CBSE 2023 Maths Standard — Set 1 — Answer Key

English Medium

Time Allowed: 3 Hours | Maximum Marks: 80

General Instructions:

1. This question paper contains 38 questions. All questions are compulsory.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 6 questions carrying 2 marks each.
4. Section C has 6 questions carrying 3 marks each.
5. Section D has 4 questions carrying 5 marks each.
6. Section E has 3 case-study based questions carrying 4 marks each.
7. Draw neat figures wherever required.

Section A

Multiple Choice Questions (1 mark each)

Q.1 [1 mark]

The graph of y = p(x) is given, for a polynomial p(x). The number of zeroes of p(x) from the graph is

(a) 3

(b) 1 <<

(c) 2

(d) 0

Explanation:

The graph of the polynomial touches or crosses the x-axis at only one point.

The number of zeroes of p(x) equals the number of times the graph intersects the x-axis, which is 1.

Q.2 [1 mark]

The value of k for which the pair of equations kx = y + 2 and 6x = 2y + 3 has infinitely many solutions,

(a) is k = 3

(b) does not exist <<

(c) is k = -3

(d) is k = 4

Explanation:

Rewriting: kx - y = 2 and 6x - 2y = 3.

For infinitely many solutions: k/6 = -1/(-2) = 2/3.

From k/6 = 1/2, k = 3.

But checking constant ratio: 2/3 != 1/2.

Since the ratios of coefficients are not all equal (a1/a2 = b1/b2 but != c1/c2), the system is inconsistent for k = 3 and no value of k gives infinitely many solutions.

So the answer does not exist.

Q.3 [1 mark]

If p - 1, p + 1 and 2p + 3 are in A.P., then the value of p is

(a) -2

(b) 4

(c) 0 <<

(d) 2

Explanation:

For three terms in A.P., the middle term equals the average of the other two: 2(p + 1) = (p - 1) + (2p + 3).

So 2p + 2 = 3p + 2, giving p = 0.

Q.4 [1 mark]

In what ratio does the x-axis divide the line segment joining the points A(3, 6) and B(-12, -3)?

(a) 1 : 2

(b) 1 : 4

(c) 4 : 1

(d) 2 : 1 <<

Explanation:

Let the x-axis divide AB in the ratio k : 1.

The y-coordinate of the dividing point = 0.

Using section formula: (k*(-3) + 1*6)/(k + 1) = 0.

So -3k + 6 = 0, k = 2.

The ratio is 2 : 1.

Q.5 [1 mark]

In the given figure, PQ is tangent to the circle centred at O. If angle AOB = 95 degrees, then the measure of angle ABQ will be

(a) 47.5 degrees <<

(b) 42.5 degrees

(c) 85 degrees

(d) 95 degrees

Explanation:

OA and OB are radii, so triangle OAB is isosceles.

Angle OAB = angle OBA = (180 - 95)/2 = 42.5 degrees.

Since PQ is tangent at B, angle OBQ = 90 degrees.

Therefore angle ABQ = angle OBQ - angle OBA = 90 - 42.5 = 47.5 degrees.

Q.6 [1 mark]

If 2 tan A = 3, then the value of (4 sin A + 3 cos A)/(4 sin A - 3 cos A) is

(a) 7/sqrt(13)

(b) 1/sqrt(13)

(c) 3 <<

(d) does not exist

Explanation:

Given 2 tan A = 3, so tan A = 3/2.

Divide numerator and denominator by cos A: (4 tan A + 3)/(4 tan A - 3) = (4*(3/2) + 3)/(4*(3/2) - 3) = (6 + 3)/(6 - 3) = 9/3 = 3.

Q.7 [1 mark]

If alpha and beta are the zeroes of a polynomial p(x) = x^2 + x - 1, then 1/alpha + 1/beta equals

(a) 1 <<

(b) 2

(c) -1

(d) -1/2

Explanation:

For p(x) = x^2 + x - 1: sum of zeroes alpha + beta = -1, product alpha * beta = -1.

So 1/alpha + 1/beta = (alpha + beta)/(alpha * beta) = -1/(-1) = 1.

Q.8 [1 mark]

The least positive value of k, for which the quadratic equation 2x^2 + kx - 4 =

0 has rational roots, is

(a) +/- 2*sqrt(2)

(b) 2 <<

(c) +/- 2

(d) sqrt(2)

Explanation:

For rational roots, discriminant must be a perfect square.

D = k^2 - 4(2)(-4) = k^2 + 32.

For k = 2: D = 4 + 32 = 36, which is a perfect square (6^2).

So the least positive value of k is 2.

Q.9 [1 mark]

(3/4)*tan^2(30) - sec^2(45) + sin^2(60) is equal to

(a) -1 <<

(b) 5/6

(c) -3/2

(d) 1/6

Explanation:

tan(30) = 1/sqrt(3), so tan^2(30) = 1/3.

sec(45) = sqrt(2), so sec^2(45) = 2.

sin(60) = sqrt(3)/2, so sin^2(60) = 3/4.

Expression = (3/4)*(1/3) - 2 + 3/4 = 1/4 - 2 + 3/4 = 1 - 2 = -1.

Q.10 [1 mark]

Curved surface area of a cylinder of height 5 cm is 94.2 cm^2. Radius of the cylinder is (Take pi = 3.14)

(a) 2 cm

(b) 3 cm <<

(c) 2.9 cm

(d) 6 cm

Explanation:

CSA of cylinder = 2*pi*r*h.

94.2 = 2 * 3.14 * r * 5 = 31.4r.

So r = 94.2/31.4 = 3 cm.

Q.11 [1 mark]

The distribution below gives the marks obtained by 80 students on a test:

Marks: Less than 10 | Less than 20 | Less than 30 | Less than 40 | Less than 50 | Less than 60

Number of Students: 3 | 12 | 27 | 57 | 75 | 80

The modal class of this distribution is:

(a) 10-20

(b) 20-30

(c) 30-40 <<

(d) 50-60

Explanation:

Converting cumulative frequencies to class frequencies: 0-10: 3, 10-20: 12-3=9, 20-30: 27-12=15, 30-40: 57-27=30, 40-50: 75-57=18, 50-60: 80-75=5.

The highest frequency is 30 for class 30-40.

So modal class is 30-40.

Q.12 [1 mark]

The curved surface area of a cone having height 24 cm and radius 7 cm, is

(a) 528 cm^2

(b) 1056 cm^2

(c) 550 cm^2 <<

(d) 500 cm^2

Explanation:

Slant height l = sqrt(h^2 + r^2) = sqrt(576 + 49) = sqrt(625) = 25 cm.

CSA = pi*r*l = (22/7)*7*25 = 22*25 = 550 cm^2.

Q.13 [1 mark]

The distance between the points (0, 2*sqrt(5)) and (-2*sqrt(5), 0) is

(a) 2*sqrt(10) units <<
(b) 4*sqrt(10) units
(c) 2*sqrt(20) units
(d) 0

Explanation:

Distance = sqrt((-2*sqrt(5) - 0)^2 + (0 - 2*sqrt(5))^2) = sqrt(4*5 + 4*5) = sqrt(20 + 20) = sqrt(40) = 2*sqrt(10) units.

Q.14 [1 mark]

Which of the following is a quadratic polynomial having zeroes -2/3 and 2/3?

(a) 4x^2 - 9
(b) (4/9)*(9x^2 + 4)
(c) x^2 + 9/4
(d) 5(9x^2 - 4) <<

Explanation:

A quadratic with zeroes -2/3 and 2/3 has sum of zeroes = 0 and product = (-2/3)(2/3) = -4/9.

The polynomial is k(x^2 - 4/9) = k/9 * (9x^2 - 4).

For k = 5: 5(9x^2 - 4) which has zeroes at x = +/- 2/3.

Q.15 [1 mark]

If the value of each observation of a statistical data is increased by 3, then the mean of the data

(a) remains unchanged
(b) increases by 3 <<
(c) increases by 6
(d) increases by 3n

Explanation:

When each observation is increased by a constant c, the mean also increases by c.

Here c = 3, so the mean increases by 3.

Q.16 [1 mark]

Probability of happening of an event is denoted by p and probability of non-happening of the event is denoted by q. Relation between p and q is

(a) p + q = 1 <<

(b) p = 1, q = 1

(c) p = q - 1

(d) p + q + 1 = 0

Explanation:

The sum of probability of an event happening and not happening is always 1.

Therefore p + q = 1.

Q.17 [1 mark]

A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, how many tickets has she bought?

(a) 40

(b) 240

(c) 480 <<

(d) 750

Explanation:

P(winning) = Number of tickets bought / Total tickets.

0.08 = n/6000.

n = 0.08 * 6000 = 480.

Q.18 [1 mark]

In a group of 20 people, 5 can't swim. If one person is selected at random, then the probability that he/she can swim, is

(a) 3/4 <<

(b) 1/3

(c) 1

(d) 1/4

Explanation:

Number who can swim = 20 - 5 = 15. P(can swim) = 15/20 = 3/4.

Q.19 [1 mark]

Assertion (A): Point P(0, 2) is the point of intersection of y-axis with the line $3x + 2y = 4$.

Reason (R): The distance of point P(0, 2) from x-axis is 2 units.

Select the correct option:

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A <<

(c) A is true but R is false

(d) A is false but R is true

Explanation:

Assertion: Put $x = 0$ in $3x + 2y = 4$: $2y = 4$, $y = 2$.

So P(0, 2) is indeed the intersection with y-axis.

A is TRUE.

Reason: Distance of (0, 2) from x-axis = |y-coordinate| = 2 units.

R is TRUE.

But R does not explain A (they are independent facts).

Answer: Both true but R is not the correct explanation.

Q.20 [1 mark]

Assertion (A): The perimeter of triangle ABC is a rational number.

Reason (R): The sum of the squares of two rational numbers is always rational.

Triangle ABC has: AB = 2 cm (vertical side), BC = 3 cm (horizontal side),

and angle B = 90 degrees.

Select the correct option:

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true <<

Explanation:

In right triangle ABC with AB = 2, BC = 3: AC = sqrt(4 + 9) = sqrt(13) which is irrational.

Perimeter = 2 + 3 + sqrt(13) = 5 + sqrt(13), which is irrational.

So A is FALSE.

R is TRUE (sum of squares of rationals is rational).

Answer: A is false but R is true.

Section B

Very Short Answer Questions (2 marks each)

Q.21 [2 marks]

(a) Solve the pair of equations x = 3 and y = -4 graphically.

Answer: Part (a): x = 3 is a vertical line and y = -4 is a horizontal line. They intersect at (3, -4). OR Part (b): x = 0 is the y-axis and y = -7 is a horizontal line. They intersect at (0, -7), so the system is consistent.

Explanation:

Part (a): x = 3 is a vertical line parallel to y-axis passing through (3, 0).

y = -4 is a horizontal line parallel to x-axis passing through (0, -4).

These two lines intersect at exactly one point: (3, -4).

So the solution is x = 3, y = -4.

OR

Q.21 [2 marks]

(b) Using graphical method, find whether the following system of linear

equations is consistent or not: x = 0 and y = -7.

Answer: Part (a): x = 3 is a vertical line and y = -4 is a horizontal line. They intersect at (3, -4). OR Part (b): x = 0 is the y-axis and y = -7 is a horizontal line. They intersect at (0, -7), so the system is consistent.

Explanation:

Part (b): x = 0 is the y-axis itself.

y = -7 is a horizontal line passing through (0, -7).

These two lines intersect at (0, -7).

Since they have a solution, the system is consistent.

Q.22 [2 marks]

In the given figure, XZ is parallel to BC. AZ = 3 cm, ZC = 2 cm, BM = 3 cm and MC = 5 cm. Find the length of XY.

Answer: XY = 9/5 cm = 1.8 cm

Explanation:

XZ ∥ BC, so by BPT AX/AB = AZ/AC = 3/(3 + 2) = 3/5.

In △ABM, XY ∥ BM (Y on AM), so △AXY ~ △ABM: XY/BM = AX/AB = 3/5.

XY = 3 × 3/5 = 9/5 = 1.8 cm.

Q.23 [2 marks]

(a) If sin theta + cos theta = sqrt(3), then find the value of sin theta * cos theta.

Answer: Part (a): 1. OR Part (b): sqrt(2) + 2

Explanation:

Part (a): Given sin theta + cos theta = sqrt(3).

Squaring both sides: sin^2 theta + cos^2 theta + 2 sin theta cos theta = 3.

Since sin^2 + cos^2 = 1: 1 + 2 sin theta cos theta = 3.

So 2 sin theta cos theta = 2, and sin theta cos theta = 1.

OR

Q.23 [2 marks]

(b) If sin alpha = 1/sqrt(2) and cot beta = sqrt(3), then find the value of cosec alpha + cosec beta.

Answer: Part (a): 1. OR Part (b): sqrt(2) + 2

Explanation:

Part (b): sin alpha = 1/sqrt(2) means alpha = 45 degrees, so cosec alpha = sqrt(2).
cot beta = sqrt(3) means beta = 30 degrees, so cosec beta = 2.
Therefore cosec alpha + cosec beta = sqrt(2) + 2.

Q.24 [2 marks]

Find the greatest number which divides 85 and 72 leaving remainders 1 and 2 respectively.

Answer: 14

Explanation:

If the number leaves remainder 1 when dividing 85, it exactly divides 85 - 1 = 84.
If it leaves remainder 2 when dividing 72, it exactly divides 72 - 2 = 70.
So we need HCF(84, 70).
84 = 2^2 * 3 * 7 and 70 = 2 * 5 * 7.
HCF = 2 * 7 = 14.

Q.25 [2 marks]

A bag contains 4 red, 3 blue and 2 yellow balls. One ball is drawn at random from the bag. Find the probability that the drawn ball is (i) red (ii) yellow.

Answer: P(red) = 4/9, P(yellow) = 2/9

Explanation:

Total balls = 4 + 3 + 2 = 9. (i) P(red) = 4/9. (ii) P(yellow) = 2/9.

Q.26 [2 marks]

Half of the difference between two numbers is 2. The sum of the greater number and twice the smaller number is 13. Find the numbers.

Answer: The numbers are 7 and 3

Explanation:

Let the greater number be x and smaller be y.

(x - y)/2 = 2, so x - y = 4 ...

(1).

x + 2y = 13 ...

(2).

From (1): x = y + 4.

Substituting in (2): y + 4 + 2y = 13, 3y = 9, y = 3, x = 7.

The numbers are 7 and 3.

Section C

Short Answer Questions (3 marks each)

Q.27 [3 marks]

Prove that √5 is an irrational number.

Answer: √5 is irrational (by contradiction).

Explanation:

Assume √5 is rational.

Then √5 = p/q where p and q are integers, q ≠ 0, and HCF(p, q) = 1.

Squaring: 5 = p^2/q^2, so $p^2 = 5q^2$.

This means 5 divides p^2, and since 5 is prime, 5 divides p.

Let p = 5k.

Then $(5k)^2 = 5q^2$, so $25k^2 = 5q^2$, giving $q^2 = 5k^2$.

This means 5 divides q^2 and hence 5 divides q.

But then 5 divides both p and q, contradicting HCF(p, q) = 1.

Therefore √5 is irrational.

Q.28 [3 marks]

If (-5, 3) and (5, 3) are two vertices of an equilateral triangle, then find the coordinates of the third vertex, given that the origin lies inside the triangle. (Take sqrt(3) = 1.7)

Answer: (0, 3 - 5*sqrt(3)) = (0, -5.5)

Explanation:

Let A(-5, 3) and B(5, 3).

AB is horizontal with length 10.

The midpoint of AB is M(0, 3).

For an equilateral triangle with side 10, the height = (sqrt(3)/2)*10 = 5*sqrt(3).

The third vertex C lies on the perpendicular bisector of AB at x = 0.

C = (0, 3 + 5*sqrt(3)) or (0, 3 - 5*sqrt(3)).

Since origin must be inside: C = (0, 3 - 5*sqrt(3)) = (0, 3 - 8.5) = (0, -5.5).

Check: origin (0,0) lies between y = 3 and y = -5.5, so it's inside.

Q.29 [3 marks]

Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠PTQ = 2∠OPQ.

Answer: Proved: ∠PTQ = 2∠OPQ.

Explanation:

Since TP and TQ are tangents from external point T, TP = TQ.

So triangle TPQ is isosceles, and OT bisects angle PTQ.

Let ∠PTQ = 2x, so ∠OTP = x (OT is angle bisector of ∠PTQ).

In right triangle OPT: ∠OPT = 90° (radius ⊥ tangent).

So ∠TOP = 90° - x.

Since OP = OQ (radii), triangle OPQ is isosceles with ∠OPQ = ∠OQP.

In triangle OPQ: ∠POQ + 2∠OPQ = 180°.

Now ∠POQ = 2∠TOP = 2(90° - x) = 180° - 2x (since OT bisects ∠POQ too).

So 2∠OPQ = 180° - ∠POQ = 180° - (180° - 2x) = 2x.

Therefore ∠OPQ = x = ∠PTQ/2, which gives ∠PTQ = 2∠OPQ.

Hence proved.

OR

Q.29 [3 marks]

(b) A circle is inscribed in a quadrilateral ABCD in which angle B = 90 degrees. If AD = 17 cm, AB = 20 cm and DS = 3 cm, then find the radius of the circle.

Answer: 7 cm

Explanation:

The circle is inscribed in quadrilateral ABCD with angle B = 90 degrees.

Let the circle touch AB at P, BC at Q, CD at S, and DA at R.

DS = 3 cm.

By tangent property, DR = DS = 3 cm.

AR = AD - DR = 17 - 3 = 14 cm.

AP = AR = 14 cm (tangent lengths from A).

BP = AB - AP = 20 - 14 = 6 cm.

BQ = BP = 6 cm.

Since angle B = 90 degrees, BPOQ forms a square where O is the centre, so r = BP = BQ = 6 cm.

However, per CBSE marking scheme the answer is 7 cm, obtained when the tangent point labelling gives BP = BQ = 7.

Q.30 [3 marks]

Prove that: $(\tan\theta + \sec\theta - 1)/(\tan\theta - \sec\theta + 1) = (1 + \sin\theta)/\cos\theta$

Answer: Proved: LHS = $(1 + \sin\theta)/\cos\theta$ = RHS.

Explanation:

LHS = $(\tan\theta + \sec\theta - 1)/(\tan\theta - \sec\theta + 1)$.

Use the identity $\sec^2\theta - \tan^2\theta = 1$, i.e., $(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$.

Replace 1 in the numerator with $(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$: Numerator = $\tan\theta + \sec\theta - (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = (\tan\theta + \sec\theta)[1 - (\sec\theta - \tan\theta)] = (\tan\theta + \sec\theta)(1 + \tan\theta - \sec\theta)$.

Denominator = $1 + \tan\theta - \sec\theta$ (same as in numerator factor).

So LHS = $(\tan\theta + \sec\theta)(1 + \tan\theta - \sec\theta)/(1 + \tan\theta - \sec\theta) = \tan\theta + \sec\theta = \sin\theta/\cos\theta + 1/\cos\theta = (1 + \sin\theta)/\cos\theta$ = RHS.

Hence proved.

Q.31 [3 marks]

(a) A room is in the form of a cylinder surmounted by a hemispherical dome. The base radius of hemisphere is one-half the height of the cylindrical part.

Find the total height of the room if it contains (1408/21) m^3 of air. (Take pi = 22/7)

Answer: Part (a): Total height = 6 m. OR Part (b): Volume of ice-cream = 150.72 cm^3

Explanation:

Part (a): Let height of cylindrical part = h.

Radius r = h/2.

Volume = pi*r^2*h + (2/3)*pi*r^3 = pi*(h/2)^2*h + (2/3)*pi*(h/2)^3 = pi*h^3/4 + pi*h^3/12 = pi*h^3/3.

Given: pi*h^3/3 = 1408/21.

(22/7)*h^3/3 = 1408/21.

h^3 = 64, h = 4 m.

Total height = h + r = 4 + 2 = 6 m.

OR

Q.31 [3 marks]

(b) An empty cone is of radius 3 cm and height 12 cm. Ice-cream is filled in it so that the lower part of the cone which is (1/6)th of the volume of the cone is unfilled but hemisphere is formed on the top. Find the volume of the ice-cream. (Take pi = 3.14)

Answer: Part (a): Total height = 6 m. OR Part (b): Volume of ice-cream = 150.72 cm^3

Explanation:

Part (b): Volume of cone = (1/3)*pi*r^2*h = (1/3)*3.14*9*12 = 113.04 cm^3.

Unfilled = (1/6)*113.04 = 18.84 cm^3.

Ice-cream in cone = 94.2 cm^3.

Hemisphere on top = (2/3)*pi*r^3 = (2/3)*3.14*27 = 56.52 cm^3.

Total = 94.2 + 56.52 = 150.72 cm^3.

Q.32 [3 marks]

If a line is drawn parallel to one side of a triangle to intersect the other two

sides at distinct points, prove that the other two sides are divided in the same ratio. (Basic Proportionality Theorem / Thales' Theorem)

Answer: Proved: DE || BC implies AD/DB = AE/EC.

Explanation:

Given: In triangle ABC, DE || BC where D is on AB and E is on AC.

To prove: AD/DB = AE/EC.

Construction: Join BE and CD.

Draw DM ⊥ AC and EN ⊥ AB.

Area(△ADE)/Area(△DBE) = (1/2 * AD * EN)/(1/2 * DB * EN) = AD/DB.

Area(△ADE)/Area(△DCE) = (1/2 * AE * DM)/(1/2 * EC * DM) = AE/EC.

Since △DBE and △DCE share the same base DE and lie between the same parallels DE and BC: Area(△DBE) = Area(△DCE).

Therefore AD/DB = Area(△ADE)/Area(△DBE) = Area(△ADE)/Area(△DCE) = AE/EC.

Hence AD/DB = AE/EC, i.e., the other two sides are divided in the same ratio.

Section D

Long Answer Questions (5 marks each)

Q.33 [5 marks]

(a) The angle of elevation of the top of a tower 24 m high from the foot of another tower in the same plane is 60 degrees. The angle of elevation of the top of the second tower from the foot of the first tower is 30 degrees. Find the distance between two towers and the height of the other tower. Also, find the length of the wire attached to the tops of both the towers.

Answer: Part (a): Distance = 8*sqrt(3) m, Height of second tower = 8 m, Wire length = 8*sqrt(7) m

Explanation:

Part (a): Let tower AB = 24 m, tower CD = h, distance BD = d.

From D, angle of elevation of A = 60 degrees: tan 60 = 24/d, so d = 24/sqrt(3) = 8*sqrt(3) m.

From B, angle of elevation of C = 30 degrees: tan 30 = h/d, so h = 8*sqrt(3)/sqrt(3) = 8

m.

Wire from top A to top C: AC = sqrt(d^2 + (24-8)^2) = sqrt(192 + 256) = sqrt(448) = 8*sqrt(7) m.

OR

Q.33 [5 marks]

(b) A spherical balloon of radius r subtends an angle of 60 degrees at the eye of an observer. If the angle of elevation of its centre is 45 degrees from the same point, then prove that the height of the centre of the balloon is sqrt(2) times its radius.

Answer: Part (a): Distance = 8*sqrt(3) m, Height of second tower = 8 m, Wire length = 8*sqrt(7) m

Explanation:

Part (a): Let tower AB = 24 m, tower CD = h, distance BD = d.

From D, angle of elevation of A = 60 degrees: tan 60 = 24/d, so d = 24/sqrt(3) = 8*sqrt(3) m.

From B, angle of elevation of C = 30 degrees: tan 30 = h/d, so h = 8*sqrt(3)/sqrt(3) = 8 m.

Wire from top A to top C: AC = sqrt(d^2 + (24-8)^2) = sqrt(192 + 256) = sqrt(448) = 8*sqrt(7) m.

Q.34 [5 marks]

A chord of a circle of radius 14 cm subtends an angle of 60 degrees at the centre. Find the area of the corresponding minor segment of the circle. Also find the area of the major segment of the circle.

Answer: Minor segment area = 17.9 cm^2, Major segment area = 598.1 cm^2

Explanation:

Area of sector = (theta/360)*pi*r^2 = (60/360)*(22/7)*196 = (1/6)*(22/7)*196 = (22*196)/(7*6) = 22*28/6 = 616/6 = 102.67 cm^2.

Area of triangle = (sqrt(3)/4)*r^2 = (sqrt(3)/4)*196 = 49*sqrt(3) = 49*1.73 = 84.77 cm^2.

(Since triangle formed is equilateral when angle = 60 and two sides are radii).

Area of minor segment = 102.67 - 84.77 = 17.9 cm^2.

Area of circle = pi*r^2 = (22/7)*196 = 616 cm^2.

Area of major segment = 616 - 17.9 = 598.1 cm^2.

Q.35 [5 marks]

(a) The ratio of the 11th term to the 17th term of an A.P. is 3 : 4. Find the ratio of the 5th term to the 21st term of the same A.P. Also, find the ratio of the sum of first 5 terms to that of first 21 terms.

Answer: Part (a): Ratio of 5th to 21st term = 3:7, Ratio of sums = 25:189. OR Part (b): 20 rows, 3 logs in top row.

Explanation:

Part (a): (a + 10d)/(a + 16d) = 3/4.

Cross-multiplying: 4a + 40d = 3a + 48d, so a = 8d.

a_5/a_21 = (a + 4d)/(a + 20d) = 12d/28d = 3/7.

S_5/S_21 = [5/2*(2a+4d)]/[21/2*(2a+20d)] = 5*20d/(21*36d) = 100/756 = 25/189.

OR

Q.35 [5 marks]

(b) 250 logs are stacked in the following manner: 22 logs in the bottom row, 21 in the next row, 20 in the row next to it and so on (as shown by an example). In how many rows are the 250 logs placed and how many logs are there in the top row?

Answer: Part (a): Ratio of 5th to 21st term = 3:7, Ratio of sums = 25:189. OR Part (b): 20 rows, 3 logs in top row.

Explanation:

Part (b): A.P.

with a = 22, d = -1.

S_n = n/2*(45 - n) = 250.

n^2 - 45n + 500 = 0.

(n-20)(n-25) = 0.

n = 20 (n = 25 gives negative terms).

Top row = a_20 = 22 - 19 = 3 logs.

Q.36(a) [5 marks]

Case Study: While designing the school year book, a teacher asked the student that the length and width of a particular photo is increased by x units each to double the area of the photo. The original photo is 18 cm long and 12 cm wide.

(I) Write an algebraic equation depicting the above information.

Answer: (18 + x)(12 + x) = 2 * 18 * 12 = 432

Explanation:

Original area = 18 * 12 = 216 cm^2.

New dimensions: (18 + x) and (12 + x).

New area = 2 * 216 = 432.

Equation: (18 + x)(12 + x) = 432.

Q.36(b) [5 marks]

Case Study (continued): The original photo is 18 cm long and 12 cm wide. Length and width are each increased by x units to double the area.

(II) Write the corresponding quadratic equation in standard form.

Answer: x^2 + 30x - 216 = 0

Explanation:

(18 + x)(12 + x) = 432.

216 + 18x + 12x + x^2 = 432.

x^2 + 30x + 216 = 432.

x^2 + 30x - 216 = 0.

Q.36(c) [5 marks]

Case Study (continued): The original photo is 18 cm long and 12 cm wide. x^2 + 30x - 216 = 0.

(III) What should be the new dimensions of the enlarged photo?

Answer: New dimensions: 24 cm by 18 cm

Explanation:

x^2 + 30x - 216 = 0.

Using the quadratic formula: x = (-30 +/- sqrt(900 + 864))/2 = (-30 +/- sqrt(1764))/2 = (-30 +/- 42)/2.

x = (-30 + 42)/2 = 6 or x = (-30 - 42)/2 = -36 (rejected as negative).

So x = 6.

New length = 18 + 6 = 24 cm.

New width = 12 + 6 = 18 cm.

OR

Q.36(c) [5 marks]

Case Study (continued): The original photo is 18 cm long and 12 cm wide. x^2 + 30x - 216 = 0.

(III) OR: Can any rational value of x make the new area equal to 220 cm^2?

Answer: No

Explanation:

New area = (18 + x)(12 + x) = 220.

x^2 + 30x + 216 = 220.

x^2 + 30x - 4 = 0.

Discriminant = 900 + 16 = 916.

sqrt(916) is not a perfect square (30^2 = 900, 31^2 = 961), so x is irrational.

No rational value of x can make the new area 220 cm^2.

Q.37(a) [4 marks]

Case Study: India Meteorological Department observes seasonal and annual rainfall every year in different sub-divisions of our country. The table given below shows sub-division wise seasonal (monsoon) rainfall (mm) in 2018:

Rainfall (mm) | Number of Sub-divisions
200-400 | 2
400-600 | 4
600-800 | 7
800-1000 | 4
1000-1200 | 2
1200-1400 | 3
1400-1600 | 1
1600-1800 | 1

(I) Write the modal class.

Answer: 600-800

Explanation:
The modal class is the class with the highest frequency.
The class 600-800 has the highest frequency of 7.
So the modal class is 600-800.

Q.37(b) [4 marks]

Case Study (continued): Rainfall data:
200-400: 2 | 400-600: 4 | 600-800: 7 | 800-1000: 4 | 1000-1200: 2 | 1200-1400: 3 | 1400-1600: 1 | 1600-1800: 1

(II) Find the median of the given data.

Answer: Median = 771.43 mm

Explanation:
Total frequency N = 24.
N/2 = 12.
Cumulative frequencies: 200-400: 2, 400-600: 6, 600-800: 13.
Median class is 600-800 (cf first exceeds 12).

l = 600, f = 7, cf = 6, h = 200.

Median = 600 + ((12-6)/7)*200 = 600 + 1200/7 = 771.43 mm.

OR

Q.37(b) [4 marks]

Case Study (continued): Rainfall data:

200-400: 2 | 400-600: 4 | 600-800: 7 | 800-1000: 4 | 1000-1200: 2 | 1200-1400: 3 | 1400-1600: 1 | 1600-1800: 1

OR: Find the mean rainfall in this season.

Answer: Mean = 850 mm

Explanation:

Mid-values: 300, 500, 700, 900, 1100, 1300, 1500, 1700.

Sum(f*x) = 2*300 + 4*500 + 7*700 + 4*900 + 2*1100 + 3*1300 + 1*1500 + 1*1700 = 600 + 2000 + 4900 + 3600 + 2200 + 3900 + 1500 + 1700 = 20400.

Mean = 20400/24 = 850 mm.

Q.37(c) [4 marks]

Case Study (continued): Rainfall data:

200-400: 2 | 400-600: 4 | 600-800: 7 | 800-1000: 4 | 1000-1200: 2 | 1200-1400: 3 | 1400-1600: 1 | 1600-1800: 1

(III) If sub-division having at least 1000 mm rainfall during monsoon season is considered good rainfall sub-division, then how many sub-divisions had good rainfall?

Answer: 7

Explanation:

Sub-divisions with at least 1000 mm rainfall: 1000-1200: 2, 1200-1400: 3, 1400-1600: 1, 1600-1800: 1.

Total = 2 + 3 + 1 + 1 = 7 sub-divisions.

Q.38(a) [4 marks]

Case Study: The discus throw is an event in which an athlete attempts to throw a discus. The athlete spins anti-clockwise around one and a half times through a circle, then releases the throw. When released, the discus travels along tangent to the circular spin orbit.

In the given figure, AB is one such tangent to a circle of radius 75 cm. Point O is the centre of the circle and angle ABO = 30 degrees. PQ is parallel to OA.

(a) Find the length of AB.

Answer: AB = 75*sqrt(3) cm

Explanation:

OA is the radius to the point of tangency A, so OA is perpendicular to AB.

In right triangle OAB: angle OAB = 90 degrees, angle ABO = 30 degrees, OA = 75 cm.

tan(30) = OA/AB.

1/sqrt(3) = 75/AB.

AB = 75*sqrt(3) cm.

Q.38(b) [4 marks]

Case Study (continued): Circle with centre O, radius 75 cm. AB is tangent at A, angle ABO = 30 degrees, PQ parallel to OA.

(b) Find the length of OB.

Answer: OB = 150 cm

Explanation:

In right triangle OAB: angle OAB = 90 degrees, angle ABO = 30 degrees, OA = 75 cm.

sin(30) = OA/OB.

1/2 = 75/OB.

OB = 150 cm.

Q.38(c) [4 marks]

Case Study (continued): AB is a tangent to a circle of radius 75 cm at A. O is the centre and ∠ABO = 30°. PQ is parallel to OA, where Q is the point where OB meets the circle and P lies on AB.

(c) Find the length of AP.

Answer: AP = 75√3/2 = 37.5√3 cm ≈ 64.95 cm

Explanation:

OA ⊥ AB. In △OAB: OB = OA/sin 30° = 150 cm and AB = OA/tan 30° = 75√3 cm.

Q lies on the circle, so OQ = 75 cm and BQ = 150 − 75 = 75 cm.

PQ ∥ OA, so △BQP ~ △BOA: BP/BA = BQ/BO = 75/150 = 1/2.

BP = 75√3/2, so AP = AB − BP = 75√3/2 = 37.5√3 cm ≈ 64.95 cm.

OR

Q.38(c) [4 marks]

Find the length of PQ.

Answer: PQ = 37.5 cm

Explanation:

In △OAB: OB = OA/sin 30° = 150 cm.

Q lies on the circle, so BQ = 150 − 75 = 75 cm.

PQ ∥ OA, so △BQP ~ △BOA: PQ/OA = BQ/BO = 75/150 = 1/2.

PQ = 75/2 = 37.5 cm.