CBSE 2020 Maths Standard — Set 1 — Answer Key

English Medium

Time Allowed: 3 Hours | Maximum Marks: 80

General Instructions:

1. This question paper contains 38 questions. All questions are compulsory.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 6 questions carrying 2 marks each.
4. Section C has 6 questions carrying 3 marks each.
5. Section D has 4 questions carrying 5 marks each.
6. Section E has 3 case-study based questions carrying 4 marks each.
7. Draw neat figures wherever required.

Section A

Multiple Choice Questions (1 mark each)

Q.1 [1 mark]

If one of the zeroes of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is

(a) 10

(b) –10 <<

(c) –7

(d) –2

Explanation:

If 2 is a zero of $x^2 + 3x + k$, then substituting $x = 2$: $(2)^2 + 3(2) + k = 0 \Rightarrow 4 + 6 + k = 0 \Rightarrow k = -10$.

Q.2 [1 mark]

The total number of factors of a prime number is

(a) 1

(b) 0

(c) 2 <<

(d) 3

Explanation:

A prime number has exactly two factors: 1 and the number itself.

So the total number of factors is 2.

Q.3 [1 mark]

The quadratic polynomial, the sum of whose zeroes is –5 and their product is 6, is

(a) $x^2 + 5x + 6$ <<

(b) $x^2 - 5x + 6$

(c) $x^2 - 5x - 6$

(d) $-x^2 + 5x + 6$

Explanation:

If sum of zeroes = –5 and product = 6, the polynomial is $x^2 - (\text{sum})x + \text{product} = x^2 - (-5)x + 6 = x^2 + 5x + 6$.

Q.4 [1 mark]

The value of k for which the system of equations $x + y - 4 = 0$ and $2x + ky = 3$, has no solution, is

(a) –2

(b) ∓2

(c) 3

(d) 2 <<

Explanation:

For no solution: $a_1/a_2 = b_1/b_2 \neq c_1/c_2$.

Here $a_1/a_2 = 1/2$, $b_1/b_2 = 1/k$.

For no solution: $1/2 = 1/k \Rightarrow k = 2$.

Check: $c_1/c_2 = -4/-3 = 4/3 \neq 1/2$.

So k = 2.

Q.5 [1 mark]

The HCF and the LCM of 12, 21, 15 respectively are

(a) 3, 140

(b) 12, 420

(c) 3, 420 <<

(d) 420, 3

Explanation:

$12 = 2^2 \times 3$, $21 = 3 \times 7$, $15 = 3 \times 5$.

HCF = 3 (common factor).

LCM = $2^2 \times 3 \times 5 \times 7 = 420$.

So HCF = 3 and LCM = 420.

Q.6 [1 mark]

The value of x for which 2x, (x + 10) and (3x + 2) are the three consecutive

terms of an AP, is

(a) 6 <<

(b) –6

(c) 18

(d) –18

Explanation:

For three consecutive terms of an AP, the middle term equals the average of the other two: $2(x + 10) = 2x + (3x + 2) \Rightarrow 2x + 20 = 5x + 2 \Rightarrow 18 = 3x \Rightarrow x = 6$.

Q.7 [1 mark]

The first term of an AP is p and the common difference is q, then its 10th term is

(a) q + 9p

(b) p − 9q

(c) p + 9q <<

(d) 2p + 9q

Explanation:

The nth term of an AP is $a + (n-1)d$.

Here $a = p$, $d = q$, $n = 10$.

So $a_{10} = p + (10-1)q = p + 9q$.

Q.8 [1 mark]

The distance between the points $(a \cos\theta + b \sin\theta, 0)$ and $(0, a \sin\theta - b \cos\theta)$, is

(a) $a^2 + b^2$

(b) $a^2 - b^2$

(c) $\sqrt{(a^2 + b^2)}$ <<

(d) $\sqrt{(a^2 - b^2)}$

Explanation:

Distance $= \sqrt{[(a \cos\theta + b \sin\theta)^2 + (a \sin\theta - b \cos\theta)^2]} = \sqrt{[a^2\cos^2\theta + 2ab \sin\theta \cos\theta +}$

$b^2\sin^2\theta + a^2\sin^2\theta - 2ab\sin\theta\cos\theta + b^2\cos^2\theta] = \sqrt{[a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)]} = \sqrt{(a^2 + b^2)}$.

Q.9 [1 mark]

If the point P(k, 0) divides the line segment joining the points A(2, −2) and B(−7, 4) in the ratio 1 : 2, then the value of k is

(a) 1

(b) 2

(c) –2

(d) –1 <<

Explanation:

By the section formula (ratio 1 : 2):

$k = (1 \times (-7) + 2 \times 2)/(1 + 2) = (-7 + 4)/3 = -1$.

Check y: $(1 \times 4 + 2 \times (-2))/3 = 0$ ✓.

So $k = -1 \rightarrow$ (d).

Q.10 [1 mark]

The value of p, for which the points A(3, 1), B(5, p) and C(7, −5) are collinear, is

(a) –2 <<

(b) 2

(c) –1

(d) 1

Explanation:

For collinear points, the area of triangle = 0.

Area $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 0$.

So $3(p - (-5)) + 5((-5) - 1) + 7(1 - p) = 0 \Rightarrow 3(p + 5) + 5(-6) + 7(1 - p) = 0 \Rightarrow 3p + 15 - 30 + 7 - 7p = 0 \Rightarrow -4p - 8 = 0 \Rightarrow p = -2$.

Q.11 [1 mark]

In Fig. 1, a triangle ABC is circumscribing a circle, the length of BC is _____ cm. (Fig. 1: the circle touches AB at P, BC at Q and AC at R; AP = 4 cm, PB

= 3 cm and AC = 11 cm.)

Answer: 10

Explanation:

Tangents drawn from an external point are equal.

From A: AR = AP = 4 cm, so CR = AC − AR = 11 − 4 = 7 cm.

From B: BQ = BP = 3 cm. From C: CQ = CR = 7 cm.

BC = BQ + QC = 3 + 7 = 10 cm.

Q.12 [1 mark]

Given ΔABC ~ ΔPQR, if AB/PQ = 1/3, then ar(ΔABC)/ar(ΔPQR) = _____

Answer: 1/9

Explanation:

For similar triangles, the ratio of their areas equals the square of the ratio of their corresponding sides.

ar(ΔABC)/ar(ΔPQR) = $(AB/PQ)^2 = (1/3)^2 = 1/9$.

Q.13 [1 mark]

ABC is an equilateral triangle of side 2a, then length of one of its altitudes is _____

Answer: $a\sqrt{3}$

Explanation:

In an equilateral triangle with side 2a, the altitude = $(\sqrt{3}/2) \times$ side = $(\sqrt{3}/2) \times 2a = a\sqrt{3}$.

Q.14 [1 mark]

(cos 80°)/(sin 10°) + cos 59° cosec 31° = _____

Answer: 2

Explanation:

cos 80° = sin 10° (complementary angles), so cos 80°/sin 10° = 1.

Also cos 59° = sin 31°, so cos 59° × cosec 31° = sin 31° × (1/sin 31°) = 1.

Therefore the answer = 1 + 1 = 2.

Q.15 [1 mark]

The value of $(\sin^2\theta + 1/(1 + \tan^2\theta)) =$ _____

Answer: 1

Explanation:

We know $1/(1 + \tan^2\theta) = 1/\sec^2\theta = \cos^2\theta$. So $\sin^2\theta + \cos^2\theta = 1$.

OR

Q.15 [1 mark]

The value of $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta) =$ _____

Answer: 1

Explanation:

$(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta) = \sec^2\theta \times (1 - \sin^2\theta) = \sec^2\theta \times \cos^2\theta = (1/\cos^2\theta) \times \cos^2\theta = 1$.

Q.16 [1 mark]

The ratio of the length of a vertical rod and the length of its shadow is 1 : √3. Find the angle of elevation of the sun at that moment.

Answer: 30°

Explanation:

Let the rod height = h and shadow length = $h\sqrt{3}$.

$\tan\theta$ = height/shadow = $h/(h\sqrt{3}) = 1/\sqrt{3}$.

So $\theta = 30°$.

Q.17 [1 mark]

Two cones have their heights in the ratio 1:3 and radii in the ratio 3:1. What is the ratio of their volumes?

Answer: 3:1

Explanation:

Volume of cone = $(1/3)\pi r^2 h$.

Ratio = $(r_1^2 \times h_1)/(r_2^2 \times h_2) = (3^2 \times 1)/(1^2 \times 3) = 9/3 = 3/1$.

So ratio is 3:1.

Q.18 [1 mark]

A letter of English alphabet is chosen at random. What is the probability that the chosen letter is a consonant?

Answer: 21/26

Explanation:

There are 26 letters in the English alphabet.

Vowels = 5 (a, e, i, o, u).

Consonants = 26 − 5 = 21.

P(consonant) = 21/26.

Q.19 [1 mark]

A die is thrown once. What is the probability of getting a number less than 3?

Answer: 1/3

Explanation:

Numbers less than 3 on a die: {1, 2}.

Total outcomes = 6.

P(less than 3) = 2/6 = 1/3.

OR

Q.19 [1 mark]

If the probability of winning a game is 0.07, what is the probability of losing it?

Answer: 0.93

Explanation:

P(losing) = 1 − P(winning) = 1 − 0.07 = 0.93.

Q.20 [1 mark]

If the mean of the first n natural numbers is 15, then find n.

Answer: 29

Explanation:

Mean of first n natural numbers = (n + 1)/2 = 15.

So n + 1 = 30, therefore n = 29.

Section B

Very Short Answer Questions (2 marks each)

Q.21 [2 marks]

Show that $(a - b)^2$, $(a^2 + b^2)$, and $(a + b)^2$ are in AP.

Answer: Proved: $(a^2 + b^2) - (a - b)^2 = 2ab = (a + b)^2 - (a^2 + b^2)$, so the terms are in AP ($d = 2ab$).

Explanation:

Three terms are in AP if the difference between consecutive terms is constant.

Second − First = $(a^2 + b^2) - (a - b)^2 = (a^2 + b^2) - (a^2 - 2ab + b^2) = 2ab$.

Third − Second = $(a + b)^2 - (a^2 + b^2) = (a^2 + 2ab + b^2) - (a^2 + b^2) = 2ab$.

Since both differences are equal (= 2ab), the three terms are in AP.

Q.22 [2 marks]

In Fig. 2, DE ∥ AC and DC ∥ AP. Prove that BE/EC = BC/CP.

Answer: Proved: BE/EC = BD/DA = BC/CP (BPT in ΔABC and ΔABP)

Explanation:

In ΔABC, DE ∥ AC, so by the Basic Proportionality Theorem: BD/DA = BE/EC … (i)

In ΔABP, DC ∥ AP, so by BPT: BD/DA = BC/CP … (ii)

From (i) and (ii): BE/EC = BC/CP. Hence proved.

OR

Q.22 [2 marks]

In Fig. 3, two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠PTQ = 2∠OPQ.

Answer: Proved: $\angle TPQ = 90° - \frac{1}{2}\angle PTQ$ and $\angle OPT = 90°$, so $\angle OPQ = \frac{1}{2}\angle PTQ$, i.e. $\angle PTQ = 2\angle OPQ$.

Explanation:

Since TP and TQ are tangents from T, TP = TQ (tangent lengths from external point are equal).

So ΔTPQ is isosceles and ∠TPQ = ∠TQP.

In ΔTPQ: ∠PTQ + ∠TPQ + ∠TQP = 180° ⇒ ∠PTQ + 2∠TPQ = 180° ⇒ ∠TPQ = (180° − ∠PTQ)/2 = 90° − ∠PTQ/2.

Also, OP ⊥ TP (radius ⊥ tangent), so ∠OPT = 90°.

∠OPQ = ∠OPT − ∠TPQ = 90° − (90° − ∠PTQ/2) = ∠PTQ/2.

Therefore ∠PTQ = 2∠OPQ. Hence proved.

Q.23 [2 marks]

The rod AC of a TV disc antenna is fixed at right angles to the wall AB and a rod CD is supporting the disc as shown in Fig. 4. If AC = 1.5 m long and CD = 3 m, find (i) tan θ (ii) sec θ + cosec θ.

Answer: tan θ = 1/√3; sec θ + cosec θ = 2 + 2/√3 = (6 + 2√3)/3

Explanation:

In right ΔCAD (∠A = 90°), θ = ∠ADC, AC = 1.5 m (opposite θ), CD = 3 m (hypotenuse).

sin θ = AC/CD = 1.5/3 = 1/2 ⇒ θ = 30°.

AD = √(3² − 1.5²) = √6.75 = 3√3/2 m.

(i) tan θ = AC/AD = 1.5 ÷ (3√3/2) = 1/√3.

(ii) sec θ + cosec θ = 2/√3 + 2 = (6 + 2√3)/3.

Q.24 [2 marks]

A piece of wire 22 cm long is bent into the form of an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle. [Use π = 22/7]

Answer: 21 cm

Explanation:

Arc length = (θ/360°) × 2πr. Here arc length = 22 cm and θ = 60°.

22 = (60/360) × 2 × (22/7) × r

22 = (1/6) × (44/7) × r

22 = (44/42) × r

22 = (22/21) × r

r = 22 × 21/22 = 21 cm.

Q.25 [2 marks]

If a number x is chosen at random from the numbers −3, −2, −1, 0, 1, 2, 3, what is the probability that $x^2 \leq 4$?

Answer: 5/7

Explanation:

Total outcomes = 7 (numbers: −3, −2, −1, 0, 1, 2, 3).

$x^2 \leq 4$ means $-2 \leq x \leq 2$.

Favourable outcomes: {−2, −1, 0, 1, 2} = 5.

$P(x^2 \leq 4) = 5/7$.

Q.26 [2 marks]

Find the mean of the following distribution:

Class: 3-5, 5-7, 7-9, 9-11, 11-13

Frequency: 5, 10, 10, 7, 8

Answer: 7.7

Explanation:

Class marks: 4, 6, 8, 10, 12.

$\Sigma f_i x_i = 5\times4 + 10\times6 + 10\times8 + 7\times10 + 8\times12 = 20 + 60 + 80 + 70 + 96 = 308$.

$\Sigma f_i = 5 + 10 + 10 + 7 + 8 = 40$.

Mean = 308/40 = 7.7.

OR

Q.26 [2 marks]

Find the mode of the following data:

Class: 0-20, 20-40, 40-60, 60-80, 80-100, 100-120, 120-140

Frequency: 6, 8, 10, 12, 6, 5, 3

Answer: 65

Explanation:

Modal class = 60-80 (highest frequency 12).

$l = 60$, $f_1 = 12$, $f_0 = 10$, $f_2 = 6$, $h = 20$.

Mode $= l + [(f_1 - f_0)/(2f_1 - f_0 - f_2)] \times h = 60 + [(12 - 10)/(24 - 10 - 6)] \times 20 = 60 + [2/8] \times 20 = 60 + 5 = 65$.

Section C

Short Answer Questions (3 marks each)

Q.27 [3 marks]

Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $f(x) = ax^2 + bx + c$, $a \neq 0$, $c \neq 0$.

Answer: $cx^2 + bx + a$

Explanation:

Let α, β be zeroes of $ax^2 + bx + c$. Then $\alpha + \beta = -b/a$, $\alpha\beta = c/a$.

Reciprocal zeroes are $1/\alpha$ and $1/\beta$.

Sum $= 1/\alpha + 1/\beta = (\alpha + \beta)/(\alpha\beta) = (-b/a)/(c/a) = -b/c$.

Product $= 1/(\alpha\beta) = a/c$.

Quadratic polynomial $= x^2 - (\text{sum})x + \text{product} = x^2 + (b/c)x + a/c = (1/c)(cx^2 + bx + a)$.

So the polynomial is $cx^2 + bx + a$.

OR

Q.27 [3 marks]

Divide the polynomial $f(x) = 3x^2 - x^3 - 3x + 5$ by the polynomial $g(x) = x - 1 - x^2$ and verify the division algorithm.

Answer: Quotient $= x - 2$, Remainder $= 3$

Explanation:

Rearranging: $f(x) = -x^3 + 3x^2 - 3x + 5$, $g(x) = -x^2 + x - 1$.

Dividing $-x^3 + 3x^2 - 3x + 5$ by $-x^2 + x - 1$:

First term: $-x^3/(-x^2) = x$.

$x \times (-x^2 + x - 1) = -x^3 + x^2 - x$.

Subtracting: $(3x^2 - x^2) + (-3x + x) + 5 = 2x^2 - 2x + 5$.

Next term: $2x^2/(-x^2) = -2$.

$-2 \times (-x^2 + x - 1) = 2x^2 - 2x + 2$.

Subtracting: $(2x^2 - 2x + 5) - (2x^2 - 2x + 2) = 3$.

Quotient = $x - 2$, Remainder = 3.

Verification: $g(x) \times q(x) + r = (-x^2 + x - 1)(x - 2) + 3 = -x^3 + 2x^2 + x^2 - 2x - x + 2 + 3 = -x^3 + 3x^2 - 3x + 5 = f(x)$.

Verified.

Q.28 [3 marks]

Determine graphically the coordinates of the vertices of a triangle, the equations of whose sides are given by $2y - x = 8$, $5y - x = 14$ and $y - 2x = 1$.

Answer: Vertices: (2, 5), (4, 3), (−4, 2) approximately

Explanation:

Solving the equations pairwise:

From $2y - x = 8$ and $5y - x = 14$: Subtracting: $3y = 6 \Rightarrow y = 2$, $x = 2(2) - 8 = -4$.

Vertex: (−4, 2).

From $2y - x = 8$ and $y - 2x = 1$: From second: $y = 2x + 1$.

Substituting: $2(2x + 1) - x = 8 \Rightarrow 4x + 2 - x = 8 \Rightarrow 3x = 6 \Rightarrow x = 2$, $y = 5$.

Vertex: (2, 5).

From $5y - x = 14$ and $y - 2x = 1$: From second: $y = 2x + 1$.

Substituting: $5(2x + 1) - x = 14 \Rightarrow 10x + 5 - x = 14 \Rightarrow 9x = 9 \Rightarrow x = 1$, $y = 3$.

Vertex: (1, 3).

The three vertices of the triangle are (−4, 2), (2, 5), and (1, 3).

OR

Q.28 [3 marks]

If 4 is a zero of the cubic polynomial $x^3 - 3x^2 - 10x + 24$, find its other two zeroes.

Answer: −3 and 2

Explanation:

Since 4 is a zero, $(x - 4)$ is a factor.

Dividing $x^3 - 3x^2 - 10x + 24$ by $(x - 4)$:

$x^3 - 3x^2 - 10x + 24 = (x - 4)(x^2 + x - 6)$.

Factoring $x^2 + x - 6 = (x + 3)(x - 2)$.

So the other two zeroes are $x = -3$ and $x = 2$.

Q.29 [3 marks]

In a flight of 600 km, an aircraft was slowed due to bad weather. Its average speed for the trip was reduced by 200 km/hr and time of flight increased by 30 minutes. Find the original duration of flight.

Answer: 1 hour

Explanation:

Let original speed = x km/hr. Then original time = 600/x hours.

New speed = $(x - 200)$ km/hr. New time = $600/(x - 200)$ hours.

Given: $600/(x - 200) - 600/x = 1/2$ (30 min = 1/2 hr)

$600[x - (x - 200)] / [x(x - 200)] = 1/2$

$600 \times 200 / [x(x - 200)] = 1/2$

$120000 \times 2 = x(x - 200)$

$x^2 - 200x - 240000 = 0$

Using quadratic formula: $x = (200 \pm \sqrt{(40000 + 960000)})/2 = (200 \pm \sqrt{1000000})/2 = (200 \pm 1000)/2$.

$x = 600$ (taking positive value).

Original duration = 600/600 = 1 hour.

Q.30 [3 marks]

Find the area of triangle PQR formed by the points P(−5, 7), Q(−4, −5) and R(4, 5).

Answer: 53 sq. units

Explanation:

Area = $\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$= \frac{1}{2}|(-5)(-5 - 5) + (-4)(5 - 7) + 4(7 - (-5))|$

= ½|(−5)(−10) + (−4)(−2) + 4(12)|

= ½|50 + 8 + 48|

= ½ × 106 = 53 sq. units.

OR

Q.30 [3 marks]

If the point C(−1, 2) divides internally the line segment joining A(2, 5) and B(x, y) in the ratio 3:4, find the coordinates of B.

Answer: (−5, −2)

Explanation:

Using section formula for internal division:

−1 = (3x + 4×2)/(3 + 4) ⇒ −7 = 3x + 8 ⇒ 3x = −15 ⇒ x = −5.

2 = (3y + 4×5)/(3 + 4) ⇒ 14 = 3y + 20 ⇒ 3y = −6 ⇒ y = −2.

So B = (−5, −2).

Q.31 [3 marks]

In Fig. 5, ∠D = ∠E and AD/DB = AE/EC, prove that BAC is an isosceles triangle.

Answer: Proved: DE ∥ BC (converse of BPT) ⇒ ∠B = ∠D = ∠E = ∠C ⇒ AB = AC, so ΔBAC is isosceles.

Explanation:

Given: ∠D = ∠E and AD/DB = AE/EC.

Since AD/DB = AE/EC, by converse of Basic Proportionality Theorem, DE ∥ BC.

Since DE ∥ BC: ∠ADE = ∠ABC (corresponding angles) and ∠AED = ∠ACB (corresponding angles).

But ∠ADE = ∠D and ∠AED = ∠E, and given ∠D = ∠E.

Therefore ∠ABC = ∠ACB.

In ΔABC, since ∠ABC = ∠ACB, the sides opposite to equal angles are equal: AB = AC.

Hence ΔBAC is an isosceles triangle.

Q.32 [3 marks]

In a triangle, if the square of one side is equal to the sum of the squares of

the other two sides, then prove that the angle opposite to the first side is a right angle.

Answer: Proved: construct ΔPQR with ∠Q = 90°, PQ = AB, QR = BC; then PR = AC, so ΔABC ≅ ΔPQR (SSS) and ∠B = ∠Q = 90°.

Explanation:

Given: In ΔABC, $AC^2 = AB^2 + BC^2$. To prove: ∠B = 90°.

Construction: Draw a ΔPQR with ∠Q = 90°, PQ = AB, QR = BC.

In ΔPQR (by Pythagoras theorem): $PR^2 = PQ^2 + QR^2 = AB^2 + BC^2$.

But given $AC^2 = AB^2 + BC^2$. Therefore $PR^2 = AC^2 \Rightarrow PR = AC$.

Now in ΔABC and ΔPQR: AB = PQ, BC = QR, AC = PR.

By SSS congruence: ΔABC ≅ ΔPQR.

Therefore ∠B = ∠Q = 90°. Hence proved.

Section D

Long Answer Questions (5 marks each)

Q.33 [5 marks]

If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

Answer: Proved: squaring gives $\sin\theta\cos\theta = 1$, so $\tan\theta + \cot\theta = 1/(\sin\theta\cos\theta) = 1$.

Explanation:

Given: $\sin\theta + \cos\theta = \sqrt{3}$.

Squaring both sides: $\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$.

$1 + 2\sin\theta\cos\theta = 3 \Rightarrow 2\sin\theta\cos\theta = 2 \Rightarrow \sin\theta\cos\theta = 1$.

Now $\tan\theta + \cot\theta = \sin\theta/\cos\theta + \cos\theta/\sin\theta = (\sin^2\theta + \cos^2\theta)/(\sin\theta\cos\theta) = 1/1 = 1$.

Hence proved.

Q.34 [5 marks]

A cone of base radius 4 cm is divided into two parts by drawing a plane through the mid-point of its height and parallel to its base. Compare the volumes of the two parts.

Answer: 1:7

Explanation:

Let the height of the original cone = h and radius = 4 cm.

The plane at h/2 creates a smaller cone on top with height h/2.

By similar triangles, the radius of the smaller cone = 4 × (h/2)/h = 2 cm.

Volume of smaller cone = (1/3)π(2)²(h/2) = (1/3)π × 4 × h/2 = 2πh/3.

Volume of original cone = (1/3)π(4)²h = 16πh/3.

Volume of frustum = 16πh/3 − 2πh/3 = 14πh/3.

Ratio of smaller cone to frustum = (2πh/3) : (14πh/3) = 2:14 = 1:7.

Q.35 [5 marks]

Show that the square of any positive integer cannot be of the form (5q + 2) or (5q + 3) for any integer q.

Answer: Proved: for a = 5m, 5m ± 1, 5m ± 2, a^2 is of the form 5q, 5q + 1 or 5q + 4 only — never 5q + 2 or 5q + 3.

Explanation:

By Euclid's division lemma, any positive integer a can be written as a = 5m + r, where r = 0, 1, 2, 3, or 4.

Case 1: a = 5m ⇒ $a^2 = 25m^2 = 5(5m^2) = 5q$, where $q = 5m^2$.

Case 2: a = 5m + 1 ⇒ $a^2 = 25m^2 + 10m + 1 = 5(5m^2 + 2m) + 1 = 5q + 1$.

Case 3: a = 5m + 2 ⇒ $a^2 = 25m^2 + 20m + 4 = 5(5m^2 + 4m) + 4 = 5q + 4$.

Case 4: a = 5m + 3 ⇒ $a^2 = 25m^2 + 30m + 9 = 5(5m^2 + 6m + 1) + 4 = 5q + 4$.

Case 5: a = 5m + 4 ⇒ $a^2 = 25m^2 + 40m + 16 = 5(5m^2 + 8m + 3) + 1 = 5q + 1$.

So a^2 can only be of the form 5q, 5q + 1, or 5q + 4.

It cannot be of the form 5q + 2 or 5q + 3.

OR

Q.35 [5 marks]

Prove that one of every three consecutive positive integers is divisible by 3.

Answer: Proved: writing n = 3q, 3q + 1 or 3q + 2, one of n, n + 1, n + 2 is always a multiple of 3.

Explanation:

Let the three consecutive positive integers be n, n+1, n+2.

By Euclid's division lemma, n = 3q + r, where r = 0, 1, or 2.

Case 1: r = 0, n = 3q. Then n is divisible by 3.

Case 2: r = 1, n = 3q + 1.

Then n + 2 = 3q + 3 = 3(q + 1), which is divisible by 3.

Case 3: r = 2, n = 3q + 2.

Then n + 1 = 3q + 3 = 3(q + 1), which is divisible by 3.

In each case, one of n, n+1, n+2 is divisible by 3. Hence proved.

Q.36 [5 marks]

The sum of four consecutive numbers in AP is 32 and the ratio of the product of the first and last terms to the product of two middle terms is 7:15. Find the numbers.

Answer: 2, 6, 10, 14 or 14, 10, 6, 2

Explanation:

Let the four consecutive AP terms be $a - 3d$, $a - d$, $a + d$, $a + 3d$.

Sum = $4a = 32 \Rightarrow a = 8$.

Product of first and last = $(8 - 3d)(8 + 3d) = 64 - 9d^2$.

Product of two middle = $(8 - d)(8 + d) = 64 - d^2$.

Given ratio: $(64 - 9d^2)/(64 - d^2) = 7/15$.

$15(64 - 9d^2) = 7(64 - d^2)$

$960 - 135d^2 = 448 - 7d^2$

$512 = 128d^2$

$d^2 = 4 \Rightarrow d = \pm 2$.

For d = 2: numbers are 2, 6, 10, 14.

For d = −2: numbers are 14, 10, 6, 2.

OR

Q.36 [5 marks]

Solve: 1 + 4 + 7 + 10 + ... + x = 287.

Answer: x = 40

Explanation:

This is an AP with a = 1, d = 3. Let x = a + (n−1)d = 1 + 3(n−1) = 3n − 2.

Sum = n/2 × (first + last) = n/2 × (1 + 3n − 2) = n(3n − 1)/2 = 287.

n(3n − 1) = 574.

$3n^2 - n - 574 = 0$.

Using quadratic formula: n = (1 ± √(1 + 6888))/6 = (1 ± √6889)/6 = (1 ± 83)/6.

n = 84/6 = 14 (taking positive value).

x = 3(14) − 2 = 40.

Section E

Case Study Based Questions (4 marks each)

Q.37 [4 marks]

Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as centre, draw another circle of radius 2 cm. Construct tangents to each circle from the centre of the other circle.

Answer: Construction: tangents from B to the 3 cm circle (length √40 ≈ 6.3 cm) and from A to the 2 cm circle (length √45 ≈ 6.7 cm), using the circle on AB as diameter.

Explanation:

Steps of construction:

1. Draw line segment AB = 7 cm.
2. With A as centre and radius 3 cm, draw a circle.
3. With B as centre and radius 2 cm, draw a circle.
4. Find midpoint M of AB.
5. With M as centre and MA as radius, draw a circle intersecting the circle with centre A at points P and Q.
6. Join BP and BQ. These are tangents from B to the circle with centre A.
7. With M as centre and MB as radius, draw a circle intersecting the circle with centre B at points R and S.
8. Join AR and AS. These are tangents from A to the circle with centre B.

Verification: Tangent from B to circle A: length = $\sqrt{(AB^2 - r^2)} = \sqrt{(49 - 9)} = \sqrt{40} = 2\sqrt{10}$ cm.

Tangent from A to circle B: length = $\sqrt{(49 - 4)} = \sqrt{45} = 3\sqrt{5}$ cm.

Q.38 [4 marks]

A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff of height 6 m. At a point on the plane, the angle of elevation of the bottom and top of the flag-staff are 30° and 45° respectively. Find the height of the tower. (Take $\sqrt{3} = 1.73$)

Answer: $3(\sqrt{3} + 1)$ m = 8.19 m

Explanation:

Let height of tower = h m, and distance of point from base = d m.

$\tan 30° = h/d \Rightarrow d = h\sqrt{3}$.

$\tan 45° = (h + 6)/d \Rightarrow d = h + 6$.

So $h\sqrt{3} = h + 6 \Rightarrow h(\sqrt{3} - 1) = 6 \Rightarrow h = 6/(\sqrt{3} - 1)$.

Rationalizing: $h = 6(\sqrt{3} + 1)/((\sqrt{3})^2 - 1^2) = 6(\sqrt{3} + 1)/2 = 3(\sqrt{3} + 1)$.

$h = 3(1.73 + 1) = 3 \times 2.73 = 8.19$ m.

Q.39 [4 marks]

A bucket in the form of a frustum of a cone of height 30 cm with radii of its lower and upper ends as 10 cm and 20 cm, respectively. Find the capacity of the bucket. Also find the cost of milk which can completely fill the bucket at the rate of Rs. 40 per litre. [Use $\pi = 22/7$]

Answer: Capacity = 22000 cm^3 = 22 litres, Cost = Rs. 880

Explanation:

Volume of frustum = $(\pi h/3)(R^2 + r^2 + Rr)$, where R = 20 cm, r = 10 cm, h = 30 cm.

$V = (22/7 \times 30/3)(20^2 + 10^2 + 20 \times 10)$

$= (22/7 \times 10)(400 + 100 + 200)$

$= (220/7)(700)$

$= 220 \times 100 = 22000\ cm^3$.

Capacity = 22000 cm^3 = 22000/1000 = 22 litres.

Cost = 22 × 40 = Rs. 880.

Q.40 [4 marks]

The following table gives production yield per hectare (in quintals) of wheat of 100 farms of a village:

Production yield (quintals/hectare): 40-45, 45-50, 50-55, 55-60, 60-65, 65-70
Number of farms: 4, 6, 16, 20, 30, 24

Change the distribution to a 'more than' type distribution and draw its ogive.

Answer: More than 40: 100, 45: 96, 50: 90, 55: 74, 60: 54, 65: 24 — plot (40, 100), (45, 96), (50, 90), (55, 74), (60, 54), (65, 24) and join for the ogive.

Explanation:

'More than' type distribution:

More than 40: 4 + 6 + 16 + 20 + 30 + 24 = 100

More than 45: 6 + 16 + 20 + 30 + 24 = 96

More than 50: 16 + 20 + 30 + 24 = 90

More than 55: 20 + 30 + 24 = 74

More than 60: 30 + 24 = 54

More than 65: 24

Plot the points (40, 100), (45, 96), (50, 90), (55, 74), (60, 54), (65, 24) on a graph and join them with a smooth curve to get the 'more than' ogive.

OR

Q.40 [4 marks]

The median of the following data is 525. Find the values of x and y, if the total frequency is 100.

Class: 0-100, 100-200, 200-300, 300-400, 400-500, 500-600, 600-700, 700-800, 800-900, 900-1000
Frequency: 2, 5, x, 12, 17, 20, y, 9, 7, 4

Answer: x = 9, y = 15

Explanation:

Total frequency = 2 + 5 + x + 12 + 17 + 20 + y + 9 + 7 + 4 = 100.

76 + x + y = 100 ⇒ x + y = 24 ... (i)

Median = 525, which lies in class 500-600.

l = 500, f = 20, cf = 2 + 5 + x + 12 + 17 = 36 + x, h = 100, N/2 = 50.

Median = l + [(N/2 − cf)/f] × h

525 = 500 + [(50 − 36 − x)/20] × 100

25 = (14 − x) × 5

5 = 14 − x

x = 9.

From (i): y = 24 − 9 = 15.