CBSE 2022 Maths Standard (Term 2) — Set 3 — Answer Key

English Medium

Time Allowed: 3 Hours | Maximum Marks: 80

General Instructions:

1. This question paper contains 38 questions. All questions are compulsory.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 6 questions carrying 2 marks each.
4. Section C has 6 questions carrying 3 marks each.
5. Section D has 4 questions carrying 5 marks each.
6. Section E has 3 case-study based questions carrying 4 marks each.
7. Draw neat figures wherever required.

Section A

Multiple Choice Questions (1 mark each)

Q.1 [1 mark]

Solve the quadratic equation for x: x^2 - 2ax - (4b^2 - a^2) = 0

Answer: x = (a + 2b) or x = (a - 2b)

Explanation:

Step 1: x^2 - 2ax - (4b^2 - a^2) = 0.

Step 2: Rewrite as x^2 - 2ax + a^2 - 4b^2 = 0.

Step 3: (x - a)^2 - (2b)^2 = 0.

Step 4: Using difference of squares: (x - a - 2b)(x - a + 2b) = 0.

Step 5: x = a + 2b or x = a - 2b.

OR

Q.1 [1 mark]

If the quadratic equation (1 + a^2)x^2 + 2abx + (b^2 - c^2) = 0 has equal and real roots, then prove that: b^2 = c^2(1 + a^2)

Answer: Proof: for equal roots, discriminant = 0. b^2 = c^2(1 + a^2).

Explanation:

Step 1: For equal roots, discriminant D = 0.

Step 2: D = (2ab)^2 - 4(1 + a^2)(b^2 - c^2) = 0.

Step 3: 4a^2b^2 = 4(1 + a^2)(b^2 - c^2).

Step 4: a^2b^2 = b^2 - c^2 + a^2b^2 - a^2c^2.

Step 5: 0 = b^2 - c^2 - a^2c^2.

Step 6: c^2 + a^2c^2 = b^2.

Step 7: c^2(1 + a^2) = b^2. Hence proved.

Q.2 [1 mark]

Find the sum of first 20 terms of an AP in which d = 5 and a_20 = 135.

Answer: 1750

Explanation:

Step 1: $a_{20} = a + 19d$. So $135 = a + 19(5) = a + 95$.

Step 2: $a = 135 - 95 = 40$.

Step 3: $S_{20} = (n/2)(a + a_n) = (20/2)(40 + 135) = 10 \times 175 = 1750$.

The sum of the first 20 terms is 1750.

Q.3 [1 mark]

Find the mode of the given frequency distribution:

Class: 15-25, 25-35, 35-45, 45-55, 55-65, 65-75

Frequency: 6, 11, 22, 23, 14, 5

Answer: 46

Explanation:

Step 1: The highest frequency is 23, so the modal class is 45-55.

Step 2: $l = 45$, $f_1 = 23$ (modal class frequency), $f_0 = 22$ (preceding class frequency), $f_2 = 14$ (succeeding class frequency), $h = 10$.

Step 3: Mode $= l + [(f_1 - f_0)/(2f_1 - f_0 - f_2)] \times h$.

Step 4: Mode $= 45 + [(23 - 22)/(2\times23 - 22 - 14)] \times 10$.

Step 5: Mode $= 45 + [1/10] \times 10 = 45 + 1 = 46$.

The mode of the distribution is 46.

Q.4 [1 mark]

150 spherical marbles, each of diameter 1.4 cm, are dropped in a cylindrical vessel of diameter 7 cm containing some water, and are completely immersed in water. Find the rise in the level of water in the cylindrical vessel.

Answer: 5.6 cm

Explanation:

Step 1: Radius of each marble $= 1.4/2 = 0.7$ cm.

Step 2: Volume of one marble $= (4/3)\pi(0.7)^3 = (4/3)\pi(0.343) = 1.372\pi/3$ cm^3.

Step 3: Volume of 150 marbles $= 150 \times (4/3)\pi(0.7)^3 = 150 \times (4/3)\pi \times 0.343 = 200 \times \pi \times 0.343 = 68.6\pi$ cm^3.

Step 4: Radius of cylinder = 7/2 = 3.5 cm.

Step 5: Volume of water displaced = π(3.5)^2 × h = 12.25πh.

Step 6: 12.25πh = 68.6π.

Step 7: h = 68.6/12.25 = 5.6 cm.

The rise in water level is 5.6 cm.

OR

Q.4 [1 mark]

Three cubes of side 6 cm each, are joined end to end as shown. Find the total surface area of the resulting cuboid.

Answer: 504 sq cm

Explanation:

Step 1: When three cubes of side 6 cm are joined end to end, the resulting cuboid has dimensions: length = 3 × 6 = 18 cm, breadth = 6 cm, height = 6 cm.

Step 2: Total surface area of cuboid = 2(lb + bh + lh).

Step 3: = 2(18×6 + 6×6 + 18×6).

Step 4: = 2(108 + 36 + 108).

Step 5: = 2 × 252 = 504 sq cm.

The total surface area is 504 sq cm.

Q.5 [1 mark]

For what value of 'n', are the nth terms of the APs: 9, 7, 5, ... and 15, 12, 9, ... the same?

Answer: n = 7

Explanation:

Step 1: For the first AP: a = 9, d = 7 - 9 = -2.

nth term = 9 + (n-1)(-2) = 9 - 2n + 2 = 11 - 2n.

Step 2: For the second AP: a = 15, d = 12 - 15 = -3.

nth term = 15 + (n-1)(-3) = 15 - 3n + 3 = 18 - 3n.

Step 3: Setting them equal: 11 - 2n = 18 - 3n.

Step 4: 3n - 2n = 18 - 11.

Step 5: n = 7.

The 7th terms of both APs are the same.

Q.6 [1 mark]

In Figure 2, PQ and PR are tangents to the circle centred at O. If ∠OPR = 45°, then prove that ORPQ is a square.

Answer: Proved: ORPQ is a square (all angles 90°, all sides equal)

Explanation:

OQ ⊥ PQ and OR ⊥ PR (radius ⊥ tangent), so ∠OQP = ∠ORP = 90°.

OP bisects ∠QPR (tangents from an external point), so ∠QPR = 2 × 45° = 90°.

Angle sum of quadrilateral ORPQ: ∠QOR = 360° − 90° − 90° − 90° = 90°.

So ORPQ is a rectangle.

In △ORP, ∠ORP = 90° and ∠OPR = 45°, so ∠POR = 45° and OR = RP.

Also OR = OQ (radii) and PR = PQ (tangents from P), so all sides are equal.

A rectangle with all sides equal is a square. Hence ORPQ is a square.

Q.7 [1 mark]

Draw a line segment AB of length 8 cm and locate a point P on AB such that AP : PB = 1 : 5.

Answer: AP = 4/3 cm, PB = 20/3 cm. Construction divides AB in ratio 1:5.

Explanation:

Construction steps:

Step 1: Draw a line segment AB = 8 cm.

Step 2: Draw a ray AX making an acute angle with AB.

Step 3: Mark 6 equal arcs (1 + 5 = 6) on AX: A1, A2, A3, A4, A5, A6.

Step 4: Join A6 to B.

Step 5: Through A1, draw a line parallel to A6B (using compass), meeting AB at point P.

Step 6: P divides AB such that AP : PB = 1 : 5.

Verification: AP = (1/6) × 8 = 4/3 cm ≈ 1.33 cm, PB = (5/6) × 8 = 20/3 cm ≈ 6.67 cm.

OR

Q.7 [1 mark]

Draw a circle of radius 3 cm. From a point P lying outside the circle at a distance of 6 cm from its centre, construct two tangents PA and PB to the circle.

Answer: Two tangents PA = PB = $3\sqrt{3}$ cm ≈ 5.20 cm constructed.

Explanation:

Construction steps:

Step 1: Draw a circle of radius 3 cm with centre O.

Step 2: Mark a point P such that OP = 6 cm.

Step 3: Find the midpoint M of OP (bisect OP).

Step 4: Draw a circle with centre M and radius MO = MP = 3 cm.

Step 5: Let this circle intersect the original circle at points A and B.

Step 6: Join PA and PB. These are the required tangents.

Verification: Length of tangent = $\sqrt{(OP^2 - r^2)} = \sqrt{(36 - 9)} = \sqrt{27} = 3\sqrt{3}$ cm ≈ 5.20 cm.

Justification: Angle OAP = 90° (angle in a semicircle of the circle with diameter OP), so PA ⊥ OA, confirming PA is a tangent.

Q.8 [1 mark]

The tops of two poles of heights 20 m and 28 m are connected with a wire. The wire is inclined to the horizontal at an angle of 30°. Find the length of the wire and the distance between the two poles.

Answer: Length of wire = 16 m, Distance = $8\sqrt{3}$ m

Explanation:

Step 1: The difference in heights of the two poles = 28 - 20 = 8 m.

Step 2: Let the length of the wire be L.

The wire makes an angle of 30° with the horizontal.

Step 3: sin 30° = (difference in height)/L = 8/L.

Step 4: 1/2 = 8/L, so L = 16 m.

Step 5: For the distance between poles: cos 30° = distance/L.

Step 6: $\sqrt{3}/2$ = distance/16.

Step 7: distance = $16 \times \sqrt{3}/2 = 8\sqrt{3}$ m ≈ 13.86 m.

The length of the wire is 16 m and the distance between the poles is $8\sqrt{3}$ m.

Q.9 [1 mark]

The weights (in kg) of 50 wild animals of a National Park were recorded and the following data was obtained:

Weight (kg): 100-110, 110-120, 120-130, 130-140, 140-150
Number of animals: 4, 12, 23, 8, 3

Find the mean weight (in kg) of animals, using assumed mean method.

Answer: 123.8 kg

Explanation:

Step 1: Let assumed mean a = 125 (midpoint of class 120-130).
Step 2: Midpoints (x_i): 105, 115, 125, 135, 145.
Step 3: Deviations $d_i = x_i - a$: -20, -10, 0, 10, 20.
Step 4: $f_i \times d_i$: 4×(-20) = -80, 12×(-10) = -120, 23×0 = 0, 8×10 = 80, 3×20 = 60.
Step 5: $\Sigma(f_i \times d_i) = -80 - 120 + 0 + 80 + 60 = -60$.
Step 6: $\Sigma f_i = 50$.
Step 7: Mean = $a + \Sigma(f_i \times d_i)/\Sigma f_i = 125 + (-60)/50 = 125 - 1.2 = 123.8$ kg.
The mean weight is 123.8 kg.

Q.10 [1 mark]

For the following frequency distribution, find the median:

Class: 1400-1550, 1550-1700, 1700-1850, 1850-2000
Frequency: 6, 13, 25, 10

Answer: 1748

Explanation:

Step 1: Total frequency N = 6 + 13 + 25 + 10 = 54. N/2 = 27.
Step 2: Cumulative frequencies: 6, 19, 44, 54.
Step 3: The median class is 1700-1850 (since cumulative frequency first exceeding 27 is 44).
Step 4: l = 1700, f = 25, cf = 19 (cumulative frequency before median class), h = 150.

Step 5: Median = l + [(N/2 - cf)/f] × h.

Step 6: Median = 1700 + [(27 - 19)/25] × 150 = 1700 + (8/25) × 150 = 1700 + 48 = 1748.

The median is 1748.

Q.11 [1 mark]

In Figure 3, two circles with centres at O and O' of radii 2r and r respectively, touch each other internally at A. A chord AB of the bigger circle meets the smaller circle at C. Show that C bisects AB.

Answer: Proved: OC ⊥ AB, so C is the midpoint of AB

Explanation:

The circles touch internally at A, so O, O' and A are collinear.

OA = 2r and O'A = r, so O'O = r: O lies on the smaller circle and AO is its diameter.

C lies on the smaller circle, so ∠ACO = 90° (angle in a semicircle).

Hence OC ⊥ AB.

AB is a chord of the bigger circle and OC is the perpendicular from its centre O to the chord, so it bisects the chord.

Therefore AC = CB, i.e. C bisects AB.

OR

Q.11 [1 mark]

In Figure 4, O is centre of a circle of radius 5 cm. PA and BC are tangents to the circle at A and B respectively. If OP = 13 cm, then find the length of tangents PA and BC.

Answer: PA = 12 cm, BC = 12 cm

Explanation:

Step 1: OA = 5 cm (radius), OP = 13 cm.

Step 2: Since PA is tangent to circle at A, angle OAP = 90°.

Step 3: In right triangle OAP: $PA^2 = OP^2 - OA^2 = 169 - 25 = 144$.

Step 4: PA = 12 cm.

Step 5: For tangent BC at B: OB = 5 cm (radius), angle OBP = 90°.

Step 6: Since OB = OA (radii) and both P and C are external points from which

tangents are drawn to the same circle, and from the figure both tangents originate from the same external point (C = P by the symmetry of Figure 4): OC = OP = 13 cm.
Step 7: $BC^2 = OC^2 - OB^2 = 169 - 25 = 144$.
Step 8: BC = 12 cm.
Therefore, PA = BC = 12 cm.

Q.12 [1 mark]

A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Ten seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Answer: 5 seconds

Explanation:
Step 1: Let the height of the tower be h, and the initial distance of the car from the foot be d.
Step 2: From the first observation: $\tan 30° = h/d$, so $d = h/\tan 30° = h\sqrt{3}$.
Step 3: After 10 seconds, let the car be at distance x from the foot: $\tan 60° = h/x$, so $x = h/\tan 60° = h/\sqrt{3}$.
Step 4: Distance covered in 10 seconds $= d - x = h\sqrt{3} - h/\sqrt{3} = h(\sqrt{3} - 1/\sqrt{3}) = h(3-1)/\sqrt{3} = 2h/\sqrt{3}$.
Step 5: Speed of car $= (2h/\sqrt{3})/10 = 2h/(10\sqrt{3}) = h/(5\sqrt{3})$.
Step 6: Time to cover remaining distance $x = h/\sqrt{3}$ from the foot:
Step 7: Time = x/speed $= (h/\sqrt{3})/(h/(5\sqrt{3})) = (h/\sqrt{3}) \times (5\sqrt{3}/h) = 5$ seconds.
The car takes 5 seconds to reach the foot of the tower.

Q.13(a) [1 mark]

Case Study: A rectangular in-ground swimming pool is installed by a family in their backyard. There is a concrete sidewalk around the pool of width x m. The outside edges of the sidewalk measure 7 m and 12 m. The area of the pool is 36 sq m.

(a) Based on the information, form a quadratic equation in terms of x.

Answer: $2x^2 - 19x + 24 = 0$

Explanation:

Step 1: The outside dimensions of the sidewalk are 7 m and 12 m.

Step 2: The pool dimensions are: length = (12 - 2x) m and width = (7 - 2x) m (subtracting sidewalk width from both sides).

Step 3: Area of pool = $(12 - 2x)(7 - 2x) = 36$.

Step 4: $84 - 24x - 14x + 4x^2 = 36$.

Step 5: $4x^2 - 38x + 84 - 36 = 0$.

Step 6: $4x^2 - 38x + 48 = 0$.

Step 7: Dividing by 2: $2x^2 - 19x + 24 = 0$.

The quadratic equation is $2x^2 - 19x + 24 = 0$.

Q.13(b) [1 mark]

Case Study: A rectangular in-ground swimming pool is installed by a family in their backyard. There is a concrete sidewalk around the pool of width x m. The outside edges of the sidewalk measure 7 m and 12 m. The area of the pool is 36 sq m.

(b) Find the width of the sidewalk around the pool.

Answer: 1.5 m

Explanation:

Step 1: From part (a), the equation is $2x^2 - 19x + 24 = 0$.

Step 2: Using the quadratic formula: $x = [19 \pm \sqrt{(361 - 192)}]/4 = [19 \pm \sqrt{169}]/4 = [19 \pm 13]/4$.

Step 3: $x = (19 + 13)/4 = 32/4 = 8$ or $x = (19 - 13)/4 = 6/4 = 1.5$.

Step 4: $x = 8$ is not possible since the outside width is only 7 m.

Step 5: Therefore $x = 1.5$ m.

The width of the sidewalk is 1.5 m.

Q.14(a) [1 mark]

Case Study: John planned a birthday party for his younger sister with his friends. They decided to make some birthday caps and to buy a cake from a bakery shop. The dimensions are:
Cake: Cylindrical shape with diameter 24 cm and height 14 cm.
Cap: Conical shape with base circumference 44 cm and height 24 cm.

(a) How many square cm paper would be used to make 4 such caps?

Answer: 2200 sq cm

Explanation:

Step 1: Base circumference of cap = 2πr = 44 cm.
Step 2: r = 44/(2π) = 44/(2 × 22/7) = 44 × 7/44 = 7 cm.
Step 3: Height h = 24 cm, radius r = 7 cm.
Step 4: Slant height l = √(r^2 + h^2) = √(49 + 576) = √625 = 25 cm.
Step 5: Curved surface area of one cap = πrl = (22/7) × 7 × 25 = 22 × 25 = 550 sq cm.
Step 6: Paper for 4 caps = 4 × 550 = 2200 sq cm.
The paper required for 4 caps is 2200 sq cm.

Q.14(b) [1 mark]

Case Study: John planned a birthday party for his younger sister with his friends. They decided to make some birthday caps and to buy a cake from a bakery shop. The dimensions are:
Cake: Cylindrical shape with diameter 24 cm and height 14 cm.
Cap: Conical shape with base circumference 44 cm and height 24 cm.

The bakery shop sells cakes by weight (0.5 kg, 1 kg, 1.5 kg, etc.). To have the required dimensions, how much cake should they order, if 650 cm^3 equals 100 g of cake?

Answer: 1 kg

Explanation:

Step 1: Diameter of cake = 24 cm, so radius = 12 cm. Height = 14 cm.

Step 2: Volume of cake = πr^2h = (22/7) × 144 × 14 = 22 × 144 × 2 = 6336 cm^3.

Step 3: Given 650 cm^3 = 100 g.

Step 4: Weight of cake = (6336 × 100)/650 = 633600/650 ≈ 975.4 g ≈ 0.975 kg.

Step 5: Since cakes are sold in 0.5 kg increments (0.5 kg, 1 kg, 1.5 kg, ...), they need to round up.

Step 6: The next available size above 0.975 kg is 1 kg.

They should order a 1 kg cake.