CBSE 2025 Maths Standard — Set 3 — Answer Key

English Medium

Time Allowed: 3 Hours | Maximum Marks: 80

General Instructions:

1. This question paper contains 38 questions. All questions are compulsory.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 6 questions carrying 2 marks each.
4. Section C has 6 questions carrying 3 marks each.
5. Section D has 4 questions carrying 5 marks each.
6. Section E has 3 case-study based questions carrying 4 marks each.
7. Draw neat figures wherever required.

Section A

Multiple Choice Questions (1 mark each)

Q.1 [1 mark]

If tan 3θ = √3, then θ/2 equals:

(a) 60°

(b) 30°

(c) 20°

(d) 10° <<

Explanation:

tan 3θ = √3 means 3θ = 60°, so θ = 20°. Therefore θ/2 = 10°.

Q.2 [1 mark]

If x is the LCM of 4, 6, 8 and y is the LCM of 3, 5, 7 and p is the LCM of x and y, then which of the following is true?

(a) p = 35x <<

(b) p = 4y

(c) p = 8x

(d) p = 16y

Explanation:

x = LCM(4, 6, 8) = 24.

y = LCM(3, 5, 7) = 105.

p = LCM(24, 105).

24 = 2^3 × 3, 105 = 3 × 5 × 7.

LCM = 2^3 × 3 × 5 × 7 = 840.

35x = 35 × 24 = 840 = p.

So p = 35x.

Q.3 [1 mark]

The value of 'k' for which the system of linear equations 6x + y = 3k and 36x + 6y = 3 have infinitely many solutions is:

(a) 6

(b) 1/6 <<

(c) 1/2

(d) 1/3

Explanation:

For infinitely many solutions: $a_1/a_2 = b_1/b_2 = c_1/c_2$.

Here $6/36 = 1/6$, $1/6 = 1/6$, and $3k/3 = k$.

So $k = 1/6$.

Q.4 [1 mark]

If α and β are the zeroes of the polynomial $p(x) = x^2 – ax – b$, then the value of $(\alpha + \beta + \alpha\beta)$ is equal to:

(a) a + b

(b) –a – b

(c) a – b <<

(d) –a + b

Explanation:

For $p(x) = x^2 – ax – b$, sum of zeroes $\alpha + \beta = a$ and product $\alpha\beta = –b$.

So $\alpha + \beta + \alpha\beta = a + (–b) = a – b$.

Q.5 [1 mark]

If $x/12 – 3/x = 0$, then the values of x are:

(a) ±6 <<

(b) ±4

(c) ±12

(d) ±3

Explanation:

$x/12 – 3/x = 0 \Rightarrow x^2/12 = 3 \Rightarrow x^2 = 36 \Rightarrow x = \pm 6$.

Q.6 [1 mark]

The line represented by $x/4 + y/6 = 1$, intersects x-axis and y-axis

respectively at P and Q. The coordinates of the mid-point of line segment PQ are:

(a) (2, 3) <<

(b) (3, 2)

(c) (2, 0)

(d) (0, 3)

Explanation:

When y = 0: x/4 = 1, so x = 4, giving P(4, 0).

When x = 0: y/6 = 1, so y = 6, giving Q(0, 6).

Mid-point = ((4+0)/2, (0+6)/2) = (2, 3).

Q.7 [1 mark]

Two of the vertices of ΔPQR are P(–1, 5) and Q(5, 2). The coordinates of a point which divides PQ in the ratio 2 : 1 are:

(a) (3, –3)

(b) (5, 5)

(c) (3, 3) <<

(d) (5, 1)

Explanation:

Using section formula for ratio 2:1: x = (2×5 + 1×(–1))/(2+1) = (10–1)/3 = 3.

y = (2×2 + 1×5)/(2+1) = (4+5)/3 = 3.

Point is (3, 3).

Q.8 [1 mark]

If tangents PA and PB drawn from an external point P to the circle with centre O are inclined to each other at an angle of 80° as shown in the given figure, then the measure of ∠POA is:

(a) 40°

(b) 50° <<

(c) 60°

(d) 80°

Explanation:

The angle between the two tangents is 80°.

The angle ∠APB = 80°.

Since OA ⊥ PA, in triangle OPA: ∠OPA = 80°/2 = 40° (OP bisects ∠APB).

So ∠POA = 90° – 40° = 50°.

Q.9 [1 mark]

(cot θ + tan θ) equals:

(a) cosec θ sec θ <<

(b) sin θ sec θ

(c) cos θ tan θ

(d) sin θ cos θ

Explanation:

cot θ + tan θ = cos θ/sin θ + sin θ/cos θ = (cos²θ + sin²θ)/(sin θ cos θ) = 1/(sin θ cos θ) = cosec θ sec θ.

Q.10 [1 mark]

If in two triangles ΔDEF and ΔPQR, ∠D = ∠Q and ∠R = ∠E, then which of the following is NOT true?

(a) DE/QR = DF/PQ

(b) EF/PR = DF/PQ

(c) EF/RP = DE/QR <<

(d) DE/PQ = EF/RP

Explanation:

Given ∠D = ∠Q and ∠R = ∠E, so ∠F = ∠P.

Therefore ΔDEF ~ ΔQPR (by AAA).

Corresponding sides: DE/QP = EF/PR = DF/QR.

Option (C) states EF/RP = DE/QR which rearranges to EF/DE = RP/QR.

From the similarity EF/PR = DE/QP, so EF/DE = PR/QP.

This means RP/QR should equal PR/QP which is not necessarily true.

So (C) is NOT true.

Q.11 [1 mark]

The measurements of ΔLMN and ΔABC are shown in the figure given below. The length of side AC is:

[Figure: ΔLMN has LM = 45 cm, MN = 63 cm, ∠M = 130°, ∠N = 28°. ΔABC has AB = 5 cm, ∠B = 130°. LN = 72 cm.]

(a) 16 cm

(b) 7 cm

(c) 8 cm <<

(d) 4 cm

Explanation:

In ΔLMN: ∠M = 130°, ∠N = 28°, so ∠L = 180° – 130° – 28° = 22°.

In ΔABC: ∠B = 130°.

Since the triangles are similar with ∠M = ∠B = 130°, corresponding sides: AB/LM = AC/LN.

So 5/45 = AC/72.

AC = 72 × 5/45 = 8 cm.

Q.12 [1 mark]

If the volumes of two cubes are in the ratio 8 : 125, then the ratio of their surface areas is:

(a) 8 : 125

(b) 4 : 25 <<

(c) 2 : 5

(d) 16 : 25

Explanation:

Ratio of volumes = 8 : 125 = $2^3 : 5^3$, so ratio of sides = 2 : 5.

Ratio of surface areas = $2^2 : 5^2$ = 4 : 25.

Q.13 [1 mark]

If the area of a sector of circle of radius 36 cm is 54π cm^2, then the length of the corresponding arc of the sector is:

(a) 8π cm

(b) 6π cm

(c) 4π cm

(d) 3π cm <<

Explanation:

Area of sector = (θ/360)πr² = 54π.

So (θ/360) × 36² = 54.

(θ/360) × 1296 = 54.

θ/360 = 54/1296 = 1/24.

Arc length = (θ/360) × 2πr = (1/24) × 2π × 36 = 72π/24 = 3π cm.

Q.14 [1 mark]

A die is thrown once. The probability of getting a number which is NOT a factor of 36, is:

(a) 1/2

(b) 2/3

(c) 1/6 <<

(d) 5/6

Explanation:

Factors of 36 from {1,2,3,4,5,6} are {1,2,3,4,6} = 5 numbers.

Non-factors = {5} = 1 number.

Probability = 1/6.

Q.15 [1 mark]

If the mean of 2, 9, x+6, 2x+3, 5, 10, 5 is 7, then the value of x is:

(a) 9

(b) 6

(c) 5

(d) 3 <<

Explanation:

Mean = (2 + 9 + x+6 + 2x+3 + 5 + 10 + 5)/7 = 7.

So (40 + 3x)/7 = 7.

40 + 3x = 49.

3x = 9.

x = 3.

Q.16 [1 mark]

AOBC is a rectangle whose three vertices are A(0, 2), O(0, 0) and B(4, 0). The square of the length of its diagonal is equal to:

(a) 36

(b) 20 <<

(c) 16

(d) 4

Explanation:

The diagonal can be AB or OC.

Using A(0,2) and B(4,0): $AB^2 = (4–0)^2 + (0–2)^2 = 16 + 4 = 20$.

Q.17 [1 mark]

Zeroes of the polynomial $p(x) = x^2 – 3\sqrt{2}\,x + 4$ are:

(a) $2, \sqrt{2}$

(b) $2\sqrt{2}, \sqrt{2}$ <<

(c) $4\sqrt{2}, –\sqrt{2}$

(d) $\sqrt{2}, 2$

Explanation:

$p(x) = x^2 – 3\sqrt{2}\,x + 4$.

Using quadratic formula or factoring: $x^2 – 3\sqrt{2}\,x + 4 = (x – 2\sqrt{2})(x – \sqrt{2}) = 0$.

Check: $2\sqrt{2} \times \sqrt{2} = 2 \times 2 = 4$ ✓.

$2\sqrt{2} + \sqrt{2} = 3\sqrt{2}$ ✓.

Zeroes are $2\sqrt{2}$ and $\sqrt{2}$.

Q.18 [1 mark]

In the given figure, in ΔABC, AD ⊥ BC and ∠BAC = 90°. If BC = 16 cm and DC = 4 cm, then the value of x is:

[Figure shows triangle ABC with AD perpendicular to BC, BC = 16 cm, DC = 4 cm, AC = x]

(a) 4 cm

(b) 5 cm

(c) 8 cm <<

(d) 3 cm

Explanation:

In right ΔABC with ∠BAC = 90° and AD ⊥ BC: By geometric mean relation, $AC^2 = DC \times BC = 4 \times 16 = 64$.

So AC = x = 8 cm.

Q.19 [1 mark]

Assertion (A): A ladder leaning against a wall, stands at a horizontal distance of 6 m from the wall. If the height of the wall up to which the ladder reaches is 8 m, then the length of the ladder is 10 m.

Reason (R): The ladder makes an angle of 60° with the ground.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is NOT the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false. <<

(d) Assertion (A) is false, but Reason (R) is true.

Explanation:

Assertion: Ladder length = $\sqrt{(6^2 + 8^2)} = \sqrt{(36 + 64)} = \sqrt{100} = 10$ m.

True.

Reason: $\tan\theta = 8/6 = 4/3$.

$\theta = \tan^{-1}(4/3) \approx 53.13°$, not 60°.

So Reason is false.

Answer: (C).

Q.20 [1 mark]

Assertion (A): If two tangents are drawn to a circle from an external point, then they subtend equal angles at the centre of the circle.

Reason (R): A parallelogram circumscribing a circle is a rhombus.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is NOT the correct explanation of the Assertion (A). <<

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

Explanation:

Assertion: Two tangents from an external point are equal in length and the line from external point to centre bisects the angle between tangents.

The two tangles OA and OB (where A, B are points of tangency) are equal (radii), and PA = PB.

So triangles OPA and OPB are congruent, meaning ∠AOP = ∠BOP.

True.

Reason: A parallelogram circumscribing a circle is indeed a rhombus.

True.

But R does not explain A.

Answer: (B).

Section B

Very Short Answer Questions (2 marks each)

Q.21 [2 marks]

If $4k = \tan^2 60° - 2\,\text{cosec}^2 30° - 2\tan^2 30°$, then find the value of k.

Answer: k = -17/12

Explanation:

$\tan^2 60° = (\sqrt{3})^2 = 3$, $\operatorname{cosec}^2 30° = 2^2 = 4$, $\tan^2 30° = (1/\sqrt{3})^2 = 1/3$.

$4k = 3 – 2(4) – 2(1/3) = 3 – 8 – 2/3 = (9 – 24 – 2)/3 = –17/3$.

$k = –17/12$.

Q.22 [2 marks]

The probability of guessing the correct answer of a certain test question is x/12. If the probability of not guessing the correct answer is 5/6, then find the value of x.

Answer: x = 2

Explanation:

P(correct) + P(not correct) = 1.

x/12 + 5/6 = 1.

x/12 = 1 – 5/6 = 1/6.

x = 12/6 = 2.

Q.23 [2 marks]

Find the smallest number which is divisible by both 644 and 462.

Answer: 21252

Explanation:

The smallest number divisible by both is their LCM.

$644 = 2^2 \times 7 \times 23$; $462 = 2 \times 3 \times 7 \times 11$.

LCM = $2^2 \times 3 \times 7 \times 11 \times 23 = 21252$.

OR

Q.23 [2 marks]

[OR] Two numbers are in the ratio 4 : 5 and their HCF is 11. Find the LCM of these numbers.

Answer: 220

Explanation:

Let the numbers be 4 × 11 = 44 and 5 × 11 = 55.

LCM = (44 × 55)/HCF = (44 × 55)/11 = 4 × 55 = 220.

Or: LCM = 4 × 5 × 11 = 220.

Q.24 [2 marks]

Find the value(s) of 'k' so that the quadratic equation $4x^2 + kx + 1 = 0$ has real and equal roots.

Answer: $k = \pm 4$

Explanation:

For real and equal roots, discriminant = 0.

$D = k^2 - 4(4)(1) = k^2 - 16 = 0$.

$k^2 = 16$.

$k = \pm 4$.

OR

Q.24 [2 marks]

[OR] If 'α' and 'β' are the zeroes of the polynomial $p(y) = y^2 - 5y + 3$, then find the value of $\alpha^4\beta^3 + \alpha^3\beta^4$.

Answer: 135

Explanation:

$\alpha + \beta = 5$, $\alpha\beta = 3$.

$\alpha^4\beta^3 + \alpha^3\beta^4 = \alpha^3\beta^3(\alpha + \beta) = (\alpha\beta)^3(\alpha + \beta) = 3^3 \times 5 = 27 \times 5 = 135$.

Q.25 [2 marks]

Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Answer: Proved: tangents at A and B are both $\perp$ to diameter AB, so they are parallel.

Explanation:

Let AB be a diameter of a circle with centre O.

Let PQ be the tangent at A and RS be the tangent at B.

Since PQ is tangent at A, OA ⊥ PQ, so ∠OAP = 90°.

Since RS is tangent at B, OB ⊥ RS, so ∠OBR = 90°.

Now ∠OAP + ∠OBR = 90° + 90° = 180°.

Since ∠OAP and ∠OBR are co-interior angles (same side of transversal AB) and their sum is 180°, PQ ∥ RS.

Hence the tangents at the ends of a diameter are parallel.

Q.26 [2 marks]

If the mid-point of the line segment joining the points A(3, 4) and B(k, 6) is P(x, y) and x + y – 10 = 0, find the value of k.

Answer: k = 7

Explanation:

Mid-point P = ((3+k)/2, (4+6)/2) = ((3+k)/2, 5).

So x = (3+k)/2 and y = 5.

Given x + y – 10 = 0: (3+k)/2 + 5 = 10.

(3+k)/2 = 5.

3+k = 10.

k = 7.

OR

Q.26 [2 marks]

[OR] Find the coordinates of the points which divide the line segment joining A(–2, 2) and B(2, 8) into four equal parts.

Answer: (-1, 7/2), (0, 5), (1, 13/2)

Explanation:

The three points dividing AB into 4 equal parts use ratios 1:3, 2:2 (i.e., 1:1), and 3:1.

For 1:3: x = (1×2 + 3×(–2))/4 = (2–6)/4 = –1, y = (1×8 + 3×2)/4 = (8+6)/4 = 14/4 = 7/2.

Point: (–1, 7/2).

For 1:1: x = (–2+2)/2 = 0, y = (2+8)/2 = 5.

Point: (0, 5).

For 3:1: x = (3×2 + 1×(–2))/4 = (6–2)/4 = 1, y = (3×8 + 1×2)/4 = (24+2)/4 = 26/4 = 13/2.

Point: (1, 13/2).

Section C

Short Answer Questions (3 marks each)

Q.27 [3 marks]

Prove that (5√3 + 2/3) is an irrational number given that √3 is an irrational number.

Answer: Proved by contradiction: if 5√3 + 2/3 were rational, √3 = (3a – 2b)/(15b) would be rational.

Explanation:

Assume (5√3 + 2/3) is rational.

Then 5√3 + 2/3 = a/b for some integers a, b (b ≠ 0).

So 5√3 = a/b – 2/3 = (3a – 2b)/(3b).

Thus √3 = (3a – 2b)/(15b).

Since a, b are integers, (3a – 2b)/(15b) is rational.

But √3 is given to be irrational.

This is a contradiction.

Hence (5√3 + 2/3) is irrational.

Q.28 [3 marks]

Prove that: √((sec A – 1)/(sec A + 1)) + √((sec A + 1)/(sec A – 1)) = 2 cosec A

Answer: Proved: LHS = 2 sec A / tan A = 2 cosec A

Explanation:

LHS = √((sec A – 1)/(sec A + 1)) + √((sec A + 1)/(sec A – 1)).

Rationalize: multiply first term by √(sec A – 1)/√(sec A – 1) and second by √(sec A + 1)/√(sec A + 1).

LHS = (sec A – 1)/√(sec²A – 1) + (sec A + 1)/√(sec²A – 1) = (sec A – 1 + sec A + 1)/√(tan²A) = 2 sec A / |tan A| = 2 sec A / tan A = 2(1/cos A)/(sin A/cos A) = 2/sin A = 2

cosec A.

Hence proved.

OR

Q.28 [3 marks]

[OR] Prove that: (1/cos A – cos A)(1/sin A – sin A) = 1/(tan A + cot A)

Answer: Proved: LHS = RHS = sin A cos A

Explanation:

LHS = (1/cos A – cos A)(1/sin A – sin A) = ((1 – cos²A)/cos A)((1 – sin²A)/sin A) = (sin²A/cos A)(cos²A/sin A) = sin A cos A.

RHS = 1/(tan A + cot A) = 1/(sin A/cos A + cos A/sin A) = 1/((sin²A + cos²A)/(sin A cos A)) = sin A cos A.

LHS = RHS.

Hence proved.

Q.29 [3 marks]

A chord of a circle of radius 10 cm subtends a right angle at the centre of the circle. Find the area of the corresponding minor segment. [Use π = 3.14]

Answer: 28.5 cm²

Explanation:

Area of minor segment = Area of sector – Area of triangle.

Area of sector = (θ/360)πr² = (90/360) × 3.14 × 100 = 78.5 cm².

Area of triangle = (1/2)r² sinθ = (1/2) × 100 × 1 = 50 cm².

Area of minor segment = 78.5 – 50 = 28.5 cm².

Q.30 [3 marks]

Three unbiased coins are tossed simultaneously. Find the probability of getting:

(a) exactly two tails

(b) at least one head

(c) at most two heads

Answer: (a) 3/8, (b) 7/8, (c) 7/8

Explanation:

Total outcomes when 3 coins are tossed = $2^3 = 8$: {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}.

(a) Exactly two tails: {HTT, THT, TTH} = 3.

P = 3/8.

(b) At least one head: All except TTT = 7.

P = 7/8.

(c) At most two heads: All except HHH = 7.

P = 7/8.

Q.31 [3 marks]

In the given figure, PC is a tangent to the circle at C. AOB is the diameter which when extended meets the tangent at P. If ∠PCA = 110°, find ∠CBA and ∠BCO.

[Figure shows circle with centre O, diameter AOB extended to P, PC tangent at C]

Answer: ∠CBA = 70°, ∠BCO = 70°

Explanation:

OC ⊥ PC (radius ⊥ tangent), so ∠OCP = 90° and ∠OCA = ∠PCA − ∠OCP = 110° − 90° = 20°.

∠ACB = 90° (angle in a semicircle), so ∠BCO = ∠ACB − ∠OCA = 90° − 20° = 70°.

In ΔOBC, OB = OC (radii), so ∠CBA = ∠OBC = ∠OCB = 70°.

Q.32 [3 marks]

The perimeter of an isosceles triangle is 32 cm. If each equal side is 5/6 th of the base, find the area of the triangle.

Answer: 48 cm^2

Explanation:

Let the base = b cm.

Each equal side = 5b/6.

Perimeter: $b + 5b/6 + 5b/6 = 32$.

$b + 10b/6 = 32$.

$b + 5b/3 = 32$.

$(3b + 5b)/3 = 32$.

$8b/3 = 32$.

$b = 12$ cm.

Equal side $= 5 \times 12/6 = 10$ cm.

Height $h = \sqrt{(10^2 – 6^2)} = \sqrt{(100 – 36)} = \sqrt{64} = 8$ cm.

Area $= (1/2) \times 12 \times 8 = 48$ cm².

Section D

Long Answer Questions (5 marks each)

Q.33 [5 marks]

The sum of the third and the seventh term of an AP is 6 and their product is 8. Find the sum of the first sixteen terms of the AP.

Answer: 76 or 20

Explanation:

Let first term = a, common difference = d.

$a_3 = a + 2d$, $a_7 = a + 6d$.

Given: $(a + 2d) + (a + 6d) = 6$, so $2a + 8d = 6$, i.e., $a + 4d = 3$, so $a = 3 – 4d$.

Also $(a + 2d)(a + 6d) = 8$.

Substituting: $(3 – 2d)(3 + 2d) = 8$.

$9 – 4d^2 = 8$.

$4d^2 = 1$.

$d^2 = 1/4$.

$d = \pm 1/2$.

If $d = 1/2$: $a = 3 – 2 = 1$.

$S_{16} = (16/2)[2(1) + 15(1/2)] = 8[2 + 7.5] = 8 \times 9.5 = 76$.

If $d = –1/2$: $a = 3 + 2 = 5$.

$S_{16} = 8[10 + 15(–1/2)] = 8[10 – 7.5] = 8 \times 2.5 = 20$.

OR

Q.33 [5 marks]

[OR] The minimum age of children eligible to participate in a painting competition is 8 years. It is observed that the age of the youngest boy was 8 years and the ages of the participants, when seated in order of age, have a common difference of 4 months. If the sum of the ages of all the participants is 168 years, find the age of the eldest participant in the painting competition.

Answer: 13 years

Explanation:

First term a = 8 years, common difference d = 4 months = 1/3 year.

Sum $S_n = 168$.

$S_n = (n/2)[2a + (n–1)d] = (n/2)[16 + (n–1)/3] = (n/2)[(48 + n – 1)/3] = (n/2)[(47 + n)/3] = n(47 + n)/6$.

So $n(47 + n) = 1008$.

$n^2 + 47n – 1008 = 0$.

Using quadratic formula: $n = (–47 + \sqrt{(2209 + 4032)})/2 = (–47 + \sqrt{6241})/2 = (–47 + 79)/2 = 16$.

Age of eldest = $a + (n–1)d = 8 + 15 \times 1/3 = 8 + 5 = 13$ years.

Q.34 [5 marks]

In the given figure, PA, QB and RC are perpendicular to AC. If PA = x units, QB = y units and RC = z units, prove that $1/x + 1/z = 1/y$.

[Figure shows points A, B, C on a line with perpendiculars PA, QB, RC]

Answer: Proved: $y/x + y/z = BC/AC + AB/AC = 1$, so $1/x + 1/z = 1/y$

Explanation:

In ΔPAC and ΔQBC: $\angle PAC = \angle QBC = 90°$ and $\angle C$ is common.

So $\Delta PAC \sim \Delta QBC$.

Hence $QB/PA = BC/AC$, i.e., $y/x = BC/AC$ ...

(1).

In ΔRCA and ΔQBA: $\angle RCA = \angle QBA = 90°$ and $\angle A$ is common.

So $\Delta RCA \sim \Delta QBA$.

Hence QB/RC = AB/AC, i.e., y/z = AB/AC ...

(2).

Adding (1) and (2): y/x + y/z = BC/AC + AB/AC = (BC + AB)/AC = AC/AC = 1.

So y/x + y/z = 1.

Dividing both sides by y: 1/x + 1/z = 1/y.

Hence proved.

OR

Q.34 [5 marks]

[OR] Sides AB and BC and median AD of triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR. Show that ΔABC ~ ΔPQR.

Answer: Proved: ΔABD ~ ΔPQM (SSS) gives ∠B = ∠Q, so ΔABC ~ ΔPQR (SAS)

Explanation:

Given: AB/PQ = BC/QR = AD/PM.

Since AD is median of ΔABC, D is midpoint of BC, so BD = BC/2.

Since PM is median of ΔPQR, M is midpoint of QR, so QM = QR/2.

So AB/PQ = 2BD/2QM = BD/QM = AD/PM.

In ΔABD and ΔPQM: AB/PQ = BD/QM = AD/PM.

By SSS similarity, ΔABD ~ ΔPQM.

Therefore ∠B = ∠Q.

Now in ΔABC and ΔPQR: AB/PQ = BC/QR and ∠B = ∠Q.

By SAS similarity, ΔABC ~ ΔPQR.

Q.35 [5 marks]

A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm, are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Answer: 100

Explanation:

Volume of cone = $(1/3)\pi r^2 h = (1/3)\pi(25)(8) = 200\pi/3$ cm³.

One-fourth of water flows out = $(1/4) \times 200\pi/3 = 50\pi/3$ cm³.

Volume of each lead shot = $(4/3)\pi(0.5)^3 = (4/3)\pi(1/8) = \pi/6$ cm³.

Number of lead shots = $(50\pi/3) \div (\pi/6) = (50/3) \times 6 = 100$.

Q.36(a) [5 marks]

[Case Study 1 - Garden Lawn] A garden designer is planning a rectangular lawn that is to be surrounded by a uniform walkway. The total area of the lawn and the walkway is 360 square metres. The width of the walkway is same on all sides. The dimensions of the lawn itself are 12 metres × 10 metres.

(i) Formulate the quadratic equation representing the total area of the lawn and the walkway, taking width of walkway = x m.

Answer: $4x^2 + 44x - 240 = 0$ (or $x^2 + 11x - 60 = 0$)

Explanation:

Total length = 12 + 2x, total width = 10 + 2x.

Total area = $(12 + 2x)(10 + 2x) = 360$.

$120 + 24x + 20x + 4x^2 = 360$.

$4x^2 + 44x + 120 = 360$.

$4x^2 + 44x - 240 = 0$.

Simplifying: $x^2 + 11x - 60 = 0$.

Q.36(b) [5 marks]

[Case Study 1 - Garden Lawn] (ii)(a) Solve the quadratic equation to find the width of the walkway 'x'.

Answer: x = 4 m

Explanation:

$x^2 + 11x - 60 = 0$.

Using quadratic formula: $x = (-11 \pm \sqrt{(121 + 240)})/2 = (-11 \pm \sqrt{361})/2 = (-11 \pm 19)/2$.

x = 8/2 = 4 or x = –30/2 = –15.

Since width must be positive, x = 4 m.

OR

Q.36(b) [5 marks]

[Case Study 1 - Garden Lawn] [OR] (ii)(b) If the cost of paving the walkway at the rate of Rs.50 per square metre is Rs.12,000, calculate the area of the walkway.

Answer: 240 m²

Explanation:

Cost = Rate × Area. 12000 = 50 × Area. Area of walkway = 12000/50 = 240 m².

Q.36(c) [5 marks]

[Case Study 1 - Garden Lawn] (iii) Find the perimeter of the lawn.

Answer: 44 m

Explanation:

The lawn dimensions are 12 m × 10 m. Perimeter = 2(12 + 10) = 2 × 22 = 44 m.

Q.37(a) [4 marks]

[Case Study 2 - Lighthouse] A lighthouse stands tall on a cliff by the sea, watching over ships that pass by. One day a ship is seen approaching the shore and from the top of the lighthouse, the angles of depression of the ship are observed to be 30° and 45° as it moves from point P to point Q. The height of the lighthouse is 50 metres.

(i) Find the distance of the ship from the base of the lighthouse when it is at point Q, where the angle of depression is 45°.

Answer: 50 m

Explanation:

Let the base of the lighthouse be A.

When angle of depression is 45°, tan 45° = height/distance = 50/QA.
So QA = 50/tan 45° = 50/1 = 50 m.

Q.37(b) [4 marks]

[Case Study 2 - Lighthouse] (ii) Find the measures of ∠PBA and ∠QBA.

Answer: ∠PBA = 30° and ∠QBA = 45°

Explanation:
The angle of depression from the top equals the angle of elevation from the base (alternate interior angles).
So ∠PBA = 30° (angle of elevation from P) and ∠QBA = 45° (angle of elevation from Q).

Q.37(c) [4 marks]

[Case Study 2 - Lighthouse] (iii)(a) Find the distance travelled by the ship between points P and Q.

Answer: 50(√3 – 1) m

Explanation:
From P: tan 30° = 50/PA.
PA = 50/tan 30° = 50√3 m.
From Q: QA = 50 m.
Distance PQ = PA – QA = 50√3 – 50 = 50(√3 – 1) m.

OR

Q.37(c) [4 marks]

[Case Study 2 - Lighthouse] [OR] (iii)(b) If the ship continues moving towards the shore and takes 10 minutes to travel from Q to A, calculate the speed of the ship in km/h, from Q to A.

Answer: 0.3 km/h

Explanation:
Distance QA = 50 m = 0.05 km.
Time = 10 minutes = 1/6 hour.

Speed = distance/time = 0.05/(1/6) = 0.05 × 6 = 0.3 km/h.

Q.38(a) [4 marks]

[Case Study 3 - Rainfall Data] The India Meteorological Department observes seasonal and annual rainfall every year in different sub-divisions of our country. It helps them to compare and analyse the results.

The table below shows sub-divisions wise seasonal (monsoon) rainfall (in mm) in 2023:

Rainfall (mm)	No. of Sub-divisions
200 – 400	3
400 – 600	4
600 – 800	7
800 – 1000	4
1000 – 1200	3
1200 – 1400	3

(i) Write the modal class.

Answer: 600 – 800

Explanation:

The modal class is the class with the highest frequency.

The class 600–800 has the highest frequency of 7.

Q.38(b) [4 marks]

[Case Study 3 - Rainfall Data] (ii)(a) Find the median of the given data.

Answer: ≈ 742.86 mm

Explanation:

N = 3 + 4 + 7 + 4 + 3 + 3 = 24, N/2 = 12.

Cumulative frequencies: 3, 7, 14, 18, 21, 24 → median class 600–800.

l = 600, cf = 7, f = 7, h = 200.

Median = 600 + ((12 – 7)/7) × 200 = 600 + 1000/7 ≈ 742.86 mm.

OR

Q.38(b) [4 marks]

[Case Study 3 - Rainfall Data] [OR] (ii)(b) Find the mean rainfall in the season.

Answer: 775 mm

Explanation:

Using mid-values: Σfx = 300×3 + 500×4 + 700×7 + 900×4 + 1100×3 + 1300×3 = 900 + 2000 + 4900 + 3600 + 3300 + 3900 = 18600.

N = 24.

Mean = 18600/24 = 775 mm.

Q.38(c) [4 marks]

[Case Study 3 - Rainfall Data] (iii) If a sub-division having at least 800 mm rainfall during monsoon season is considered a good rainfall sub-division, then how many sub-divisions had good rainfall?

Answer: 10

Explanation:

Sub-divisions with at least 800 mm rainfall: 800–1000: 4, 1000–1200: 3, 1200–1400: 3.

Total = 4 + 3 + 3 = 10.