Chapter 14: Math of Space: Surface Area and Volume

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Exercise 14.1 — Exercise Set 14.1

Q1 The volume of a cube is 64 cm³. What is its total surface area?

SOLUTION

Given: Volume of cube = 64 cm³
Volume of cube = a^3, so $a^3 = 64$
Side a = $\sqrt[3]{64} = 4$ cm

Total surface area of a cube = $6a^2$
TSA = $6 \times 4^2 = 6 \times 16 = 96$ cm²

The total surface area of the cube is 96 cm².

Q2 How many small cubes with side 20 cm can be packed tight in a cubical box with side 2 m?

SOLUTION

Side of large cubical box = 2 m = 200 cm
Side of each small cube = 20 cm

Number of small cubes along each edge = 200 ÷ 20 = 10

Total number of small cubes = 10 × 10 × 10 = 1000

1000 small cubes can be packed in the box.

Q3 The dimensions of a godown are 40 m × 25 m × 10 m. If it is filled with cuboidal boxes, each of dimensions 2 m × 1.25 m × 1 m, then find the number of boxes.

SOLUTION

Volume of godown = 40 × 25 × 10 = 10000 m³

Volume of each cuboidal box = 2 × 1.25 × 1 = 2.5 m³

Number of boxes = Volume of godown ÷ Volume of one box
= 10000 ÷ 2.5 = 4000

The number of boxes that can be stored is 4000.

Q4 Two cubes each of volume 125 cm³ are joined end to end. Find the surface area of the resulting cuboid.

SOLUTION

Volume of each cube = 125 cm³
Side of each cube: $a^3 = 125 \Rightarrow a = 5$ cm

When two cubes of side 5 cm are joined end to end, the resulting cuboid has:
Length l = 5 + 5 = 10 cm, Width w = 5 cm, Height h = 5 cm

TSA of cuboid = 2(lw + wh + lh)
= 2(10×5 + 5×5 + 10×5)
= 2(50 + 25 + 50)
= 2 × 125 = 250 cm²

The surface area of the resulting cuboid is 250 cm².

Q5 A cube of side 4 cm is cut into cubes of side 1 cm. What is the ratio of the surface areas of the original cube and all the cut-out cubes?

SOLUTION

Surface area of original cube (side 4 cm) = 6 × 4² = 6 × 16 = 96 cm²

Number of small cubes formed = 4³ = 64
Surface area of each small cube (side 1 cm) = 6 × 1² = 6 cm²
Total surface area of all small cubes = 64 × 6 = 384 cm²

Ratio = SA of original cube : Total SA of all cut-out cubes
= 96 : 384 = 1 : 4

The ratio of the surface areas is 1 : 4.

Q6 The surface areas of the three faces of a cuboid that meet at one of the corners of the cuboid are 6 cm², 15 cm², and 10 cm² respectively. What is the volume of the cuboid?

SOLUTION

Let the dimensions of the cuboid be l, w, and h.
The three faces at one corner have areas: lw = 6, lh = 15, wh = 10

Multiplying all three: (lw)(lh)(wh) = 6 × 15 × 10 = 900
So $(lwh)^2 = 900$
Therefore $lwh = \sqrt{900} = 30$ cm³

The volume of the cuboid is 30 cm³.

Q7 A cube of side 5 cm is painted on all its faces. If it is sliced into 1 cm³ cubes, how many of these 1 cm³ cubes have (i) exactly three faces painted? (ii) exactly two faces painted? (iii) exactly one face painted? (iv) no face painted?

SOLUTION

Total number of 1 cm³ cubes = $5^3 = 125$

(i) Exactly three faces painted:
These are the corner cubes of the large cube.
A cube has 8 corners, so 8 small cubes have exactly 3 faces painted.

(ii) Exactly two faces painted:
These are the edge cubes (not corners). Each edge of the large cube has $5 - 2 = 3$ interior positions.
A cube has 12 edges, so number = $12 \times 3 = 36$ small cubes.

(iii) Exactly one face painted:
These are the face-interior cubes. Each face has a $3 \times 3 = 9$ inner grid.
A cube has 6 faces, so number = $6 \times 9 = 54$ small cubes.

(iv) No face painted:
These are the interior cubes, forming a $3 \times 3 \times 3$ inner cube.
Number = $3^3 = 27$ small cubes.

Verification: $8 + 36 + 54 + 27 = 125$ ✓

Q8 Find a cuboid with edges whose lengths are integers (in cm), given that it has a total surface area of exactly 100 cm². (i) Is there more than one such cuboid? (ii) Can you find them all? (iii) Show that you have found them all.

SOLUTION

TSA = $2(lw + wh + lh) = 100 \Rightarrow lw + wh + lh = 50$
We need positive integers $l \le w \le h$ with $lw + wh + lh = 50$.

Systematic search with $l \le w \le h$:
If $l = 1$: $w + wh + h = 50 \Rightarrow (w+1)(h+1) = 51 = 3 \times 17$
Factorisation 3×17: $w+1 = 3$, $h+1 = 17 \Rightarrow w = 2$, $h = 16$ ✓ (1, 2, 16)
Factorisation 1×51: $w+1 = 1 \rightarrow w = 0$, not a positive integer.

If $l = 2$: $2w + 2h + wh = 50 \Rightarrow (w+2)(h+2) = 54 = 2\times27, 3\times18, 6\times9$
$w+2=6$, $h+2=9 \Rightarrow w=4$, $h=7$ ✓ (2, 4, 7)
$w+2=3$, $h+2=18 \Rightarrow w=1$, but $l=2 > w=1$ violates $l \le w$, skip.
$w+2=2$, $h+2=27 \Rightarrow w=0$, not valid.

If $l = 3$: $3w + 3h + wh = 50 \Rightarrow (w+3)(h+3) = 59$, which is prime.
Only factorisation: 1×59, giving $w+3=1 \rightarrow$ invalid.

If $l = 4$: $4w + 4h + wh = 50 \Rightarrow (w+4)(h+4) = 66 = 6\times11$
$w+4=6$, $h+4=11 \Rightarrow w=2$, but $l=4 > w=2$, violates $l \le w$.

No valid solution.

For $l \geq 5$: $lw + wh + lh \geq 3l^2 \geq 75 > 50$, impossible.

(i) Yes, there is more than one such cuboid.
(ii) The two solutions are (1, 2, 16) and (2, 4, 7).
(iii) The search above is exhaustive for $l \leq w \leq h$ with $l \geq 1$, since $l \geq 5$ is impossible, showing all solutions have been found.

Exercise 14.2 — Exercise Set 14.2

Q1 Two cylinders, A and B, are given. The radius of cylinder B is twice that of cylinder A, and the height of cylinder B is half that of cylinder A. Find the ratio of the curved surface area of A to the curved surface area of B. Also find the ratio of the volume of A to the volume of B.

SOLUTION

Let radius of A = r and height of A = h.
Then radius of B = 2r and height of B = h/2.

CSA of A = $2\pi rh$
CSA of B = $2\pi(2r)(h/2) = 2\pi rh$
Ratio CSA(A) : CSA(B) = $2\pi rh : 2\pi rh = 1 : 1$

Volume of A = $\pi r^2 h$
Volume of B = $\pi(2r)^2(h/2) = \pi \times 4r^2 \times h/2 = 2\pi r^2 h$
Ratio V(A) : V(B) = $\pi r^2 h : 2\pi r^2 h = 1 : 2$

The ratio of curved surface areas is 1 : 1, and the ratio of volumes is 1 : 2.

Q2 The radii of two cylinders are in the ratio 2:3, and their heights are in the ratio 3:2. Find (a) the ratio of their volumes, and (b) the ratio of their curved surface areas.

SOLUTION

Let radii be $r_1 = 2k$ and $r_2 = 3k$; heights $h_1 = 3m$ and $h_2 = 2m$.

(a) Volume of cylinder = $\pi r^2 h$
$V_1/V_2 = (\pi r_1^2 h_1)/(\pi r_2^2 h_2) = (2^2 \times 3)/(3^2 \times 2) = (4 \times 3)/(9 \times 2) = 12/18 = 2/3$
Ratio $V_1 : V_2 = 2 : 3$

(b) CSA of cylinder = $2\pi rh$
$CSA_1/CSA_2 = (r_1 h_1)/(r_2 h_2) = (2 \times 3)/(3 \times 2) = 6/6 = 1$
Ratio $CSA_1 : CSA_2 = 1 : 1$

(a) The ratio of volumes is 2 : 3. (b) The ratio of curved surface areas is 1 : 1.

Q3 The edge of a cube measures r cm. The largest possible right circular cylinder is cut out of the cube. What do you think is the volume of the cylinder (in cm^3)?

SOLUTION

The cube has edge r cm.
The largest cylinder that fits inside the cube has:
Height = r cm (same as cube edge)
Diameter = r cm (the circle fits within the square face), so radius = r/2

Volume of cylinder = $\pi \times (r/2)^2 \times r = \pi \times r^2/4 \times r = \pi r^3/4$

The volume of the largest cylinder is $\pi r^3/4$ cm^3.

Q4 The radius of the base of a cylinder is increased by 10%. At the same time, the height of the cylinder is decreased by x%. Given that the volume of the cylinder remains unchanged, find the value of x.

SOLUTION

Original volume $V = \pi r^2 h$.
New radius = 1.1r (10% increase), new height = $(1 - x/100)h$.

New volume = $\pi(1.1r)^2(1 - x/100)h = \pi \times 1.21r^2 \times (1 - x/100)h$

Setting new volume equal to original volume:
$1.21(1 - x/100) = 1$
$1 - x/100 = 1/1.21 = 100/121$
$x/100 = 1 - 100/121 = 21/121$
$x = 2100/121 \approx 17.36$

The value of x is $2100/121 \approx 17.36$.

Q5 A solid metallic cube of side 12 cm is melted and recast into solid cylindrical rods, each having radius 2 cm and height 12 cm. Find: (i) the volume of the cube, (ii) the volume of one cylindrical rod, (iii) the approximate number of complete cylindrical rods that can be formed. ($\pi \approx 22/7$)

SOLUTION

(i) Volume of the metallic cube = $\text{side}^3 = 12^3 = 1728\ \text{cm}^3$

(ii) Volume of one cylindrical rod = $\pi r^2 h = (22/7) \times 2^2 \times 12$
$= (22/7) \times 4 \times 12 = (22 \times 48)/7 = 1056/7 \approx 150.86\ \text{cm}^3$

(iii) Number of rods = Volume of cube ÷ Volume of one rod
$= 1728 \div (1056/7) = 1728 \times 7/1056 = 12096/1056 = 126/11 \approx 11.45$

The number of complete cylindrical rods that can be formed is 11.

Exercise 14.3 — Exercise Set 14.3

Q1 Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m.

SOLUTION

Slant height l = 21 m, diameter = 24 m, so radius r = 12 m

TSA of cone = πr(l + r) = (22/7) × 12 × (21 + 12)
= (22/7) × 12 × 33
= (22 × 12 × 33)/7
= 8712/7
= 1244.57 m²

The total surface area of the cone is 8712/7 m² ≈ 1244.57 m².

Q2 Find the curved surface area of a right circular cone whose slant height is 10 cm and base radius is 7 cm.

SOLUTION

Slant height l = 10 cm, base radius r = 7 cm

CSA of cone = πrl = (22/7) × 7 × 10
= 22 × 10 = 220 cm²

The curved surface area of the cone is 220 cm².

Q3 The height of a cone is 16 cm, and its base radius is 12 cm. Find the curved surface area and the total surface area of the cone.

SOLUTION

Height h = 16 cm, base radius r = 12 cm

Slant height l = √(h² + r²) = √(16² + 12²) = √(256 + 144) = √400 = 20 cm

CSA = πrl = (22/7) × 12 × 20 = (22 × 240)/7 = 5280/7 ≈ 754.29 cm²

TSA = πr(l + r) = (22/7) × 12 × (20 + 12) = (22/7) × 12 × 32
= (22 × 384)/7 = 8448/7 ≈ 1206.86 cm²

CSA ≈ 754.29 cm², TSA ≈ 1206.86 cm².

Q4 A cone has a height of 15 cm. If its volume is 1570 cm³, find the radius of the base.

SOLUTION

Height h = 15 cm, Volume V = 1570 cm³ (using π ≈ 3.14)

Volume of cone = (1/3)πr²h
1570 = (1/3) × 3.14 × r² × 15

$1570 = 3.14 \times 5 \times r^2$
$1570 = 15.7 \times r^2$
$r^2 = 1570/15.7 = 100$
$r = 10$ cm

The radius of the base is 10 cm.

Q5 The curved surface area of a cone is 308 cm² and its slant height is 14 cm. Find (i) the radius of the base, (ii) the total surface area of the cone.

SOLUTION

CSA = 308 cm², slant height l = 14 cm

(i) CSA = πrl
$308 = (22/7) \times r \times 14$
$308 = 44r$
$r = 308/44 = 7$ cm

(ii) TSA = $\pi r(l + r) = (22/7) \times 7 \times (14 + 7)$
$= (22/7) \times 7 \times 21 = 22 \times 21 = 462$ cm²

(i) The radius of the base is 7 cm. (ii) The total surface area is 462 cm².

Q6 A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.

SOLUTION

Base radius r = 7 cm, height h = 24 cm

Slant height $l = \sqrt{(r^2 + h^2)} = \sqrt{(7^2 + 24^2)} = \sqrt{(49 + 576)} = \sqrt{625} = 25$ cm

Curved surface area of one cap = $\pi rl = (22/7) \times 7 \times 25 = 22 \times 25 = 550$ cm²

Area of sheet required for 10 caps = $10 \times 550 = 5500$ cm²

The area of sheet required to make 10 caps is 5500 cm².

Q7 What length of tarpaulin 3 m wide is required to make a conical tent of height 8 m and base radius 6 m? Assume that the extra length of material that is required for stitching margins and wastage in cutting is 20 cm.

SOLUTION

Height h = 8 m, base radius r = 6 m

Slant height $l = \sqrt{(h^2 + r^2)} = \sqrt{(8^2 + 6^2)} = \sqrt{(64 + 36)} = \sqrt{100} = 10$ m

Curved surface area of cone = $\pi rl = 3.14 \times 6 \times 10 = 188.4$ m²

Tarpaulin is 3 m wide. Length needed for the tent = CSA/width = 188.4/3 = 62.8 m

Extra length for stitching and wastage = 20 cm = 0.2 m

Total length of tarpaulin required = 62.8 + 0.2 = 63 m

The required length of tarpaulin is 63 m.

Q8 A right triangle with sides 6 cm, 8 cm and 10 cm is rotated through 360° about the side of 8 cm. Find the volume and the curved surface area of the solid so formed.

SOLUTION

The triangle has sides 6 cm, 8 cm, 10 cm.
Since $6^2 + 8^2 = 36 + 64 = 100 = 10^2$, the right angle is between sides 6 cm and 8 cm.

Rotating about the side of 8 cm forms a cone with: height h = 8 cm, radius r = 6 cm, slant height l = 10 cm.

Volume = $(1/3)\pi r^2 h = (1/3) \times 3.14 \times 6^2 \times 8$
$= (1/3) \times 3.14 \times 36 \times 8 = (1/3) \times 904.32 = 301.44$ cm³

CSA = $\pi r l = 3.14 \times 6 \times 10 = 188.4$ cm²

Volume = 301.44 cm³, Curved surface area = 188.4 cm².

Q9 Suppose you have a cup in the shape of a right circular cone. Fill it with water to half the depth of the cone. What fraction of the volume of the cup is occupied by the water?

SOLUTION

Let the cone have radius r and height h.

When water fills the cone to depth h/2, the water surface forms a smaller cone similar to the cup.
By similar triangles, the radius of the water surface = $r \times (h/2)/h = r/2$.

Volume of water = $(1/3)\pi(r/2)^2(h/2) = (1/3)\pi \times r^2/4 \times h/2 = (1/3)\pi r^2 h \times 1/8$

Volume of cup = $(1/3)\pi r^2 h$

Fraction = Volume of water / Volume of cup = 1/8

The water occupies 1/8 of the volume of the cup.

Exercise 14.4 — Exercise Set 14.4

Q1 A ball bearing has a radius of 0.7 cm. Find its surface area.

SOLUTION

Radius r = 0.7 cm = 7/10 cm

Surface area of sphere = $4\pi r^2$
$= 4 \times (22/7) \times (7/10)^2$
$= 4 \times (22/7) \times 49/100$
$= (4 \times 22 \times 49)/(7 \times 100)$
$= (4 \times 22 \times 7)/100$
$= 616/100$
$= 6.16\ cm^2$

The surface area of the ball bearing is 6.16 cm^2.

Q2 Two solid spheres made of the same metal have weights 5920 g and 740 g. Determine the radius of the larger sphere, if the diameter of the smaller one is 5 cm.

SOLUTION

Since the spheres are made of the same metal, their volumes are proportional to their weights.

V_large/V_small = 5920/740 = 8

$(4/3)\pi R^3 / (4/3)\pi r^3 = 8 \Rightarrow (R/r)^3 = 8 \Rightarrow R/r = 2$

Diameter of smaller sphere = 5 cm, so r = 2.5 cm
R = 2 × r = 2 × 2.5 = 5 cm

The radius of the larger sphere is 5 cm.

Q3 The diameter of the moon is approximately one fourth the diameter of the earth. Given that the moon and earth are both roughly spherical, find the ratio of their surface areas.

SOLUTION

Let diameter of Earth = D, so radius of Earth = D/2.
Diameter of Moon = D/4, so radius of Moon = D/8.

SA of Moon / SA of Earth = $4\pi(D/8)^2 / 4\pi(D/2)^2$
$= (D/8)^2 / (D/2)^2$
$= (1/8)^2 / (1/2)^2$
$= (1/64) / (1/4)$
$= 1/64 \times 4/1 = 1/16$

The ratio of the surface area of the Moon to the surface area of the Earth is 1 : 16.

Q4 Find (i) the curved surface area and (ii) the total surface area of a hemisphere of radius 21 cm.

SOLUTION

Radius r = 21 cm

(i) Curved surface area of hemisphere = $2\pi r^2$
$= 2 \times (22/7) \times 21^2$
$= 2 \times (22/7) \times 441$
$= 2 \times 22 \times 63$
$= 2772\ cm^2$

(ii) Total surface area of hemisphere = $3\pi r^2$
$= 3 \times (22/7) \times 441$
$= 3 \times 22 \times 63$
$= 4158\ cm^2$

(i) CSA = 2772 cm^2. (ii) TSA = 4158 cm^2.

Q5 Metal spheres, each of radius 2 cm, are packed into a rectangular box of internal dimensions 16 cm × 8 cm × 8 cm. When 16 spheres are packed, the box is filled with preservative liquid. Find the volume of this liquid. Round your answer to the nearest integer.

SOLUTION

Volume of the rectangular box = $16 \times 8 \times 8 = 1024\ cm^3$

Volume of one metal sphere = $(4/3)\pi r^3 = (4/3) \times (22/7) \times 2^3$
$= (4/3) \times (22/7) \times 8 = (4 \times 22 \times 8)/(3 \times 7) = 704/21\ cm^3$

Volume of 16 spheres = $16 \times 704/21 = 11264/21 \approx 536.38\ cm^3$

Volume of preservative liquid = Volume of box − Volume of spheres
$= 1024 - 11264/21 = (21504 - 11264)/21 = 10240/21 \approx 487.62\ cm^3$

The volume of the liquid is approximately 488 cm^3.

Q6 The radius of a sphere is increased by 10%. Show that the volume increases by approximately 33.1%.

SOLUTION

Let original radius = r. New radius = 1.1r (10% increase).

Original volume $V = (4/3)\pi r^3$
New volume $V' = (4/3)\pi(1.1r)^3 = (4/3)\pi r^3 \times (1.1)^3$

$(1.1)^3 = 1.1 \times 1.1 \times 1.1 = 1.21 \times 1.1 = 1.331$

$V'/V = 1.331$

Percentage increase = $(1.331 - 1) \times 100\% = 33.1\%$

Hence the volume increases by approximately 33.1%. ∎

Q7 The radius of a sphere is increased by x%. The volume of the sphere increases by 72.8%. Find the value of x.

SOLUTION

Let original radius = r. New radius = r(1 + x/100).

New volume / Original volume = 1 + 72.8/100 = 1.728

$(1 + x/100)^3 = 1.728$

Taking cube root: $1 + x/100 = \sqrt[3]{1.728} = 1.2$
(Since $1.2^3 = 1.2 \times 1.2 \times 1.2 = 1.44 \times 1.2 = 1.728$ ✓)

x/100 = 0.2 => x = 20

The value of x is 20.

Q8 The hemispherical dome of a building needs to be painted. If the circumference of the base of the dome is 35.2 m, find the cost of painting it, given the cost of painting is 10 per 100 cm².

SOLUTION

Circumference of base = $2\pi r = 35.2$ m
$r = 35.2 / (2 \times 22/7) = 35.2 \times 7/44 = 246.4/44 = 5.6$ m

Curved surface area of hemispherical dome = $2\pi r^2$
$= 2 \times (22/7) \times 5.6^2$
$= 2 \times (22/7) \times 31.36$
$= (2 \times 22 \times 31.36)/7$
$= 1379.84/7 = 197.12$ m²

Converting to cm²: 197.12 m² = 197.12 × 10000 = 19,71,200 cm²

Cost of painting = (19,71,200 ÷ 100) × 10 = 19,712 × 10 = 1,97,120

The cost of painting the dome is 1,97,120.

Exercises — End-of-Chapter Exercises

Q1 Estimate how many scoops of ice cream can be obtained from a cuboidal container of dimensions 10 cm × 15 cm × 20 cm.

SOLUTION

Volume of the cuboidal container = 10 × 15 × 20 = 3000 cm³

Estimate: A typical ice cream scoop has radius ≈ 2 cm.
Volume of one spherical scoop = (4/3)π × 2³ = (4/3) × 3.14 × 8 ≈ 33.5 cm³

Upper bound number of scoops = 3000/33.5 ≈ 90

Accounting for the fact that spherical scoops leave gaps and the scoop is not a perfect sphere (it's typically a hemisphere atop a rounded base), a reasonable estimate is approximately 50–70 scoops.

Estimated number of scoops: approximately 50–90 (depending on scoop size).

Q2 A cube of integer side length a is made of unit cubes. Write an expression giving the number of unit cubes to be added to make a cube of side length a + 1.

SOLUTION

Volume of cube of side a = a^3
Volume of cube of side a + 1 = $(a + 1)^3$

Number of unit cubes to be added = $(a + 1)^3 - a^3$
= $a^3 + 3a^2 + 3a + 1 - a^3$
= $3a^2 + 3a + 1$

The number of unit cubes to be added is $3a^2 + 3a + 1$.

Q3 Solve the following: (i) Could a person drink enough water in a lifetime to fill an entire room the size of your classroom? (ii) Estimate the number of bricks used to build the walls of your classroom.

SOLUTION

(i) Estimating water drunk in a lifetime:
Average water intake ≈ 2 litres/day.
Lifespan ≈ 70 years = 70 × 365 days ≈ 25,550 days.
Total water ≈ 2 × 25,550 = 51,100 litres = 51.1 m³

A typical classroom: 10 m × 8 m × 3 m = 240 m³
51.1 m³ << 240 m³.
No, a person cannot drink enough water in a lifetime to fill a classroom.

(ii) Estimating bricks in classroom walls:
Assume classroom: length 10 m, width 8 m, height 3 m.

Perimeter of walls = 2(10 + 8) = 36 m.
Wall area = 36 × 3 = 108 m², minus door (2×1 m) and 2 windows (1.2×1 m each) ≈ 108 − 2 − 2.4 ≈ 103.6 m².
Wall thickness ≈ 0.23 m (one-brick thick).
Volume of brickwork ≈ 103.6 × 0.23 ≈ 23.8 m³.
Volume of one standard brick ≈ 0.23 × 0.115 × 0.075 ≈ 0.00199 m³.
Number of bricks ≈ 23.8/0.00199 ≈ 12,000 bricks.

Approximately 10,000–15,000 bricks (estimate depends on actual dimensions).

Q4 You are given two options: (A) one chocolate cube of side 50 mm, (B) hundred chocolate cubes of side 10 mm. Which option gives you more chocolate?

SOLUTION

Volume of option A (one cube, side 50 mm) = 50^3 = 125,000 mm³

Volume of option B (100 cubes, each of side 10 mm) = 100 × 10^3 = 100 × 1000 = 100,000 mm³

125,000 > 100,000

Option A gives more chocolate.

Q5 If all the people in the world are crowded together in one location, how much area would it cover?

SOLUTION

Estimate: World population ≈ 8 billion = 8 × 10^9 people.

Standing space per person ≈ 0.5 m × 0.5 m = 0.25 m² (a tightly packed crowd).

Total area ≈ 8 × 10^9 × 0.25 = 2 × 10^9 m² = 2000 km²

This is roughly the area of a small city like Delhi (about 1484 km²) or a large city.

If we use a more generous personal space of 0.5 m²: Total ≈ 4000 km², about the size of the Indian state of Goa.

The entire world population would fit in an area of approximately 2000–4000 km².

Q6 A school provides milk to its students in cylindrical glasses, each with a diameter of 7 cm. If each glass is filled with milk to a height of 12 cm, how many litres of milk are needed to serve 1600 students? Use 1000 cm³ = 1 litre.

SOLUTION

Diameter = 7 cm, so radius r = 3.5 cm; height h = 12 cm

Volume of milk in one glass = $\pi r^2 h$ = (22/7) × $(3.5)^2$ × 12
= (22/7) × (49/4) × 12

= (22 × 49 × 12)/(7 × 4)
= (22 × 7 × 12)/4
= 22 × 21 = 462 cm³

Total volume for 1600 students = 1600 × 462 = 739,200 cm³

Converting to litres: 739,200 ÷ 1000 = 739.2 litres

739.2 litres of milk are needed.

Q7 The surface area of a sphere of radius 5 cm is five times the area of the curved surface of a cone of radius 4 cm. Find the height and the volume of the cone.

SOLUTION

Surface area of sphere = $4\pi r^2 = 4\pi \times 5^2 = 100\pi$ cm²

CSA of cone = πrl, where r = 4 cm

Given: $100\pi = 5 \times \pi rl = 5 \times \pi \times 4 \times l = 20\pi l$
=> 100 = 20l => l = 5 cm

Slant height l = 5 cm, radius r = 4 cm
Height $h = \sqrt{(l^2 - r^2)} = \sqrt{(25 - 16)} = \sqrt{9} = 3$ cm

Volume of cone = $(1/3)\pi r^2 h = (1/3) \times \pi \times 4^2 \times 3 = (1/3) \times \pi \times 48 = 16\pi$
≈ 16 × 3.14 = 50.24 cm³

Height of cone = 3 cm; Volume of cone = 16π ≈ 50.24 cm³.

Q8 Take Earth to be a perfect sphere with radius 6370 km. Take Jupiter to be a perfect sphere with radius 69,900 km. Take the Sun to be a perfect sphere with radius 6,95,700 km. Compute approximately: (i) the ratio of the volume of Jupiter to the volume of the Earth; (ii) the ratio of the volume of the Sun to the volume of the Earth. (Calculator can be used.)

SOLUTION

Volume of a sphere is proportional to r^3.

(i) $V_{Jupiter} / V_{Earth} = (r_J/r_E)^3 = (69900/6370)^3$
69900/6370 ≈ 10.97
$(10.97)^3 \approx 1321$

(ii) $V_{Sun} / V_{Earth} = (r_S/r_E)^3 = (695700/6370)^3$
695700/6370 ≈ 109.22
$(109.22)^3 \approx 1,302,700$

(i) The volume of Jupiter is approximately 1321 times the volume of the Earth.
(ii) The volume of the Sun is approximately 1,302,700 times the volume of the Earth.

Q9 Show that the volume of a sphere is equal to 2/3 of the volume of the smallest cylinder which encloses it.

SOLUTION

The smallest cylinder that encloses a sphere of radius r has:
base radius = r (cylinder must be at least as wide as the sphere)
height = 2r (cylinder must be at least as tall as the sphere's diameter)

Volume of this cylinder = $\pi r^2 \times 2r = 2\pi r^3$

Volume of sphere = $(4/3)\pi r^3$

Ratio = Volume of sphere / Volume of cylinder
$= (4/3)\pi r^3 / (2\pi r^3)$
$= (4/3) / 2$
$= 4/6 = 2/3$

Hence, the volume of the sphere = (2/3) × volume of the enclosing cylinder. ∎

Q10 Suppose the Earth is perfectly spherical. A string is wrapped tightly around the equator of the Earth. Another string, 1 metre longer, is placed around the Earth so that it forms a larger circle, staying the same distance above the ground everywhere. How high above the ground is the second string? Repeat this for: (i) the Moon (ii) Jupiter (iii) a volleyball. Are you surprised by the three answers? Why or why not?

SOLUTION

Let the Earth's radius = R km. The first string has length $C_1 = 2\pi R$.
The second string is 1 m = 0.001 km longer: $C_2 = 2\pi R + 0.001$.

If the second string forms a circle of radius R + h above ground:
$2\pi(R + h) = 2\pi R + 0.001$
$2\pi h = 0.001$
$h = 0.001/(2\pi) = 1/(2\pi)$ m ≈ 0.1592 m ≈ 15.9 cm

This result is completely independent of R — the height h = 1/(2π) metres regardless of the sphere's size.

(i) For the Moon: h = 1/(2π) m ≈ 15.9 cm — the same.
(ii) For Jupiter: h = 1/(2π) m ≈ 15.9 cm — the same.
(iii) For a volleyball: h = 1/(2π) m ≈ 15.9 cm — the same.

Yes, this is very surprising! The height is always about 16 cm regardless of the size of the sphere, because the relationship between circumference and radius is linear (C = 2πR).

Q11 We have a cylinder with a base radius of r cm and height h cm. A square pyramid is fitted inside it. The square base of the pyramid lies on the base of the cylinder, its corners on the boundary of the cylinder. The apex of the pyramid lies on the top of the cylinder. Find the ratio of the volume of this pyramid to the volume of the cylinder.

SOLUTION

The cylinder has base radius r and height h.

The square base of the pyramid has its four corners on the circular base of the cylinder (a circle of radius r).
The diagonal of the square equals the diameter of the circle = 2r.
If the side of the square is s, then $s\sqrt{2} = 2r$, so $s = r\sqrt{2}$.

Area of the square base = $s^2 = (r\sqrt{2})^2 = 2r^2$

The apex is at height h (top of the cylinder).

Volume of pyramid = (1/3) × base area × height = $(1/3) \times 2r^2 \times h = 2r^2h/3$

Volume of cylinder = $\pi r^2 h$

Ratio = $(2r^2h/3) / (\pi r^2 h) = 2/(3\pi)$

The ratio of the volume of the pyramid to the volume of the cylinder is $2 : 3\pi$.

Q12 What is the change in volume when: (i) the length of a cuboid with dimensions l, w, h is increased by 1 unit? (ii) the radius of a cylinder with dimensions r, h is increased by 1 unit? *(iii) the radius of a sphere is decreased by 1 unit?

SOLUTION

(i) Original volume = lwh
New volume = $(l + 1)wh = lwh + wh$
Change in volume = wh cubic units.
Answer: (c) wh cubic units.

(ii) Original volume = $\pi r^2 h$
New volume = $\pi(r + 1)^2 h = \pi(r^2 + 2r + 1)h = \pi r^2 h + (2\pi rh + \pi h)$
Change in volume = $(2\pi rh + \pi h) = \pi h(2r + 1)$ cubic units.
Answer: (e) $2\pi rh + \pi h$ cubic units.

*(iii) Original volume = $(4/3)\pi r^3$
New volume = $(4/3)\pi(r - 1)^3 = (4/3)\pi(r^3 - 3r^2 + 3r - 1)$
Decrease in volume = $(4/3)\pi[r^3 - (r - 1)^3] = (4/3)\pi(3r^2 - 3r + 1)$
The volume decreases by $(4/3)\pi(3r^2 - 3r + 1)$ cubic units.

Q13 Given a cube with volume V, express its total surface area S in terms of V.

SOLUTION

Let side of cube = a.
Volume: V = a³ => a = V^(1/3)

Total surface area: S = 6a²
= 6 × (V^(1/3))²
= 6V^(2/3)

The total surface area is S = 6V^(2/3).

Q14 Given a cube with total surface area S, express its volume V in terms of S.

SOLUTION

Let side of cube = a.
TSA: S = 6a² => a² = S/6 => a = √(S/6)

Volume: V = a³ = (√(S/6))³ = (S/6)^(3/2)

We can also write: V = (S/6)^(3/2) = S^(3/2) / 6^(3/2) = S√S / (6√6)

The volume is V = (S/6)^(3/2).

Q15 A cylindrical glass of height 25 cm and radius 4 cm has water up to a height of 16 cm. A crow wants to drink water from this glass. The water must be at a height of 20 cm for the crow to reach it. There are some marbles lying around. How many marbles of radius 1 cm should the crow drop into the glass to make the water reach the required height?

SOLUTION

The water level needs to rise from 16 cm to 20 cm — a rise of 4 cm.

Volume of water needed to fill this rise = π × r² × 4 = π × 4² × 4 = 64π cm³

This rise is caused by the marbles displacing water upward.
Volume displaced by n marbles = n × (4/3)π × 1³ = (4n/3)π cm³

Setting displaced volume equal to volume needed for the rise:
(4n/3)π = 64π
4n/3 = 64
n = 64 × 3/4 = 48

The crow should drop 48 marbles into the glass.

Q16 Find the volume of ink in a new ball point pen. Take the necessary measurements and make approximations, as needed.

SOLUTION

The ink in a ballpoint pen is stored in a thin cylindrical refill tube.

Typical measurements of a refill:
Inner radius $r \approx 0.05$ cm (a standard ink tube has very small bore)
Length of ink column $l \approx 7$ cm (the usable length)

Volume of ink $= \pi r^2 l \approx 3.14 \times (0.05)^2 \times 7$
$= 3.14 \times 0.0025 \times 7$
$\approx 0.055\ cm^3 \approx 0.055$ mL

This is a guesstimate; actual ink volumes in ballpoint pens range from about 0.05 to 0.1 mL depending on pen brand and refill size.

Estimated volume of ink ≈ 0.05–0.1 cm^3 (0.05–0.1 mL).

Q17 Solve: (i) A ball of chapati dough of radius 6 cm is prepared. Estimate how many chapatis can be made from it? *(ii) Cut a coconut/muskmelon in half and find out the approximate volume of edible coconut flesh/fruit by taking the necessary measurements.

SOLUTION

(i) Volume of dough ball (sphere, r = 6 cm):
$V = (4/3)\pi r^3 = (4/3) \times 3.14 \times 6^3 = (4/3) \times 3.14 \times 216 = 904.3\ cm^3$

A chapati is a thin disc. Typical dimensions:
Radius ≈ 10 cm, thickness ≈ 0.2 cm
Volume of one chapati $= \pi \times 10^2 \times 0.2 = 3.14 \times 100 \times 0.2 = 62.8\ cm^3$

Number of chapatis ≈ 904.3/62.8 ≈ 14 chapatis

Approximately 12–15 chapatis can be made (depending on thickness and radius).

*(ii) Measuring a coconut half:
Model the inner (edible) part as a hemisphere of inner radius r_1 and the outer shell as a hemisphere of outer radius r_2.
Measure r_2 (outer radius) ≈ 6 cm and r_1 (inner radius) ≈ 5 cm (shell thickness ≈ 1 cm).
Volume of edible flesh $= (2/3)\pi(r_2^3 - r_1^3) = (2/3) \times 3.14 \times (216 - 125)$
$= (2/3) \times 3.14 \times 91 \approx 190\ cm^3$ for one half.
(Actual measurements should be taken from the real fruit.)

Q18 Looking at the Ganita Manjari, Grade 9, Part 2 textbook, Sheela wonders: (i) If all the pages of this textbook were laid out side-by-side on the floor would they cover the entire classroom floor? (ii) What is the maximum number of textbooks that can fit in an empty storeroom of dimensions 15 ft × 20 ft × 30 ft?

SOLUTION

(i) Estimating page coverage:
The textbook (Part 2) has approximately 150 pages = 75 leaves (both sides).
Page size ≈ 24 cm × 18 cm = 432 cm².
Total area of all pages = 150 × 432 = 64,800 cm² ≈ 6.48 m²

A typical classroom floor area ≈ 10 m × 8 m = 80 m².
6.48 m² << 80 m².
No, the pages would cover only about 8% of the classroom floor.

(ii) Storeroom: 15 ft × 20 ft × 30 ft
Converting: 1 ft ≈ 30 cm
Storeroom volume ≈ 450 × 600 × 900 cm = 2.43×10^8 cm³
Textbook dimensions ≈ 24 cm × 18 cm × 1.5 cm (thickness), Volume ≈ 648 cm³
Maximum number = 2.43×10^8 / 648 ≈ 3,75,000 textbooks

Approximately 3–4 lakh textbooks can fit in the storeroom.

Q19 If the entire human population decided to climb into one giant cube, how long would the side have to be?

SOLUTION

World population ≈ 8×10^9 people.

Average volume per person (including some space): model a person as roughly 0.5 m × 0.4 m × 1.7 m ≈ 0.34 m³, but when packed tightly in a cube (like stacking boxes), allow about 0.065 m³ per person.

Total volume of cube needed ≈ $8 \times 10^9 \times 0.065 = 5.2 \times 10^8$ m³

Side of cube = $(5.2 \times 10^8)^{(1/3)}$
≈ $(5.2)^{(1/3)} \times 10^{(8/3)}$
≈ $1.73 \times 10^{2.67}$
≈ 1.73 × 464 ≈ 800 m

The side of the cube would need to be approximately 800 m (about 0.8 km).

Q20 The Earth's surface has an estimated volume of 1.38 billion km³ of water. Suppose the Earth is a perfect sphere and all this water forms a uniform layer completely covering the Earth's surface, like a thin water bubble. Estimate the thickness of this water layer. The Earth's radius is ~6371 km. (i) Write an expression that gives the thickness of this water layer. (ii) Simplify the expression in (i) using a calculator.

SOLUTION

Let t = thickness of water layer, R = radius of Earth = 6371 km.

(i) Volume of water = Volume of spherical shell
$V = (4/3)\pi(R + t)^3 - (4/3)\pi R^3$
Since $t << R$, we approximate: $V \approx 4\pi R^2 \times t$
So $t \approx V / (4\pi R^2)$

This is the expression for the thickness.

(ii) Substituting values: $V = 1.38 \times 10^9$ km³, R = 6371 km
$t = 1.38 \times 10^9 / (4 \times 3.14159 \times 6371^2)$
$= 1.38 \times 10^9 / (4 \times 3.14159 \times 40{,}589{,}641)$
$= 1.38 \times 10^9 / (5.1006 \times 10^8)$
≈ 2.706 km
≈ 2706 m ≈ 2.7 km

The uniform water layer would be approximately 2.7 km thick.

Q21 Give the dimension of a cuboid whose volume is halved when its surface area is doubled.

SOLUTION

This is an open-ended question: we look for a pair of cuboids where, going from the first to the second, the volume is halved while the surface area is doubled.
We need a cuboid A with volume V and surface area S, and a cuboid B with volume V/2 and surface area 2S.

A valid example:
Cuboid A = 2 cm × 2 cm × 24 cm
Volume V = 2 × 2 × 24 = 96 cm³
Surface area S = 2(2×2 + 2×24 + 2×24) = 2(4 + 48 + 48) = 200 cm²

Cuboid B = 0.25 cm × 8 cm × 24 cm
Volume = 0.25 × 8 × 24 = 48 cm³ = V/2 ✓ (halved)
Surface area = 2(0.25×8 + 8×24 + 0.25×24) = 2(2 + 192 + 6) = 400 cm² = 2S ✓ (doubled)

So from cuboid A to cuboid B the volume halves while the surface area doubles.

One such pair of dimensions is A = 2 × 2 × 24 and B = 0.25 × 8 × 24 (both in cm).

Q22 Project: Find the volume of your house making necessary approximations. Present how you solved it.

SOLUTION

This is a project/activity question. A suggested approach:

Step 1: Measure or estimate the dimensions of each room and other spaces.
Typical room: length × width × height (e.g., 4 m × 3 m × 3 m = 36 m³ per room).

Step 2: List all spaces — living room, bedrooms, kitchen, bathroom, corridors, balcony.
Example for a 2-BHK flat:

Living room: 5 × 4 × 3 = 60 m³
Bedroom 1: 4 × 3 × 3 = 36 m³
Bedroom 2: 3.5 × 3 × 3 = 31.5 m³
Kitchen: 3 × 2.5 × 3 = 22.5 m³
Bathroom: 2 × 1.5 × 3 = 9 m³
Corridor: 4 × 1 × 3 = 12 m³

Step 3: Sum all volumes.
Total ≈ 60 + 36 + 31.5 + 22.5 + 9 + 12 = 171 m³

Step 4: Subtract the volume of walls (approx. 10–15% of total) if required.
Net interior volume ≈ 150–155 m³

Actual answer will vary based on your home's dimensions.