**Series : EF3GH**

**SET~3**

रोल नं.
**Roll No.**

प्रश्न-पत्र कोड
**Q.P. Code 30/3/3**

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।
Candidates must write the Q.P. Code on the title page of the answer-book.

**नोट**

(I) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं।

(II) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।

(III) कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं।

(IV) **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें।**

(V) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।

**NOTE**

(I) Please check that this question paper contains **23** printed pages.

(II) Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(III) Please check that this question paper contains **38** questions.

(IV) **Please write down the Serial Number of the question in the answer-book at the given place before attempting it.**

(V) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.

#

# गणित (मानक)

# MATHEMATICS (STANDARD)

*निर्धारित समय : **3** घण्टे* | *अधिकतम अंक : **80***

*Time allowed : **3** hours* | *Maximum Marks : **80***

• • •

***सामान्य निर्देश :***

*निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख़्ती से पालन कीजिए :*

*(i)* *इस प्रश्न-पत्र में **38** प्रश्न हैं। **सभी** प्रश्न **अनिवार्य** हैं।*

*(ii)* *यह प्रश्न-पत्र **पाँच** खण्डों में विभाजित है – **क, ख, ग, घ** एवं **ङ**।*

*(iii)* ***खण्ड क** में प्रश्न संख्या **1** से **18** तक बहुविकल्पीय (MCQ) तथा प्रश्न संख्या **19** एवं **20** अभिकथन एवं तर्क आधारित **1** अंक के प्रश्न हैं ।*

*(iv)* ***खण्ड ख** में प्रश्न संख्या **21** से **25** तक अति लघु-उत्तरीय (VSA) प्रकार के **2** अंकों के प्रश्न हैं ।*

*(v)* ***खण्ड ग** में प्रश्न संख्या **26** से **31** तक लघु-उत्तरीय (SA) प्रकार के **3** अंकों के प्रश्न हैं ।*

*(vi)* ***खण्ड घ** में प्रश्न संख्या **32** से **35** तक दीर्घ-उत्तरीय (LA) प्रकार के **5** अंकों के प्रश्न हैं ।*

*(vii)* ***खण्ड ङ** में प्रश्न संख्या **36** से **38** तक प्रकरण अध्ययन आधारित **4** अंकों के प्रश्न हैं। प्रत्येक प्रकरण अध्ययन में आंतरिक विकल्प **2** अंकों के प्रश्न में दिया गया है।*

*(viii)* *प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 2 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 3 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।*

*(ix)* *जहाँ आवश्यक हो स्वच्छ आकृतियाँ बनाइए। जहाँ आवश्यक हो $\pi = \frac{22}{7}$ लीजिए, यदि अन्यथा न दिया गया हो।*

*(x)* *कैल्कुलेटर का उपयोग **वर्जित** है।*

## खण्ड क

*इस खण्ड में **20** बहुविकल्पीय प्रश्न (MCQ) हैं, जिनमें प्रत्येक प्रश्न **1** अंक का है।* *20×1=20*

**1.** एक पहिए का व्यास 63 cm है। इस पहिए के द्वारा 100 चक्करों में तय की गई दूरी है :

(A) 99 m (B) 198 m
(C) 63 m (D) 136 m

**2.** सात प्रेक्षणों का माध्य 17 है। यदि पहले चार प्रेक्षणों का माध्य 15 तथा आखिरी चार प्रेक्षणों का माध्य 18 है, तो चौथा प्रेक्षण है :

(A) 14 (B) 13
(C) 12 (D) 10

• • •

***General Instructions :***

*Read the following instructions very carefully and strictly follow them :*

*(i)* *This question paper contains **38** questions. **All** questions are **compulsory**.*

*(ii)* *This question paper is divided into **five** Sections – **A**, **B**, **C**, **D** and **E**.*

*(iii)* *In **Section A**, Questions no. **1** to **18** are Multiple Choice Questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

*(iv)* *In **Section B**, Questions no. **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*

*(v)* *In **Section C**, Questions no. **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

*(vi)* *In **Section D**, Questions no. **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.*

*(vii)* *In **Section E**, Questions no. **36** to **38** are Case Study based questions carrying **4** marks each. Internal choice is provided in **2** marks questions in each case study.*

*(viii)* *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.*

*(ix)* *Draw neat diagrams wherever required. Take $\pi = \frac{22}{7}$ wherever required, if not stated.*

*(x)* *Use of calculator is **not** allowed.*

## SECTION A

*This section has **20** Multiple Choice Questions (MCQs) carrying **1** mark each.* *20×1=20*

**1.** The diameter of a wheel is 63 cm. The distance travelled by the wheel in 100 revolutions is :

(A) 99 m (B) 198 m

(C) 63 m (D) 136 m

**2.** The mean of seven observations is 17. If the mean of the first four observations is 15 and that of the last four observations is 18, then the fourth observation is :

(A) 14 (B) 13

(C) 12 (D) 10

• • •

**3.** बिंदु (4, 0) की x-अक्ष से दूरी है :

(A) 4 इकाई (B) 16 इकाई

(C) 0 इकाई (D) $4\sqrt{2}$ इकाई

**4.** यदि $\alpha$ तथा $\beta$ बहुपद $p(x) = kx^2 - 30x + 45k$ के शून्यक हैं तथा $\alpha + \beta = \alpha\beta$ है, तो 'k' का मान है :

(A) $-\frac{2}{3}$ (B) $-\frac{3}{2}$

(C) $\frac{3}{2}$ (D) $\frac{2}{3}$

**5.** एक गोले की त्रिज्या 'r', एक शंकु के आधार की त्रिज्या के समान है। यदि उनके आयतन भी समान हैं, तो शंकु की ऊँचाई है :

(A) r (B) 2r

(C) 3r (D) 4r

**6.** यदि किसी लाटरी में, 10 स्थानों पर इनाम हैं तथा 30 स्थान खाली हैं, तो एक इनाम जीतने की प्रायिकता है :

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$

(C) $\frac{3}{4}$ (D) $\frac{2}{3}$

**7.** यदि $\tan 3\theta = \sqrt{3}$ है, तो $\frac{\theta}{2}$ बराबर है :

(A) 60° (B) 30°

(C) 20° (D) 10°

**8.** यदि दो त्रिभुजों $\Delta$ DEF तथा $\Delta$ PQR में, $\angle D = \angle Q$ तथा $\angle R = \angle E$ है, तो निम्नलिखित में से कौन-सा सही ***नहीं*** है ?

(A) $\frac{DE}{QR} = \frac{DF}{PQ}$ (B) $\frac{EF}{PR} = \frac{DF}{PQ}$

(C) $\frac{EF}{RP} = \frac{DE}{QR}$ (D) $\frac{DE}{PQ} = \frac{EF}{RP}$

• • •

**3.** The distance of the point (4, 0) from x-axis is :

(A) 4 units (B) 16 units

(C) 0 units (D) $4\sqrt{2}$ units

**4.** If $\alpha$ and $\beta$ are zeroes of the polynomial $p(x) = kx^2 - 30x + 45k$ and $\alpha + \beta = \alpha\beta$, then the value of 'k' is :

(A) $-\frac{2}{3}$ (B) $-\frac{3}{2}$

(C) $\frac{3}{2}$ (D) $\frac{2}{3}$

**5.** The radii 'r' of a sphere and that of the base of a cone are same. If their volumes are also same, then the height of the cone is :

(A) r (B) 2r

(C) 3r (D) 4r

**6.** If in a lottery, there are 10 prizes and 30 blanks, then the probability of winning a prize is :

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$

(C) $\frac{3}{4}$ (D) $\frac{2}{3}$

**7.** If $\tan 3\theta = \sqrt{3}$, then $\frac{\theta}{2}$ equals :

(A) $60°$ (B) $30°$

(C) $20°$ (D) $10°$

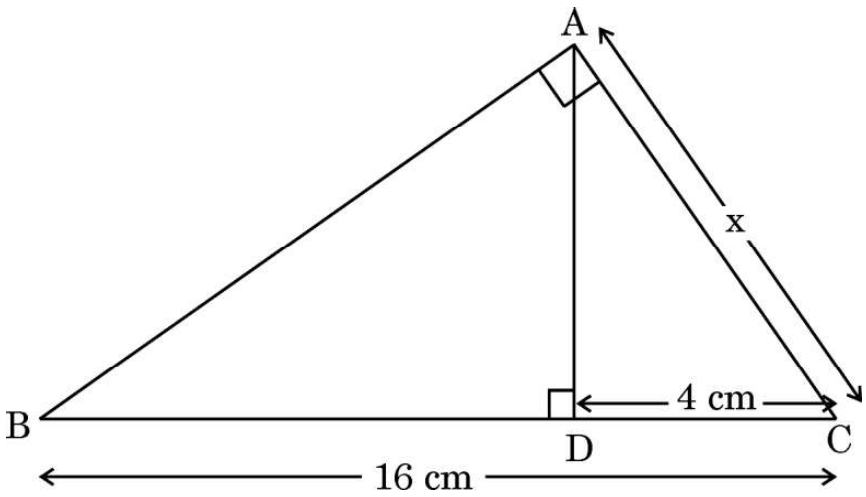
**8.** If in two triangles $\Delta$ DEF and $\Delta$ PQR, $\angle$ D = $\angle$ Q and $\angle$ R = $\angle$ E, then which of the following is ***not*** true ?

(A) $\frac{DE}{QR} = \frac{DF}{PQ}$ (B) $\frac{EF}{PR} = \frac{DF}{PQ}$

(C) $\frac{EF}{RP} = \frac{DE}{QR}$ (D) $\frac{DE}{PQ} = \frac{EF}{RP}$

• • •

**9.** दी गई आकृति में, $\Delta$ ABC में, AD $\perp$ BC तथा $\angle$ BAC = 90° है। यदि BC = 16 cm तथा DC = 4 cm है, तो x का मान है :




(A) 4 cm (B) 5 cm

(C) 8 cm (D) 3 cm

**10.** $\Delta$ PQR के दो शीर्ष P(–1, 5) तथा Q(5, 2) हैं। रेखाखण्ड PQ को 2 : 1 के अनुपात में बाँटने वाले बिंदु के निर्देशांक हैं :

(A) (3, – 3)

(B) (5, 5)

(C) (3, 3)

(D) (5, 1)

**11.** (cot $\theta$ + tan $\theta$) बराबर है :

(A) cosec $\theta$ sec $\theta$ (B) sin $\theta$ sec $\theta$

(C) cos $\theta$ tan $\theta$ (D) sin $\theta$ cos $\theta$

**12.** बहुपद $p(x) = x^2 - 3\sqrt{2}\,x + 4$ के शून्यक हैं :

(A) $2, \sqrt{2}$

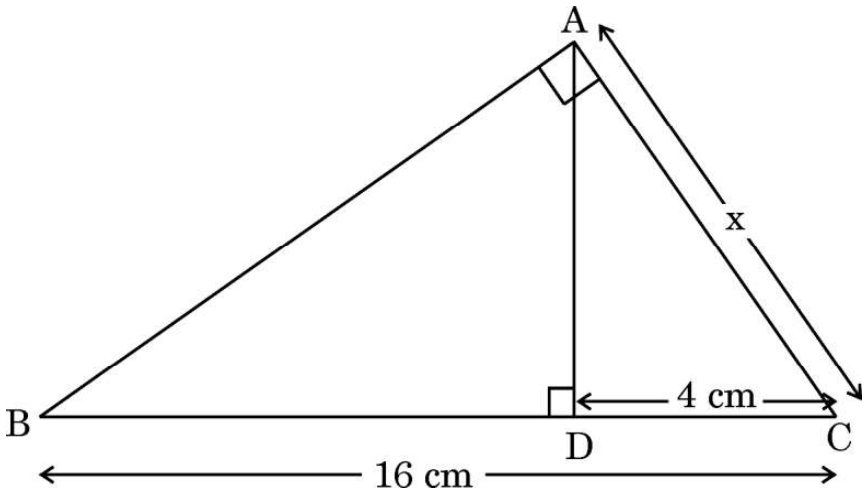
(B) $2\sqrt{2}, \sqrt{2}$

(C) $4\sqrt{2}, -\sqrt{2}$

(D) $\sqrt{2}, 2$

• • •

**9.** In the given figure, in $\Delta$ ABC, AD $\perp$ BC and $\angle$ BAC = 90°. If BC = 16 cm and DC = 4 cm, then the value of x is :




(A) 4 cm (B) 5 cm

(C) 8 cm (D) 3 cm

**10.** Two of the vertices of $\Delta$ PQR are P(–1, 5) and Q(5, 2). The coordinates of a point which divides PQ in the ratio 2 : 1 are :

(A) (3, – 3)

(B) (5, 5)

(C) (3, 3)

(D) (5, 1)

**11.** (cot $\theta$ + tan $\theta$) equals :

(A) cosec $\theta$ sec $\theta$ (B) sin $\theta$ sec $\theta$

(C) cos $\theta$ tan $\theta$ (D) sin $\theta$ cos $\theta$

**12.** Zeroes of the polynomial $p(x) = x^2 - 3\sqrt{2}\,x + 4$ are :

(A) $2, \sqrt{2}$

(B) $2\sqrt{2}, \sqrt{2}$

(C) $4\sqrt{2}, -\sqrt{2}$

(D) $\sqrt{2}, 2$

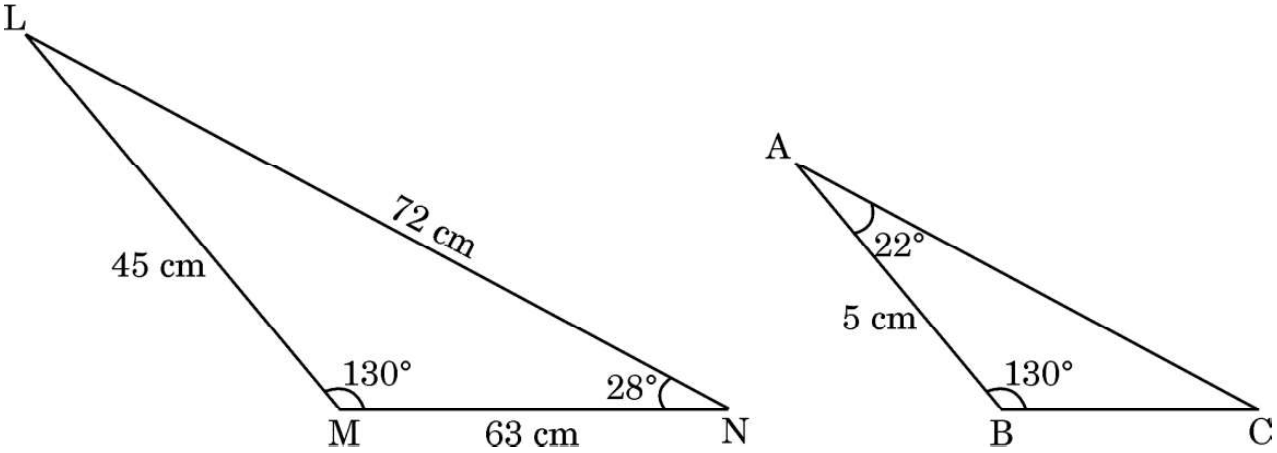
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**13.** 'k' का वह मान जिसके लिए रैखिक समीकरण निकाय $6x + y = 3k$ तथा $36x + 6y = 3$ के अपरिमित रूप से अनेक हल हैं, है :

(A) 6 (B) $\frac{1}{6}$

(C) $\frac{1}{2}$ (D) $\frac{1}{3}$

**14.** नीचे दी गई आकृति में दो त्रिभुजों LMN तथा ABC के माप दिए गए हैं। भुजा AC की लंबाई है :




(A) 16 cm (B) 7 cm

(C) 8 cm (D) 4 cm

**15.** $\frac{x}{4} + \frac{y}{6} = 1$ द्वारा निरूपित रेखा, x-अक्ष तथा y-अक्ष को क्रमश: P तथा Q पर काटती है। रेखाखण्ड PQ के मध्य-बिंदु के निर्देशांक हैं :

(A) (2, 3) (B) (3, 2)

(C) (2, 0) (D) (0, 3)

**16.** यदि 4, 6, 8 का ल.स. (LCM) x है, 3, 5, 7 का ल.स. y है तथा x और y का ल.स. p है, तो निम्नलिखित में से कौन-सा सही है ?

(A) $p = 35x$ (B) $p = 4y$

(C) $p = 8x$ (D) $p = 16y$

**17.** यदि $\frac{x}{12} - \frac{3}{x} = 0$ है, तो x के मान हैं :

(A) $\pm 6$ (B) $\pm 4$

(C) $\pm 12$ (D) $\pm 3$

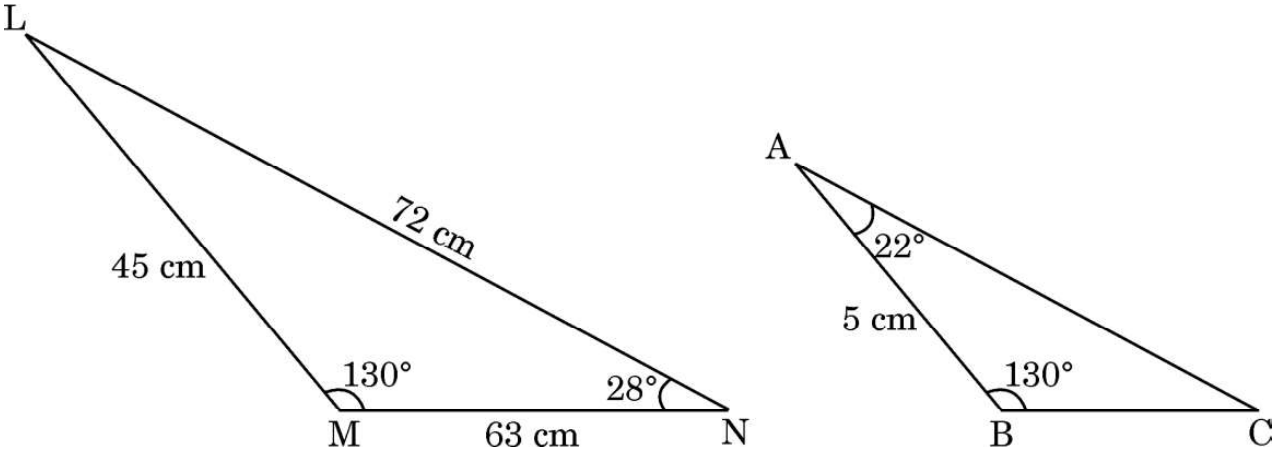
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**13.** The value of 'k' for which the system of linear equations $6x + y = 3k$ and $36x + 6y = 3$ have infinitely many solutions is :

(A) 6 (B) $\frac{1}{6}$

(C) $\frac{1}{2}$ (D) $\frac{1}{3}$

**14.** The measurements of $\Delta$ LMN and $\Delta$ ABC are shown in the figure given below. The length of side AC is :




(A) 16 cm (B) 7 cm

(C) 8 cm (D) 4 cm

**15.** The line represented by $\frac{x}{4} + \frac{y}{6} = 1$, intersects x-axis and y-axis respectively at P and Q. The coordinates of the mid-point of line segment PQ are :

(A) (2, 3) (B) (3, 2)

(C) (2, 0) (D) (0, 3)

**16.** If x is the LCM of 4, 6, 8 and y is the LCM of 3, 5, 7 and p is the LCM of x and y, then which of the following is true ?

(A) $p = 35x$ (B) $p = 4y$

(C) $p = 8x$ (D) $p = 16y$

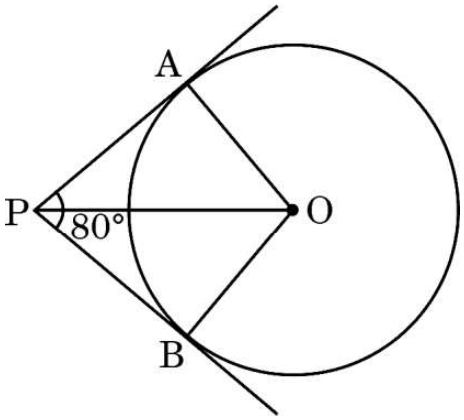
**17.** If $\frac{x}{12} - \frac{3}{x} = 0$, then the values of x are :

(A) $\pm 6$ (B) $\pm 4$

(C) $\pm 12$ (D) $\pm 3$

• • •

**18.** दी गई आकृति में केंद्र O वाले वृत्त पर एक बाह्य बिंदु P से यदि PA तथा PB स्पर्श-रेखाएँ खींची गई हैं जिनके बीच का कोण 80° है, तो $\angle$ POA की माप है :




(A) 40° (B) 50°

(C) 60° (D) 80°

*प्रश्न संख्या **19** और **20** अभिकथन एवं तर्क आधारित प्रश्न हैं। दो कथन दिए गए हैं, जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है। इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (A), (B), (C) और (D) में से चुनकर दीजिए।*

(A) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है।

(B) अभिकथन (A) और तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या ***नहीं*** करता है।

(C) अभिकथन (A) सही है, परन्तु तर्क (R) ग़लत है।

(D) अभिकथन (A) ग़लत है, परन्तु तर्क (R) सही है।

**19.** *अभिकथन (A)* : एक बाह्य बिंदु से वृत्त पर खींची गई दो स्पर्श-रेखाएँ, वृत्त के केंद्र पर समान कोण अन्तरित करती हैं।

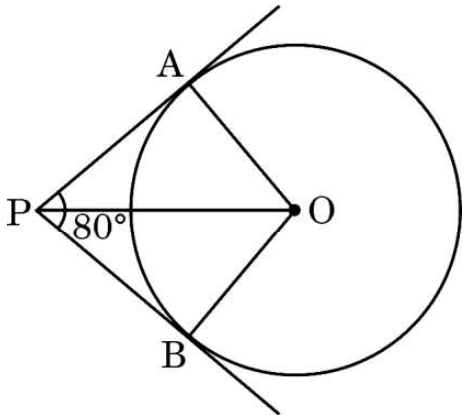
*तर्क (R)* : एक वृत्त के परिगत समांतर चतुर्भुज, एक समचतुर्भुज होता है।

**20.** *अभिकथन (A)* : दीवार के साथ लगी हुई एक सीढ़ी के पाद की दीवार से क्षैतिज दूरी 6 m है। यदि यह दीवार पर 8 m ऊँचाई तक पहुँचती है, तो सीढ़ी की लंबाई 10 m है।

*तर्क (R)* : सीढ़ी, भूमि से 60° का कोण बनाती है।

• • •

**18.** If tangents PA and PB drawn from an external point P to the circle with centre O are inclined to each other at an angle of 80° as shown in the given figure, then the measure of ∠ POA is :




(A) 40° (B) 50°

(C) 60° (D) 80°

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

**19.** *Assertion (A) :* If two tangents are drawn to a circle from an external point, then they subtend equal angles at the centre of the circle.

*Reason (R):* A parallelogram circumscribing a circle is a rhombus.

**20.** *Assertion (A) :* A ladder leaning against a wall, stands at a horizontal distance of 6 m from the wall. If the height of the wall up to which the ladder reaches is 8 m, then the length of the ladder is 10 m.

*Reason (R):* The ladder makes an angle of 60° with the ground.

• • •

## खण्ड ख

*इस खण्ड में **5** अति लघु-उत्तरीय (VSA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के **2** अंक हैं।* *5×2=10*

**21.** यदि $\tan A + \cot A = 6$ है, तो $\tan^2 A + \cot^2 A - 4$ का मान ज्ञात कीजिए।

**22.** (क) यदि द्विघात समीकरण $4x^2 + kx + 1 = 0$ के मूल वास्तविक और समान हैं, तो 'k' का/के मान ज्ञात कीजिए।

**अथवा**

(ख) यदि '$\alpha$' तथा '$\beta$', बहुपद $p(y) = y^2 - 5y + 3$ के शून्यक हैं, तो $\alpha^4\beta^3 + \alpha^3\beta^4$ का मान ज्ञात कीजिए।

**23.** एक निश्चित परीक्षा प्रश्न के सही उत्तर का अनुमान लगाने की प्रायिकता $\frac{x}{12}$ है। यदि इसी प्रश्न के सही उत्तर का अनुमान न लगा पाने की प्रायिकता $\frac{5}{6}$ है, तो x का मान ज्ञात कीजिए।

**24.** सिद्ध कीजिए कि वृत्त के किसी व्यास के सिरों पर खींची गई स्पर्श-रेखाएँ परस्पर समांतर होती हैं।

**25.** (क) वह छोटी से छोटी संख्या ज्ञात कीजिए जो 644 तथा 462 दोनों से विभाजित हो।

**अथवा**

(ख) दो संख्याएँ 4 : 5 के अनुपात में हैं तथा उनका म.स. (HCF) 11 है। इन संख्याओं का ल.स. (LCM) ज्ञात कीजिए।

## खण्ड ग

*इस खण्ड में **6** लघु-उत्तरीय (SA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के **3** अंक हैं।* *6×3=18*

**26.** 52 पत्तों की ताश की गड्डी में से सभी हुकुम के तस्वीर वाले पत्ते निकाल दिए गए तथा शेष गड्डी को अच्छी प्रकार से फेंटा गया। शेष गड्डी में से एक पत्ता यादृच्छया निकालने पर निम्नलिखित के प्राप्त होने की प्रायिकता ज्ञात कीजिए :

(क) एक तस्वीर वाला पत्ता

(ख) एक इक्का अथवा गुलाम वाला पत्ता

• • •

## SECTION B

*This section has **5** Very Short Answer (VSA) type questions carrying **2** marks each.* *5×2=10*

**21.** If $\tan A + \cot A = 6$, then find the value of $\tan^2 A + \cot^2 A - 4$.

**22.** (a) Find the value(s) of 'k' so that the quadratic equation $4x^2 + kx + 1 = 0$ has real and equal roots.

**OR**

(b) If '$\alpha$' and '$\beta$' are the zeroes of the polynomial $p(y) = y^2 - 5y + 3$, then find the value of $\alpha^4\beta^3 + \alpha^3\beta^4$.

**23.** The probability of guessing the correct answer of a certain test question is $\frac{x}{12}$. If the probability of not guessing the correct answer is $\frac{5}{6}$, then find the value of x.

**24.** Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

**25.** (a) Find the smallest number which is divisible by both 644 and 462.

**OR**

(b) Two numbers are in the ratio 4 : 5 and their HCF is 11. Find the LCM of these numbers.

## SECTION C

*This section has **6** Short Answer (SA) type questions carrying **3** marks each.* *6×3=18*

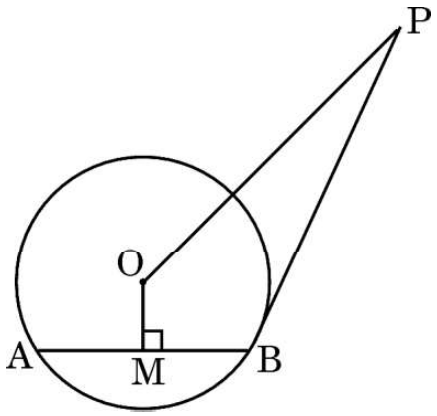
**26.** All face cards of spades are removed from a pack of 52 playing cards and the remaining pack is shuffled well. A card is then drawn at random from the remaining pack. Find the probability of getting :

(a) a face card

(b) an ace or a jack

• • •

**27.** दी गई आकृति में, केंद्र O वाले वृत्त के बिंदु B पर PB एक स्पर्श-रेखा है। AB वृत्त की एक जीवा है जिसकी लंबाई 24 cm है तथा यह जीवा वृत्त के केंद्र से 5 cm की दूरी पर है। यदि स्पर्श-रेखा PB की लंबाई 20 cm है, तो OP की लंबाई ज्ञात कीजिए।




**28.** 10 cm त्रिज्या वाले वृत्त की एक जीवा, वृत्त के केंद्र पर समकोण अंतरित करती है। संगत लघु वृत्तखण्ड का क्षेत्रफल ज्ञात कीजिए। [$\pi = 3{\cdot}14$ प्रयोग कीजिए]

**29.** सिद्ध कीजिए कि $\left(5\sqrt{3} + \frac{2}{3}\right)$ एक अपरिमेय संख्या है, जबकि दिया गया है कि $\sqrt{3}$ एक अपरिमेय संख्या है।

**30.** (क) सिद्ध कीजिए कि : $\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2 \operatorname{cosec} A$

**अथवा**

(ख) सिद्ध कीजिए कि : $\left(\frac{1}{\cos A} - \cos A\right)\left(\frac{1}{\sin A} - \sin A\right) = \frac{1}{\tan A + \cot A}$

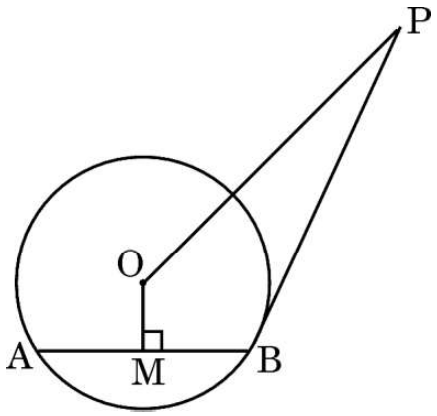
**31.** (क) यदि बिंदुओं A(3, 4) तथा B(k, 6) को मिलाने वाले रेखाखण्ड का मध्य-बिंदु P(x, y) है तथा $x + y - 10 = 0$ है, तो k का मान ज्ञात कीजिए।

**अथवा**

(ख) बिंदुओं A(– 2, 2) तथा B(2, 8) को मिलाने वाले रेखाखण्ड को चार समान भागों में बाँटने वाले बिंदुओं के निर्देशांक ज्ञात कीजिए।

• • •

**27.** In the given figure, PB is a tangent to the circle with centre O at B. AB is a chord of the circle of length 24 cm and at a distance of 5 cm from the centre of the circle. If the length PB of the tangent is 20 cm, find the length of OP.




**28.** A chord of a circle of radius 10 cm subtends a right angle at the centre of the circle. Find the area of the corresponding minor segment. [Use $\pi = 3{\cdot}14$]

**29.** Prove that $\left(5\sqrt{3} + \frac{2}{3}\right)$ is an irrational number given that $\sqrt{3}$ is an irrational number.

**30.** (a) Prove that : $\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2 \operatorname{cosec} A$

**OR**

(b) Prove that : $\left(\frac{1}{\cos A} - \cos A\right)\left(\frac{1}{\sin A} - \sin A\right) = \frac{1}{\tan A + \cot A}$

**31.** (a) If the mid-point of the line segment joining the points A(3, 4) and B(k, 6) is P(x, y) and $x + y - 10 = 0$, find the value of k.

**OR**

(b) Find the coordinates of the points which divide the line segment joining A(– 2, 2) and B(2, 8) into four equal parts.

• • •

## खण्ड घ

*इस खण्ड में **4** दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के **5** अंक हैं।* *4×5=20*

**32.** एक ठोस खिलौना, एक अर्धगोले पर अध्यारोपित एक लंब-वृत्तीय शंकु के आकार का है। शंकु की ऊँचाई 2 cm है तथा इसके आधार का व्यास 4 cm है। इस खिलौने का आयतन ज्ञात कीजिए। खिलौने का पृष्ठीय क्षेत्रफल भी ज्ञात कीजिए। ($\pi = 3{\cdot}14$ लीजिए)

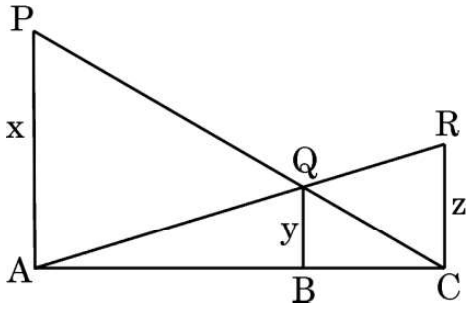
**33.** एक कक्षा के विद्यार्थियों को एकसमान संख्या में पंक्तियों में खड़ा किया गया। यदि प्रत्येक पंक्ति में 3 विद्यार्थी अधिक लिए जाएँ, तो 1 पंक्ति कम हो जाती है। यदि प्रत्येक पंक्ति में 3 विद्यार्थी कम लिए जाएँ, तो 2 पंक्तियाँ अधिक हो जाती हैं। कक्षा में विद्यार्थियों की कुल संख्या ज्ञात कीजिए।

**34.** (क) एक समांतर श्रेढ़ी के तीसरे तथा सातवें पदों का योगफल 6 है तथा उनका गुणनफल 8 है। इस समांतर श्रेढ़ी के प्रथम 16 पदों का योगफल ज्ञात कीजिए।

**अथवा**

(ख) एक पेंटिंग प्रतियोगिता में भाग लेने के लिए बच्चों की न्यूनतम आयु 8 वर्ष है। यह देखा गया कि भाग लेने वालों में से सबसे छोटे बच्चे की आयु 8 वर्ष थी तथा भाग लेने वालों को आयु के अनुसार क्रम में बिठाने पर सार्व अंतर 4 माह का था। यदि सभी भाग लेने वालों की आयु का योगफल 168 वर्ष है, तो पेंटिंग प्रतियोगिता में भाग लेने वाले सबसे बड़े बच्चे की आयु ज्ञात कीजिए।

**35.** (क) दी गई आकृति में, PA, QB तथा RC भुजा AC पर लंबवत हैं। यदि PA = x इकाई, QB = y इकाई तथा RC = z इकाई है, तो सिद्ध कीजिए कि $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$.




**अथवा**

(ख) एक त्रिभुज ABC की भुजाएँ AB और BC तथा माध्यिका AD एक अन्य त्रिभुज PQR की क्रमश: भुजाओं PQ और QR तथा माध्यिका PM के समानुपाती हैं। दर्शाइए कि $\Delta$ ABC ~ $\Delta$ PQR है।

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## SECTION D

*This section has **4** Long Answer (LA) type questions carrying **5** marks each.* $4 \times 5 = 20$

**32.** A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. Also, find the surface area of the toy. (Take $\pi = 3{\cdot}14$)

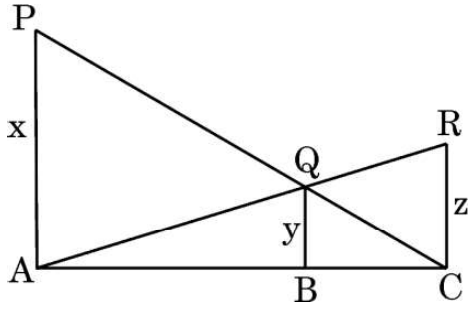
**33.** The students of a class are made to stand equally in rows. If 3 students are extra in each row, there would be 1 row less. If 3 students are less in a row, there would be 2 more rows. Find the number of students in the class.

**34.** (a) The sum of the third term and the seventh term of an AP is 6 and their product is 8. Find the sum of the first sixteen terms of the AP.

**OR**

(b) The minimum age of children eligible to participate in a painting competition is 8 years. It is observed that the age of the youngest boy was 8 years and the ages of the participants, when seated in order of age, have a common difference of 4 months. If the sum of the ages of all the participants is 168 years, find the age of the eldest participant in the painting competition.

**35.** (a) In the given figure, PA, QB and RC are perpendicular to AC. If PA = x units, QB = y units and RC = z units, prove that $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$.




**OR**

(b) Sides AB and BC and median AD of triangle ABC are respectively proportional to sides PQ and QR and median PM of $\Delta$ PQR. Show that $\Delta$ ABC ~ $\Delta$ PQR.

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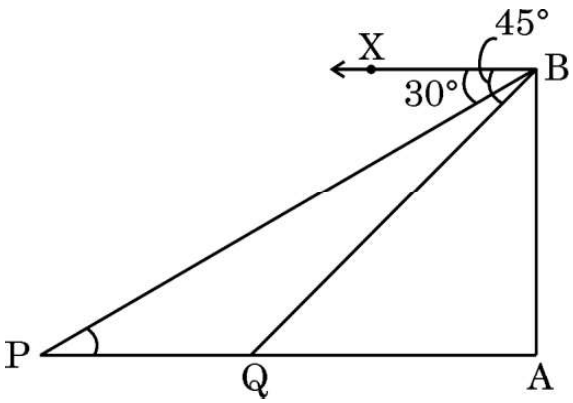
## खण्ड ङ

*इस खण्ड में **3** प्रकरण अध्ययन आधारित प्रश्न हैं जिनमें प्रत्येक के **4** अंक हैं।* *3×4=12*

### प्रकरण अध्ययन – 1

**36.** समुद्र के किनारे एक चट्टान पर एक लाइटहाउस खड़ा है, जो पास से गुज़रने वाले जहाजों पर नज़र रखता है। एक दिन लाइटहाउस के शिखर से एक जहाज को किनारे की ओर आते देखने पर पाया कि जैसे ही यह बिंदु P से Q की तरफ आता है, तो इसका अवनमन कोण 30° से 45° में बदल जाता है। लाइटहाउस की ऊँचाई 50 मीटर है।






उपर्युक्त दी गई जानकारी के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) जहाज जब बिंदु Q पर है तो इसकी लाइटहाउस के आधार से दूरी ज्ञात कीजिए, जबकि यहाँ का अवनमन कोण 45° है। *1*

(ii) $\angle$ PBA तथा $\angle$ QBA की माप ज्ञात कीजिए। *1*

(iii) (क) जहाज द्वारा बिंदु P से Q तक तय की गई दूरी ज्ञात कीजिए। *2*

**अथवा**

(ख) यदि यह जहाज किनारे की ओर चलता रहे तथा Q से A तक आने में 10 मिनट ले, तो इसकी गति Q से A तक km/h में परिकलित कीजिए। *2*

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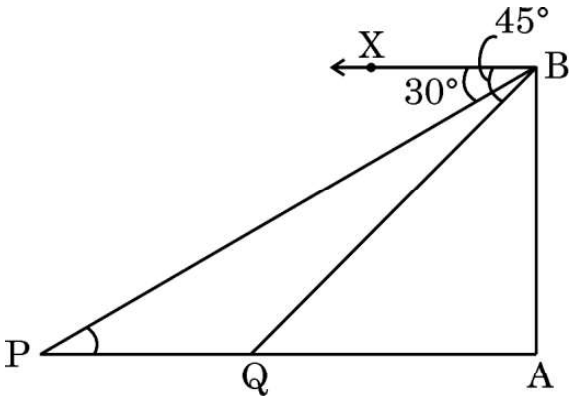
## SECTION E

*This section has **3** Case Study based questions carrying **4** marks each.* *3×4=12*

### Case Study – 1

**36.** A lighthouse stands tall on a cliff by the sea, watching over ships that pass by. One day a ship is seen approaching the shore and from the top of the lighthouse, the angles of depression of the ship are observed to be 30° and 45° as it moves from point P to point Q. The height of the lighthouse is 50 metres.






Based on the information given above, answer the following questions :

(i) Find the distance of the ship from the base of the lighthouse when it is at point Q, where the angle of depression is 45°. *1*

(ii) Find the measures of ∠ PBA and ∠ QBA. *1*

(iii) (a) Find the distance travelled by the ship between points P and Q. *2*

**OR**

(b) If the ship continues moving towards the shore and takes 10 minutes to travel from Q to A, calculate the speed of the ship in km/h, from Q to A. *2*

• • •

## प्रकरण अध्ययन – 2

**37.** भारत मौसम विज्ञान विभाग प्रत्येक वर्ष देश के विभिन्न उप-विभागों में मौसमी और वार्षिक वर्षा का निरीक्षण करता है। इससे उन्हें परिणामों की तुलना और विश्लेषण करने में सहायता मिलती है।

निम्नलिखित सारणी में उप-विभागों में 2023 में मौसमी वर्षा (mm में) दर्शाई गई है।

| *वर्षा (mm में)* | *उप-विभागों की संख्या* |
|---|---|
| 200 – 400 | 3 |
| 400 – 600 | 4 |
| 600 – 800 | 7 |
| 800 – 1000 | 4 |
| 1000 – 1200 | 3 |
| 1200 – 1400 | 3 |

उपर्युक्त दी गई जानकारी के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) बहुलक वर्ग लिखिए। *1*

(ii) (क) दिए गए आँकड़ों का माध्यक ज्ञात कीजिए। *2*

**अथवा**

(ख) इस मौसम में वर्षा का माध्य ज्ञात कीजिए। *2*

(iii) यदि वर्षा के मौसम के दौरान कम-से-कम 800 mm वर्षा वाले उप-विभाग को अच्छी वर्षा वाला उप-विभाग माना जाता है, तो कितने उप-विभागों में अच्छी वर्षा हुई ? *1*

• • •

## Case Study – 2

**37.** The India Meteorological Department observes seasonal and annual rainfall every year in different sub-divisions of our country. It helps them to compare and analyse the results.

The table below shows sub-divisions wise seasonal (monsoon) rainfall (in mm) in 2023.

| *Rainfall (mm)* | *No. of Sub-divisions* |
|---|---|
| 200 – 400 | 3 |
| 400 – 600 | 4 |
| 600 – 800 | 7 |
| 800 – 1000 | 4 |
| 1000 – 1200 | 3 |
| 1200 – 1400 | 3 |

Based on the information given above, answer the following questions :

(i) Write the modal class. *1*

(ii) (a) Find the median of the given data. *2*

**OR**

(b) Find the mean rainfall in the season. *2*

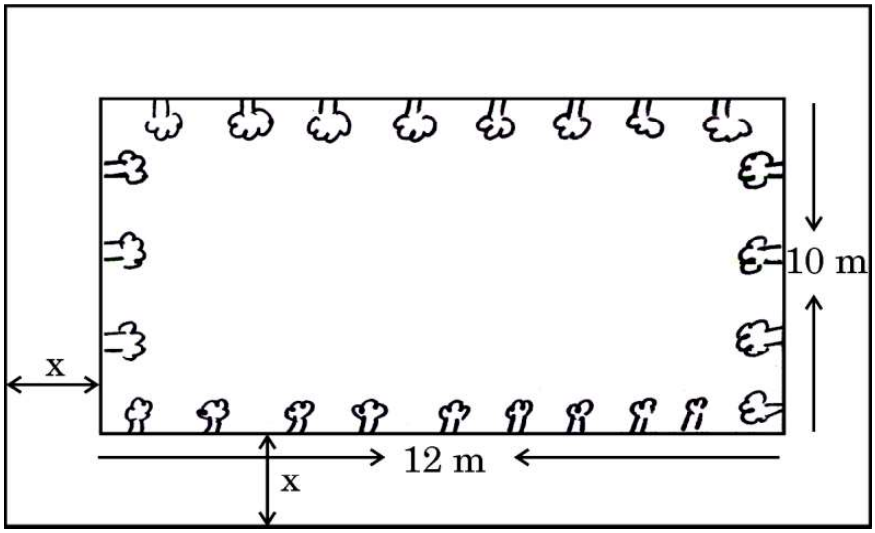
(iii) If a sub-division having at least 800 mm rainfall during monsoon season is considered a good rainfall sub-division, then how many sub-divisions had good rainfall ? *1*

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## प्रकरण अध्ययन – 3

**38.** एक उद्यान डिज़ाइनर एक आयताकार लॉन की योजना बना रहा है, जिसके चारों ओर एकसमान रास्ता बनाया जाना है।






लॉन और रास्ते का कुल क्षेत्रफल 360 वर्ग मीटर है। रास्ते की चौड़ाई हर तरफ से समान है। लॉन की विमाएँ 12 मीटर × 10 मीटर हैं।

उपर्युक्त दी गई जानकारी के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) रास्ते की चौड़ाई को x मीटर लेकर लॉन तथा रास्ते के कुल क्षेत्रफल को निरूपित करने वाला द्विघात समीकरण बनाइए। *1*

(ii) (क) रास्ते की चौड़ाई 'x' ज्ञात करने के लिए द्विघात समीकरण हल कीजिए। *2*

**अथवा**

(ख) यदि रास्ते पर टाइलें लगाने का खर्च ₹ 50 प्रति वर्ग मीटर की दर से ₹ 12,000 आता है, तो रास्ते का क्षेत्रफल परिकलित कीजिए। *2*

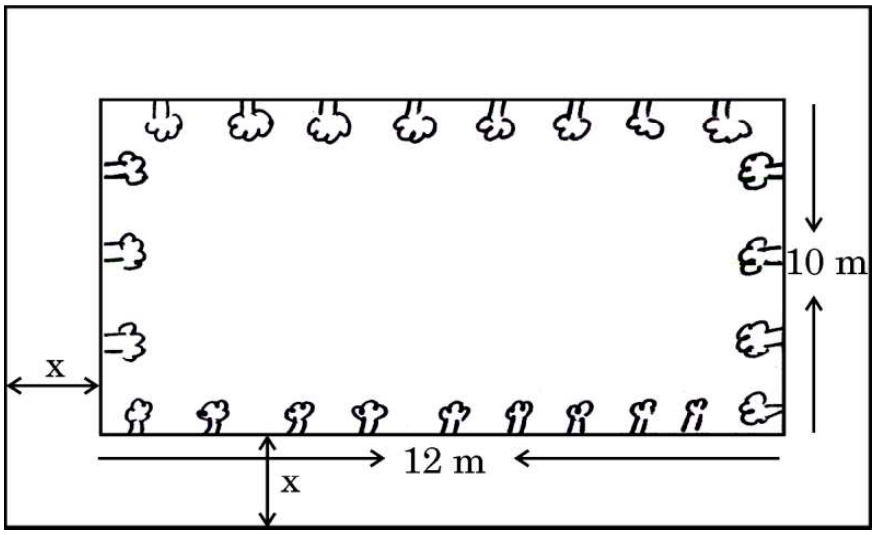
(iii) लॉन का परिमाप ज्ञात कीजिए। *1*

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## Case Study – 3

**38.** A garden designer is planning a rectangular lawn that is to be surrounded by a uniform walkway.






The total area of the lawn and the walkway is 360 square metres. The width of the walkway is same on all sides. The dimensions of the lawn itself are 12 metres by 10 metres.

Based on the information given above, answer the following questions :

(i) Formulate the quadratic equation representing the total area of the lawn and the walkway, taking width of walkway = x m. *1*

(ii) (a) Solve the quadratic equation to find the width of the walkway 'x'. *2*

**OR**

(b) If the cost of paving the walkway at the rate of ₹ 50 per square metre is ₹ 12,000, calculate the area of the walkway. *2*

(iii) Find the perimeter of the lawn. *1*

**Marking Scheme**

**Strictly Confidential**

**(For Internal and Restricted use only)**

**Secondary School Examination, 2025**

**MATHEMATICS (Standard) (Q.P. CODE 30/3/3)**

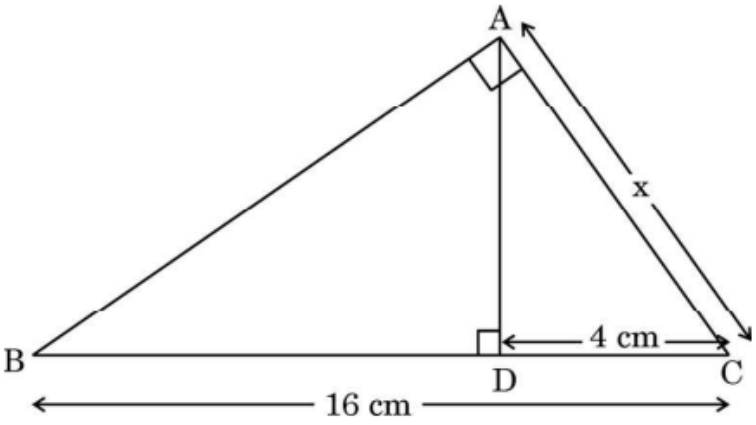
**General Instructions: -**

| | |
|---|---|
| **1.** | You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. |
| **2.** | **"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. It's leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc. may invite action under various rules of the Board and IPC."** |
| **3.** | Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-X, while evaluating the competency-based questions, please try to understand given answer and even if reply is not from Marking Scheme but correct competency is enumerated by the candidate, due marks should be awarded.** |
| **4.** | The Marking scheme carries only suggested value points for the answers.<br>These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly. |
| **5.** | The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators. |
| **6.** | Evaluators will mark (✓) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓) while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.** |
| **7.** | If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totalled up and written on the left-hand margin and encircled. This may be followed strictly. |

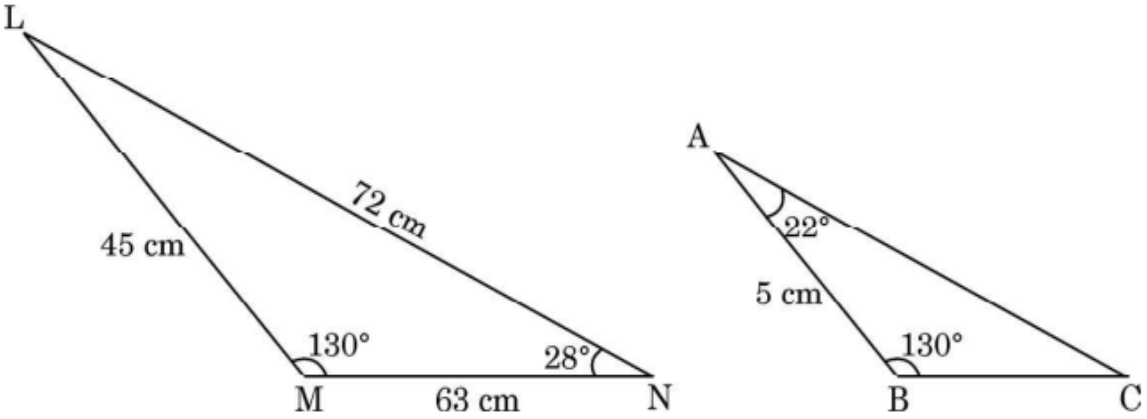
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| **8.** | If a question does not have any parts, marks must be awarded on the left-hand margin and encircled. This may also be followed strictly. |
| **9.** | If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question". |
| **10.** | No marks to be deducted for the cumulative effect of an error. It should be penalized only once. |
| **11.** | A full scale of marks ____80____ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it. |
| **12.** | Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper. |
| **13.** | Ensure that you do not make the following common types of errors committed by the Examiner in the past: -<br>● Leaving answer or part thereof unassessed in an answer book.<br>● Giving more marks for an answer than assigned to it.<br>● Wrong totalling of marks awarded to an answer.<br>● Wrong transfer of marks from the inside pages of the answer book to the title page.<br>● Wrong question wise totalling on the title page.<br>● Wrong totalling of marks of the two columns on the title page.<br>● Wrong grand total.<br>● Marks in words and figures not tallying/not same.<br>● Wrong transfer of marks from the answer book to online award list.<br>● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)<br>Half or a part of answer marked correct and the rest as wrong, but no marks awarded. |
| **14.** | While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks. |
| **15.** | Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously. |
| **16.** | The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation. |
| **17.** | Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totalled and written in figures and words. |
| **18.** | The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme. |

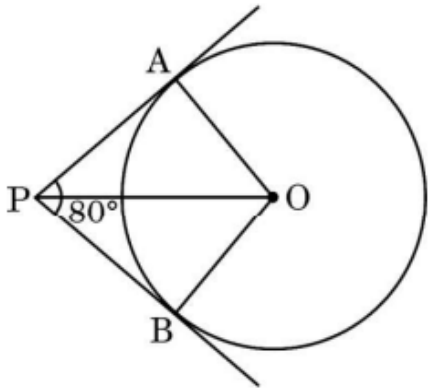
**MARKING SCHEME**
**MATHEMATICS (Subject Code–041)**
**(PAPER CODE: 30/3/3)**

| Q. No. | EXPECTED OUTCOMES/VALUE POINTS | Marks |
|---|---|---|
| | **SECTION A**<br>**This section has 20 Multiple Choice Questions (MCQs) carrying 1 mark each.** | |
| **1.** | The diameter of a wheel is 63 cm. The distance travelled by the wheel in 100 revolutions is :<br>(A) 99 m (B) 198 m<br>(C) 63 m (D) 136 m | |
| **Sol.** | (B) 198 m | **1** |
| **2.** | The mean of seven observations is 17. If the mean of the first four observations is 15 and that of the last four observations is 18, then the fourth observation is :<br>(A) 14 (B) 13<br>(C) 12 (D) 10 | |
| **Sol.** | (B) 13 | **1** |
| **3.** | The distance of the point (4, 0) from x-axis is :<br>(A) 4 units (B) 16 units<br>(C) 0 units (D) $4\sqrt{2}$ units | |
| **Sol.** | (C) 0 units | **1** |
| **4.** | If $\alpha$ and $\beta$ are zeroes of the polynomial $p(x) = kx^2 - 30x + 45k$ and $\alpha + \beta = \alpha\beta$, then the value of 'k' is :<br>(A) $-\frac{2}{3}$ (B) $-\frac{3}{2}$<br>(C) $\frac{3}{2}$ (D) $\frac{2}{3}$ | |
| **Sol.** | (D) $\frac{2}{3}$ | **1** |
| **5.** | The radii 'r' of a sphere and that of the base of a cone are same. If their volumes are also same, then the height of the cone is :<br>(A) r (B) 2r<br>(C) 3r (D) 4r | |
| **Sol.** | (D) 4r | **1** |

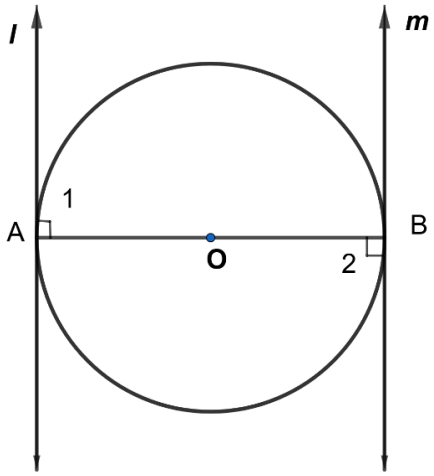
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| **6.** | If in a lottery, there are 10 prizes and 30 blanks, then the probability of winning a prize is :<br>(A) $\frac{1}{4}$ (B) $\frac{1}{3}$<br>(C) $\frac{3}{4}$ (D) $\frac{2}{3}$ | |
| **Sol.** | (A) $\frac{1}{4}$ | **1** |
| **7.** | If $\tan 3\theta = \sqrt{3}$, then $\frac{\theta}{2}$ equals :<br>(A) $60°$ (B) $30°$<br>(C) $20°$ (D) $10°$ | |
| **Sol.** | (D) $10°$ | **1** |
| **8.** | If in two triangles $\Delta$ DEF and $\Delta$ PQR, $\angle$ D = $\angle$ Q and $\angle$ R = $\angle$ E, then which of the following is ***not*** true ?<br>(A) $\frac{DE}{QR} = \frac{DF}{PQ}$ (B) $\frac{EF}{PR} = \frac{DF}{PQ}$<br>(C) $\frac{EF}{RP} = \frac{DE}{QR}$ (D) $\frac{DE}{PQ} = \frac{EF}{RP}$ | |
| **Sol.** | (D) $\frac{DE}{PQ} = \frac{EF}{RP}$ | **1** |
| **9.** | In the given figure, in $\Delta$ ABC, AD $\perp$ BC and $\angle$ BAC = $90°$. If BC = 16 cm and DC = 4 cm, then the value of x is :<br><br><br>(A) 4 cm (B) 5 cm<br>(C) 8 cm (D) 3 cm | |
| **Sol.** | (C) 8 cm | **1** |

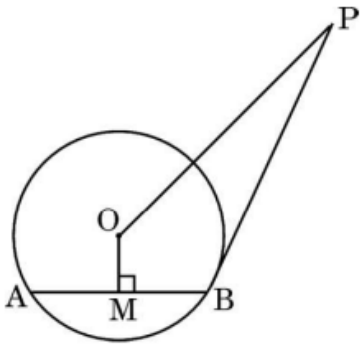
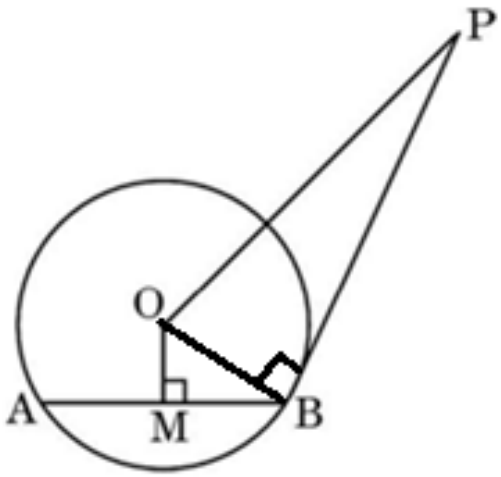
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| **10.** | Two of the vertices of $\Delta$ PQR are P(–1, 5) and Q(5, 2). The coordinates of a point which divides PQ in the ratio 2 : 1 are :<br>(A) $(3, -3)$<br>(B) $(5, 5)$<br>(C) $(3, 3)$<br>(D) $(5, 1)$ | |
| **Sol.** | (C) $(3, 3)$ | **1** |
| **11.** | $(\cot\theta + \tan\theta)$ equals :<br>(A) $\operatorname{cosec}\theta \sec\theta$ (B) $\sin\theta \sec\theta$<br>(C) $\cos\theta \tan\theta$ (D) $\sin\theta \cos\theta$ | |
| **Sol.** | (A) $\operatorname{cosec}\theta \sec\theta$ | **1** |
| **12.** | Zeroes of the polynomial $p(x) = x^2 - 3\sqrt{2}x + 4$ are :<br>(A) $2, \sqrt{2}$<br>(B) $2\sqrt{2}, \sqrt{2}$<br>(C) $4\sqrt{2}, -\sqrt{2}$<br>(D) $\sqrt{2}, 2$ | |
| **Sol.** | (B) $2\sqrt{2}, \sqrt{2}$ | **1** |
| **13.** | The value of 'k' for which the system of linear equations $6x + y = 3k$ and $36x + 6y = 3$ have infinitely many solutions is :<br>(A) 6 (B) $\frac{1}{6}$<br>(C) $\frac{1}{2}$ (D) $\frac{1}{3}$ | |
| **Sol.** | (B) $\frac{1}{6}$ | **1** |

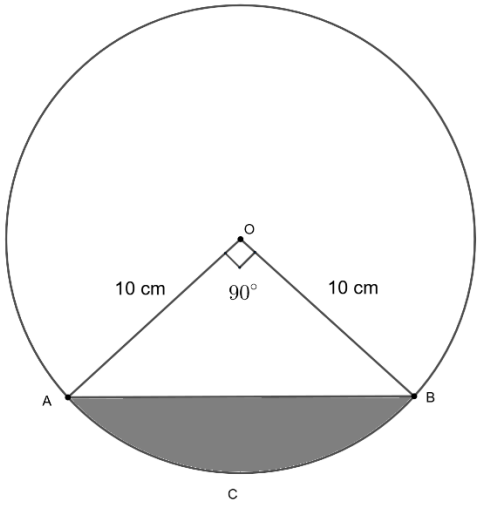
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| **14.** | The measurements of $\Delta$ LMN and $\Delta$ ABC are shown in the figure given below. The length of side AC is :<br><br><br>(A) 16 cm (B) 7 cm<br>(C) 8 cm (D) 4 cm | |
| **Sol.** | (C) 8 cm | **1** |
| **15.** | The line represented by $\frac{x}{4} + \frac{y}{6} = 1$, intersects x-axis and y-axis respectively at P and Q. The coordinates of the mid-point of line segment PQ are :<br>(A) (2, 3) (B) (3, 2)<br>(C) (2, 0) (D) (0, 3) | |
| **Sol.** | (A) (2, 3) | **1** |
| **16.** | If x is the LCM of 4, 6, 8 and y is the LCM of 3, 5, 7 and p is the LCM of x and y, then which of the following is true ?<br>(A) $p = 35x$ (B) $p = 4y$<br>(C) $p = 8x$ (D) $p = 16y$ | |
| **Sol.** | (A) $p = 35x$ | **1** |
| **17.** | If $\frac{x}{12} - \frac{3}{x} = 0$, then the values of x are :<br>(A) $\pm 6$ (B) $\pm 4$<br>(C) $\pm 12$ (D) $\pm 3$ | |
| **Sol.** | (A) $\pm 6$ | **1** |

| | | |
|---|---|---|
| **18.** | If tangents PA and PB drawn from an external point P to the circle with centre O are inclined to each other at an angle of 80° as shown in the given figure, then the measure of ∠ POA is :<br><br><br>(A) 40° (B) 50°<br>(C) 60° (D) 80° | |
| **Sol.** | (B) 50° | **1** |
| | *Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below.*<br>(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).<br>(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).<br>(C) Assertion (A) is true, but Reason (R) is false.<br>(D) Assertion (A) is false, but Reason (R) is true. | |
| **19.** | *Assertion (A) :* If two tangents are drawn to a circle from an external point, then they subtend equal angles at the centre of the circle.<br>*Reason (R):* A parallelogram circumscribing a circle is a rhombus. | |
| **Sol.** | (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). | **1** |
| **20.** | *Assertion (A) :* A ladder leaning against a wall, stands at a horizontal distance of 6 m from the wall. If the height of the wall up to which the ladder reaches is 8 m, then the length of the ladder is 10 m.<br>*Reason (R):* The ladder makes an angle of 60° with the ground. | |
| **Sol.** | (C) Assertion (A) is true, but Reason (R) is false. | **1** |

| | | |
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| | **SECTION B**<br>**This section has 5 Very Short Answer (VSA) type questions carrying 2 marks each.** | |
| **21.** | If $\tan A + \cot A = 6$, then find the value of $\tan^2 A + \cot^2 A - 4$. | |
| **Sol.** | $(\tan A + \cot A)^2 = 36$<br>$\tan^2 A + \cot^2 A + 2\tan A \cot A = 36$<br>$\tan^2 A + \cot^2 A = 34$<br>$\therefore \tan^2 A + \cot^2 A - 4 = 30$ | ½<br>½<br>½<br>½ |
| **22. (a)** | Find the value(s) of 'k' so that the quadratic equation $4x^2 + kx + 1 = 0$ has real and equal roots. | |
| **Sol.** | For real and equal roots, $D = 0$<br>$k^2 - 16 = 0$<br>$k = \pm 4$ | ½<br>**1**<br>½ |
| | **OR** | |
| **22. (b)** | If '$\alpha$' and '$\beta$' are the zeroes of the polynomial $p(y) = y^2 - 5y + 3$, then find the value of $\alpha^4\beta^3 + \alpha^3\beta^4$. | |
| **Sol.** | $\alpha + \beta = 5$<br>$\alpha\beta = 3$<br>$\alpha^4\beta^3 + \alpha^3\beta^4 = (\alpha\beta)^3(\alpha + \beta)$<br>$= 27 \times 5 = 135$ | ½<br>½<br>½<br>½ |
| **23.** | The probability of guessing the correct answer of a certain test question is $\frac{x}{12}$. If the probability of not guessing the correct answer is $\frac{5}{6}$, then find the value of x. | |
| **Sol.** | $\frac{x}{12} + \frac{5}{6} = 1$<br>$x = 2$ | **1**<br>**1** |
| **24.** | Prove that the tangents drawn at the ends of a diameter of a circle are parallel. | |

| | | |
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| **Sol.** | <br>Tangents $l$ and $m$ are drawn at the end points A and B of the diameter AB of the circle<br>$\angle 1 = 90°, \angle 2 = 90°$<br>$\therefore\ \angle 1 = \angle 2$<br>But these are alternate interior angles.<br>$\therefore l \parallel m$ | Correct figure ½ Mark<br><br>½<br>½<br><br>½ |
| **25. (a)** | Find the smallest number which is divisible by both 644 and 462. | |
| **Sol.** | $462 = 2 \times 3 \times 7 \times 11$<br>$644 = 2^2 \times 7 \times 23$<br>$\text{LCM}(462, 644) = 2^2 \times 3 \times 7 \times 11 \times 23 = 21252$<br>$\therefore$ Smallest number which is divisible by both 462 and 644 is 21252 | ½<br>½<br>**1** |
| | **OR** | |
| **25. (b)** | Two numbers are in the ratio 4 : 5 and their HCF is 11. Find the LCM of these numbers. | |
| **Sol.** | Let the two numbers be 4x and 5x where x is common factor<br>Now HCF $=$ 11<br>$\therefore$ x $= 11$<br>Numbers are 44 and 55<br>$\text{LCM}(44, 55) = \frac{44 \times 55}{11} = 220$ | ½<br>**1**<br>½ |

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| | **SECTION C**<br>**This section has 6 Short Answer (SA) type questions carrying 3 marks each.** | |
| **26.** | All face cards of spades are removed from a pack of 52 playing cards and the remaining pack is shuffled well. A card is then drawn at random from the remaining pack. Find the probability of getting :<br>(a) a face card<br>(b) an ace or a jack | |
| **Sol.** | After removing face cards of spades, total number of cards $= 52 - 3 = 49$<br>(a) P(a face card) $= \frac{9}{49}$<br>(b) P(an ace or a jack) $= \frac{7}{49}$ *or* $\frac{1}{7}$ | **1**<br>**1**<br>**1** |
| **27.** | In the given figure, PB is a tangent to the circle with centre O at B. AB is a chord of the circle of length 24 cm and at a distance of 5 cm from the centre of the circle. If the length PB of the tangent is 20 cm, find the length of OP.<br><br> | |
| **Sol.** | <br><br>Join OB | ½ |

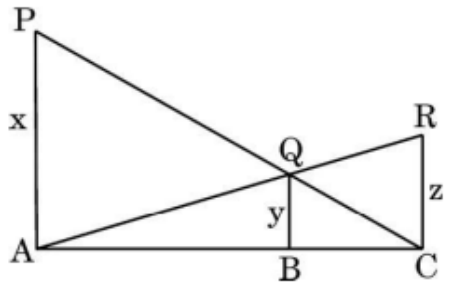
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| | $AB = 24$ cm, OM $= 5$ cm, PB $= 20$ cm | |
| | AM $=$ MB $= 12$ cm | ½ |
| | In $\Delta$OMB, OB $= \sqrt{5^2 + 12^2} = 13$ cm | **1** |
| | As PB is tangent $\Rightarrow$ PB $\perp$ OB | |
| | In rt $\Delta$OBP, OP $= \sqrt{13^2 + 20^2} = \sqrt{569}$ cm | **1** |
| **28.** | A chord of a circle of radius 10 cm subtends a right angle at the centre of the circle. Find the area of the corresponding minor segment. [Use $\pi = 3{\cdot}14$] | |
| **Sol.** |  | |
| | Area of minor segment ACB $=$ Area of sector OACB $-$ Area of right $\Delta$OAB | |
| | Area of sector OACB $= \frac{90}{360} \times 3.14 \times 10 \times 10$ | ½ |
| | $= 78.5$ cm$^2$ | ½ |
| | Area of right $\Delta$OAB $= \frac{1}{2} \times 10 \times 10$ | ½ |
| | $= 50$ cm$^2$ | ½ |
| | Area of minor segment ACB $= (78.5 - 50)$ | ½ |
| | $= 28.5$ cm$^2$ | ½ |

| | | |
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| **29.** | Prove that $\left(5\sqrt{3}+\frac{2}{3}\right)$ is an irrational number given that $\sqrt{3}$ is an irrational number. | |
| **Sol.** | Let $5\sqrt{3}+\frac{2}{3}$ be a rational number.<br>$\therefore 5\sqrt{3}+\frac{2}{3}=\frac{a}{b}$ where a and b are integers and $b \neq 0$.<br>$5\sqrt{3}=\frac{a}{b}-\frac{2}{3}$<br>$\sqrt{3}=\frac{3a-2b}{15b}$<br>$3a-2b$ and $15b$ are integers.<br>$\therefore$ RHS is rational.<br>But LHS $=\sqrt{3}$ is an irrational number which is contradiction to our supposition.<br>Hence $5\sqrt{3}+\frac{2}{3}$ is an irrational number. | **1**<br>**1**<br>½<br>½ |
| **30. (a)** | Prove that : $\sqrt{\frac{\sec A-1}{\sec A+1}}+\sqrt{\frac{\sec A+1}{\sec A-1}}=2\,\text{cosec}\,A$ | |
| **Sol.** | $\text{LHS}=\frac{\sec A-1+\sec A+1}{\sqrt{\sec^2 A-1}}$<br>$=\frac{2\sec A}{\tan A}$<br>$=2\,\text{cosec}A=\text{RHS}$ | **1**<br>**1**<br>**1** |
| | **OR** | |
| **30. (b)** | Prove that : $\left(\frac{1}{\cos A}-\cos A\right)\left(\frac{1}{\sin A}-\sin A\right)=\frac{1}{\tan A+\cot A}$ | |
| **Sol.** | $\text{LHS}=\left(\frac{1-\cos^2 A}{\cos A}\right)\left(\frac{1-\sin^2 A}{\sin A}\right)$<br>$=\frac{\sin^2 A}{\cos A}.\frac{\cos^2 A}{\sin A}$ | **1**<br>½ |

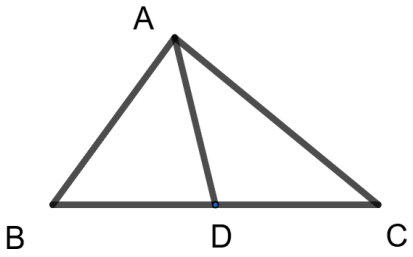
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|---|---|---|
| | $= \text{sinA}.\text{cosA}$<br>$\text{RHS} = \frac{\text{sinA}.\text{cosA}}{\sin^2 A + \cos^2 A}$<br>$= \text{sinA}.\text{cosA}$<br>$\therefore \text{LHS} = \text{RHS}$ | ½<br>½<br>½ |
| **31. (a)** | If the mid-point of the line segment joining the points A(3, 4) and B(k, 6) is P(x, y) and $x + y - 10 = 0$, find the value of k. | |
| **Sol.** | $P(x, y)$ is the mid $-$ point<br>$\therefore (x, y) = (\frac{3+k}{2}, \frac{4+6}{2})$<br>$x = \frac{3+k}{2}, y = 5$<br>$x + y - 10 = 0$<br>$\frac{3+k}{2} + 5 - 10 = 0$<br>$k = 7$ | **1**<br>**1**<br>**1** |
| | **OR** | |
| **31. (b)** | Find the coordinates of the points which divide the line segment joining A(– 2, 2) and B(2, 8) into four equal parts. | |
| **Sol.** | **1 : 1 : 1 : 1**<br>A (–2, 2) P Q R B (2, 8)<br>Coordinates of mid – point Q of AB $= (0, 5)$<br>Coordinates of mid – point P of AQ $= \left(-1, \frac{7}{2}\right)$<br>Coordinates of mid – point R of BQ $= \left(1, \frac{13}{2}\right)$ | **1**<br>**1**<br>**1** |

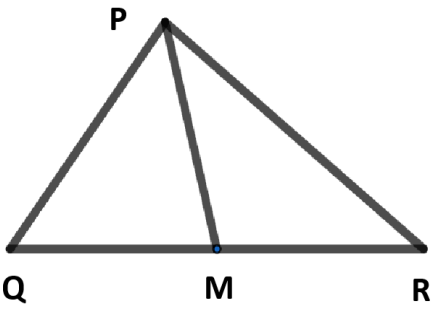
| | | |
|---|---|---|
| | **SECTION D**<br>**This section has 4 Long Answer (LA) type questions carrying 5 marks each.** | |
| **32.** | A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. Also, find the surface area of the toy. (Take $\pi = 3{\cdot}14$) | |
| **Sol.** | Height of the cone $(h)$ =2 cm | |
| | Radius of the hemisphere = radius of the base of the cone $= r = 2$ cm | ½ |
| | Volume of the toy $= \frac{1}{3} \times 3.14 \times (2)^2 \times 2 + \frac{2}{3} \times 3.14 \times (2)^3$ | **1** |
| | $= 25.12\ \text{cm}^3$ | **1** |
| | Slant heightof the cone $(l) = \sqrt{2^2 + 2^2} = 2\sqrt{2}$ cm | ½ |
| | Surface area of the toy $= 3.14 \times 2 \times 2\sqrt{2} + 2 \times 3.14 \times (2)^2$ | **1** |
| | $= 12.56\left(2 + \sqrt{2}\right)\ \text{cm}^2$ | **1** |
| **33.** | The students of a class are made to stand equally in rows. If 3 students are extra in each row, there would be 1 row less. If 3 students are less in a row, there would be 2 more rows. Find the number of students in the class. | |
| **Sol.** | Let number of students in each row be $x$ and the number of rows be $y$ | |
| | $\therefore$ Total number of students $= xy$ | ½ |
| | ATQ, $(x + 3)(y - 1) = xy$ | **1** |
| | $\Rightarrow x - 3y + 3 = 0$ | ½ |
| | Also, $(x - 3)(y + 2) = xy$ | **1** |
| | $\Rightarrow 2x - 3y - 6 = 0$ | ½ |
| | On solving these equations, we get | |
| | $x = 9$ and $y = 4$ | **1** |
| | $\therefore$ Number of students in the class $= xy = 9 \times 4 = 36$ | ½ |
| **34. (a)** | The sum of the third term and the seventh term of an AP is 6 and their product is 8. Find the sum of the first sixteen terms of the AP. | |

| | | |
|---|---|---|
| **Sol.** | Let first term = a and common difference = d | |
| | ATQ, $(a + 2d) + (a + 6d) = 6$ | |
| | $a + 4d = 3$ | ½ |
| | $a = 3 - 4d$ | |
| | Also, $(a + 2d)(a + 6d) = 8$ | ½ |
| | $(3 - 4d + 2d)(3 - 4d + 6d) = 8$ | |
| | $9 - 4d^2 = 8$ | |
| | $d = \pm\frac{1}{2}$ | **1** |
| | When $d = \frac{1}{2} \Longrightarrow a = 1$ | ½ |
| | $S_{16} = \frac{16}{2}\left[2 \times 1 + 15 \times \frac{1}{2}\right]$ | ½ |
| | $= 76$ | ½ |
| | When $d = -\frac{1}{2} \Longrightarrow a = 5$ | ½ |
| | $S_{16} = \frac{16}{2}\left[2 \times 5 + 15 \times (-\frac{1}{2})\right]$ | ½ |
| | $= 20$ | ½ |
| | **OR** | |
| **34. (b)** | The minimum age of children eligible to participate in a painting competition is 8 years. It is observed that the age of the youngest boy was 8 years and the ages of the participants, when seated in order of age, have a common difference of 4 months. If the sum of the ages of all the participants is 168 years, find the age of the eldest participant in the painting competition. | |
| **Sol.** | The ages of the participants form the following AP | |
| | $8, 8\frac{1}{3}, 8\frac{2}{3}, 9, \cdots$ | |
| | where first term = 8 and common difference $= \frac{1}{3}$ | **1** |
| | Let the number of participants be n | |
| | $S_n = \frac{n}{2}\left[2 \times 8 + (n - 1)\frac{1}{3}\right] = 168$ | **1** |

| | | |
|---|---|---|
| | $n^2 + 47n - 1008 = 0$<br>$\Rightarrow n = 16$<br>$\therefore$ the age of the eldest participant $= 8 + 15 \times \frac{1}{3} = 13$ years | **1**<br>**1**<br>**1** |
| **35. (a)** | In the given figure, PA, QB and RC are perpendicular to AC. If PA = x units, QB = y units and RC = z units, prove that $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$.<br><br> | |
| **Sol.** | $\Delta ABQ \sim \Delta ACR$<br>$\frac{AB}{AC} = \frac{QB}{RC} = \frac{y}{z}$ ... ... ... ... ... ... ... ... ... ... (i)<br>Similarly, $\Delta CBQ \sim \Delta CAP$<br>$\frac{BC}{AC} = \frac{QB}{PA} = \frac{y}{x}$ ... ... ... ... ... ... ... ... ... ... (ii)<br>On adding (i) & (ii), we get<br>$\frac{AB}{AC} + \frac{BC}{AC} = \frac{y}{z} + \frac{y}{x}$<br>$\frac{AB + BC}{AC} = y\left(\frac{1}{z} + \frac{1}{x}\right)$<br>$\frac{AC}{AC} = y\left(\frac{1}{z} + \frac{1}{x}\right)$<br>$\therefore \frac{1}{x} + \frac{1}{z} = \frac{1}{y}$ | **1**<br>½<br>½<br>½<br>**1**<br>**1**<br>½ |
| | **OR** | |
| **35. (b)** | Sides AB and BC and median AD of triangle ABC are respectively proportional to sides PQ and QR and median PM of $\Delta$ PQR. Show that $\Delta$ ABC ~ $\Delta$ PQR. | |

| Sol. | | Correct figure<br>1 mark |
|---|---|---|




In $\Delta ABD$ and $\Delta PQM$

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$ (given)

$\frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{AD}{PM}$ **1**

$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

$\therefore \Delta ABD \sim \Delta PQM$ **1**

$\therefore \angle B = \angle Q$ ½

In $\Delta ABC$ and $\Delta PQR$

$\frac{AB}{PQ} = \frac{BC}{QR}$ and $\angle B = \angle Q$ **1**

$\Delta ABC \sim \Delta PQR$ ½

## SECTION E

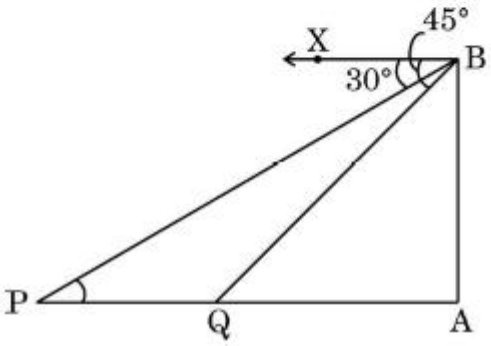
**This section has 3 Case Study based questions carrying 4 marks each.**

**36.**

### Case Study – 1

A lighthouse stands tall on a cliff by the sea, watching over ships that pass by. One day a ship is seen approaching the shore and from the top of the lighthouse, the angles of depression of the ship are observed to be 30° and 45° as it moves from point P to point Q. The height of the lighthouse is 50 metres.






Based on the information given above, answer the following questions :

(i) Find the distance of the ship from the base of the lighthouse when it is at point Q, where the angle of depression is 45°.

(ii) Find the measures of ∠ PBA and ∠ QBA.

(iii) (a) Find the distance travelled by the ship between points P and Q.

**OR**

(b) If the ship continues moving towards the shore and takes 10 minutes to travel from Q to A, calculate the speed of the ship in km/h, from Q to A.

**Sol.**

(i) $\angle AQB = \angle QBX = 45°$ and $\angle APB = \angle PBX = 30°$

In $\Delta AQB, \tan 45° = \frac{50}{AQ}$ ½

$AQ = 50$ m ½

(ii) $\angle PBA = 60°$ ½

| | | |
|---|---|---|
| | $\angle QBA = 45°$ | ½ |
| | $(iii)(a)$ In $\Delta APB, \tan 30° = \frac{50}{AP}$ | ½ |
| | $AP = 50\sqrt{3}$ m | ½ |
| | Distance travelled by the ship $= PQ = 50\sqrt{3} - 50 = 50(\sqrt{3} - 1)$ m | **1** |
| | *or* 36.5 m | |
| | OR | |
| | $(iii)$(b) Speed of the ship $= \frac{50 \text{ metres}}{10 \text{ minutes}}$ | **1** |
| | $= 0.3$ km/h | **1** |
| **37.** | **Case Study – 2**<br>The India Meteorological Department observes seasonal and annual rainfall every year in different sub-divisions of our country. It helps them to compare and analyse the results. | |

The table below shows sub-divisions wise seasonal (monsoon) rainfall (in mm) in 2023.

| *Rainfall (mm)* | *No. of Sub-divisions* |
|---|---|
| 200 – 400 | 3 |
| 400 – 600 | 4 |
| 600 – 800 | 7 |
| 800 – 1000 | 4 |
| 1000 – 1200 | 3 |
| 1200 – 1400 | 3 |

Based on the information given above, answer the following questions :

(i) Write the modal class.

(ii) (a) Find the median of the given data.

**OR**

(b) Find the mean rainfall in the season.

(iii) If a sub-division having at least 800 mm rainfall during monsoon season is considered a good rainfall sub-division, then how many sub-divisions had good rainfall ?

**Sol.**

(i) Modal Class $= 600 - 800$ — **1**

(ii)(a)

| Rainfall (mm) | No. of Sub-divisions ($f_i$) | $cf$ |
|---|---|---|
| 200–400 | 3 | 3 |
| 400–600 | 4 | 7 |
| 600–800 | 7 | 14 |
| 800–1000 | 4 | 18 |
| 1000–1200 | 3 | 21 |
| 1200–1400 | 3 | 24 |

**Correct table ½ mark**

$N = 24$

Median Class $= 600 - 800$ — ½

$\text{Median} = 600 + \frac{12 - 7}{7} \times 200$ — ½

$= \frac{5200}{7}$ or 742.8 mm $(approx.)$ ½

OR

(ii)(b)

| Rainfall (mm) | No. of Sub-divisions ($f_i$) | $x_i$ | $f_i x_i$ |
|---|---|---|---|
| 200−400 | 3 | 300 | 900 |
| 400−600 | 4 | 500 | 2000 |
| 600−800 | 7 | 700 | 4900 |
| 800−1000 | 4 | 900 | 3600 |
| 1000−1200 | 3 | 1100 | 3300 |
| 1200−1400 | 3 | 1300 | 3900 |
| | $\sum f_i = 24$ | | $\sum f_i x_i = 18600$ |

**Correct table 1 Mark**

Mean $= \frac{18600}{24} = 775$ **1**

∴ Mean rainfall $= 775$ mm

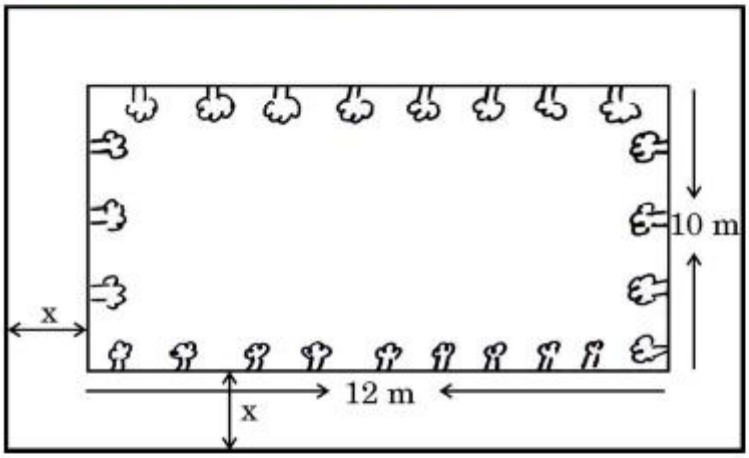
(iii) Required number of sub − divisions $= 4 + 3 + 3 = 10$ **1**

**38.**

**Case Study – 3**

A garden designer is planning a rectangular lawn that is to be surrounded by a uniform walkway.






The total area of the lawn and the walkway is 360 square metres. The width of the walkway is same on all sides. The dimensions of the lawn itself are 12 metres by 10 metres.

Based on the information given above, answer the following questions :

(i) Formulate the quadratic equation representing the total area of the lawn and the walkway, taking width of walkway = x m.

(ii) (a) Solve the quadratic equation to find the width of the walkway 'x'.

**OR**

(b) If the cost of paving the walkway at the rate of ₹ 50 per square metre is ₹ 12,000, calculate the area of the walkway.

(iii) Find the perimeter of the lawn.

**Sol.**

$(i)\ (12 + 2x)(10 + 2x) = 360$ ½

$4x^2 + 44x - 240 = 0\ \ or\ \ x^2 + 11x - 60 = 0$ ½

$(ii)(a)\ (x + 15)(x - 4) = 0$ **1**

$x = 4$

$\therefore$ width of the walkway $= 4$ m **1**

OR

| | | |
|---|---|---|
| | (ii)(b) Area of the walkway $= \frac{12000}{50}$<br>$= 240\ \text{m}^2$<br>(iii) Perimeter of the lawn $= 2(12 + 10) = 44$ m | **1**<br>**1**<br>**1** |