTION

The x-intercept is the point where the line crosses the x-axis, i.e., where y = 0.
Set y = 0 in y = mx + c:
0 = mx + c
mx = −c
x = −c/m (provided m ≠ 0)
The x-intercept of the line y = mx + c is x = −c/m.

Q6 A person is choosing between two mobile plans. Plan A: 50 monthly fee + 0.20 per minute. Plan B: 30 monthly fee + 0.30 per minute. For how many minutes is Plan A cheaper? For how many minutes is Plan B cheaper? Find the number of minutes at which both plans cost the same.

SOLUTION

Let t = number of minutes of call time per month.
Cost of Plan A = 50 + 0.20t
Cost of Plan B = 30 + 0.30t

Both plans cost the same when:
50 + 0.20t = 30 + 0.30t
20 = 0.10t
t = 200 minutes

At 200 minutes, both plans cost the same: 50 + 0.20(200) = 50 + 40 = 90.

For t < 200 minutes: Plan B is cheaper (lower monthly fee dominates).
For t > 200 minutes: Plan A is cheaper (lower per-minute rate dominates).
At t = 200 minutes: both plans cost 90.

Q7 How many lines exist that (i) have a given slope? (ii) have a given slope and pass through a given point?

SOLUTION

(i) Lines with a given slope m are of the form y = mx + d, where d can be any real number. There are infinitely many lines with a given slope (parallel lines all have the same slope).

(ii) Given a slope m and a specific point (x_0, y_0), the line is uniquely determined by:
$y - y_0 = m(x - x_0)$
There is exactly one line that has a given slope and passes through a given point.

Q8 For what values of p does the pair of equations have a unique solution? 4x + py + 8 = 0; 2x + 2y + 2 = 0.

SOLUTION

For a unique solution, we need $a_1/a_2 \neq b_1/b_2$.
Here $a_1 = 4$, $b_1 = p$, $a_2 = 2$, $b_2 = 2$.
$a_1/a_2 = 4/2 = 2$ and $b_1/b_2 = p/2$.
For a unique solution: $2 \neq p/2 \Rightarrow p \neq 4$.
The pair has a unique solution for all values of p except p = 4.

Q9 Find the values of a and b for which the following system has infinitely many solutions: (a + b)x − 2by = 5a + 2b + 1; 3x − y = 14.

SOLUTION

For infinitely many solutions: $a_1/a_2 = b_1/b_2 = c_1/c_2$.
Here $a_1 = a + b$, $b_1 = -2b$, $c_1 = -(5a + 2b + 1)$; $a_2 = 3$, $b_2 = -1$, $c_2 = -14$.

From $a_1/a_2 = b_1/b_2$:
$(a + b)/3 = -2b/-1 = 2b$
$a + b = 6b \Rightarrow a = 5b$ ...(1)

From $b_1/b_2 = c_1/c_2$:
$2b = (5a + 2b + 1)/14$
$28b = 5a + 2b + 1$
$26b = 5a + 1$ ...(2)

Substituting (1) in (2): $26b = 5(5b) + 1 = 25b + 1 \Rightarrow b = 1$
From (1): $a = 5(1) = 5$

Therefore a = 5 and b = 1.

Q10 Find the value of k for which the following system represents a pair of coincident lines: x + 2y = 3; (k − 1)x + (k + 1)y = k + 3.

SOLUTION

For coincident lines: $a_1/a_2 = b_1/b_2 = c_1/c_2$.
Here $a_1 = 1$, $b_1 = 2$, $c_1 = -3$; $a_2 = k - 1$, $b_2 = k + 1$, $c_2 = -(k + 3)$.

From $a_1/a_2 = b_1/b_2$:
$1/(k - 1) = 2/(k + 1)$
$k + 1 = 2(k - 1)$
$k + 1 = 2k - 2$
$k = 3$

Verify with c_1/c_2: $c_1/c_2 = -3/-(3 + 3) = 3/6 = 1/2$.
$a_1/a_2 = 1/(3 - 1) = 1/2$ ✓ and $b_1/b_2 = 2/(3 + 1) = 2/4 = 1/2$ ✓

All three ratios are equal when k = 3. Therefore k = 3.

Q11 (i) Robot 1 starts from origin, moves 3 right and 4 up repeatedly. Robot 2 starts from (10, 0), moves 1 right and 2 up repeatedly. Do their paths intersect? (ii) Robot 1 starts from (3, 0), moves 5 right and 5 up. Robot 2 starts from (7, 0), moves 5 right and 3 down. Do their paths intersect?

SOLUTION

(i) Robot 1's path passes through (0, 0) with slope 4/3:
Equation: $y = (4/3)x$, i.e., $4x - 3y = 0$.

Robot 2's path passes through (10, 0) with slope 2/1 = 2:

Equation: $y - 0 = 2(x - 10) \Rightarrow y = 2x - 20$, i.e., $2x - y = 20$.

Solving: from $4x - 3y = 0 \Rightarrow y = (4/3)x$.
Substituting in $2x - y = 20$: $2x - (4/3)x = 20 \Rightarrow (2/3)x = 20 \Rightarrow x = 30$.
$y = (4/3)(30) = 40$.

The paths intersect at (30, 40).

(ii) Robot 1's path passes through (3, 0) with slope 5/5 = 1:
Equation: $y - 0 = 1(x - 3) \Rightarrow y = x - 3$, i.e., $x - y = 3$.

Robot 2's path passes through (7, 0) with slope −3/5:
Equation: $y - 0 = (-3/5)(x - 7) \Rightarrow 5y = -3x + 21 \Rightarrow 3x + 5y = 21$.

Solving: from $x - y = 3 \Rightarrow x = y + 3$.
Substituting in $3x + 5y = 21$: $3(y + 3) + 5y = 21 \Rightarrow 3y + 9 + 5y = 21 \Rightarrow 8y = 12 \Rightarrow y = 3/2$.
$x = 3/2 + 3 = 9/2$.

The paths intersect at (9/2, 3/2), i.e., (4.5, 1.5).

Q12 ★ At a certain time, Jacob's watch reads 'a' minutes after two o'clock. Fifteen minutes later, it reads 'b' minutes after three o'clock. He noticed that a is six times greater than b. What time was it when he looked at his watch the second time?

SOLUTION

The first observation: time = 2 hours a minutes.
The second observation (15 minutes later): time = 3 hours b minutes.

Elapsed time in minutes: $(3 \times 60 + b) - (2 \times 60 + a) = 15$
$180 + b - 120 - a = 15$
$60 + b - a = 15$
$a - b = 45$ ...(1)

Also $a = 6b$ ...(2)

Substituting (2) in (1): $6b - b = 45 \Rightarrow 5b = 45 \Rightarrow b = 9$
$a = 6(9) = 54$

At the second look, the time was 3 hours and 9 minutes, i.e., 3:09.

Q13 ★ The sum of the digits of a two-digit number is 15. The number obtained by interchanging the digits exceeds the given number by 9. Find the number.

SOLUTION

Let the tens digit = x and units digit = y.
$x + y = 15$ ...(1)

The reversed number exceeds the original by 9:
$(10y + x) - (10x + y) = 9$

9y − 9x = 9
y − x = 1 ...(2)

Adding (1) and (2): 2y = 16 => y = 8
From (1): x = 15 − 8 = 7

The two-digit number is 78.
Verification: 7 + 8 = 15 ✓ and 87 − 78 = 9 ✓

Q14 ★ In a cyclic quadrilateral ABCD, ∠A = (x + 7)°, ∠B = (y + 8)°, ∠C = (3y + 23)°, ∠D = (4x + 12)°. Find all four angles.

SOLUTION

In a cyclic quadrilateral, opposite angles are supplementary (sum to 180°).

∠A + ∠C = 180°:
(x + 7) + (3y + 23) = 180
x + 3y + 30 = 180
x + 3y = 150 ...(1)

∠B + ∠D = 180°:
(y + 8) + (4x + 12) = 180
4x + y + 20 = 180
4x + y = 160 ...(2)

From (1): x = 150 − 3y. Substituting in (2):
4(150 − 3y) + y = 160
600 − 12y + y = 160
−11y = −440
y = 40

x = 150 − 3(40) = 150 − 120 = 30

∠A = 30 + 7 = 37°
∠B = 40 + 8 = 48°
∠C = 3(40) + 23 = 143°
∠D = 4(30) + 12 = 132°

The four angles are ∠A = 37°, ∠B = 48°, ∠C = 143°, ∠D = 132°.

Q15 ★ A train moving with uniform speed for a certain distance takes 6 hours less if its speed is increased by 6 km/h. It would have taken 6 hours more had its speed been decreased by 4 km/h. Find the distance and the speed of the train.

SOLUTION

Let the original speed = v km/h and distance = d km.

At increased speed (v + 6): time decreases by 6 hours:
$d/v - d/(v + 6) = 6$
$d[(v + 6 - v) / (v(v + 6))] = 6$
$6d / (v(v + 6)) = 6$
$d = v(v + 6)$ ...(1)

At decreased speed (v − 4): time increases by 6 hours:
$d/(v - 4) - d/v = 6$
$d[(v - (v - 4)) / (v(v - 4))] = 6$
$4d / (v(v - 4)) = 6$
$2d = 3v(v - 4)$ ...(2)

Substituting (1) in (2):
$2v(v + 6) = 3v(v - 4)$
Dividing both sides by v ($v \neq 0$):
$2(v + 6) = 3(v - 4)$
$2v + 12 = 3v - 12$
$v = 24$ km/h

From (1): $d = 24(24 + 6) = 24 \times 30 = 720$ km.

Verification: normal time = 720/24 = 30 h. At 30 km/h: 720/30 = 24 h (6 less ✓). At 20 km/h: 720/20 = 36 h (6 more ✓).
The speed of the train is 24 km/h and the distance is 720 km.

Q16 ★ The age of a father is equal to the sum of the ages of his four children. After 20 years, the sum of the ages of the children will be twice the age of the father. Find the age of the father.

SOLUTION

Let the father's current age = F years and the sum of the four children's current ages = S years.

Currently: $F = S$ ...(1)

After 20 years, father's age = F + 20; sum of children's ages = S + 4(20) = S + 80.
$S + 80 = 2(F + 20)$
$S + 80 = 2F + 40$
$S - 2F = -40$ ...(2)

Substituting (1) (F = S) in (2):
$F - 2F = -40$
$-F = -40$

F = 40

Verification: S = 40. After 20 years, children's sum = 40 + 80 = 120 = 2(40 + 20) = 2(60) ✓
The age of the father is 40 years.