CBSE Class 12 Mathematics 2025 — Set 1 — Answer Key

English Medium

Time Allowed: 3 Hours | Maximum Marks: 80

General Instructions:

1. This question paper contains 38 questions. All questions are compulsory.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 6 questions carrying 2 marks each.
4. Section C has 6 questions carrying 3 marks each.
5. Section D has 4 questions carrying 5 marks each.
6. Section E has 3 case-study based questions carrying 4 marks each.
7. Draw neat figures wherever required.

Section A

Multiple Choice Questions (1 mark each)

Q.1 [1 mark]

If A = [[-1, 0, 0], [0, 1, 0], [0, 0, 1]], then A^{-1} is

(a) , [0, -1, 0], [0, 0, -1]]

(b) , [0, -1, 0], [0, 0, -1]]

(c) , [0, -1, 0], [0, 0, 1]]

(d) , [0, 1, 0], [0, 0, 1]] <<

Explanation:

A is diagonal with entries (-1, 1, 1).

Inverse of a diagonal matrix inverts each diagonal entry: (1/-1, 1/1, 1/1) = (-1, 1, 1).

So A^{-1} = A = [[-1,0,0],[0,1,0],[0,0,1]] (option D).

Q.2 [1 mark]

If vector a = 3î + 2ĵ - k̂ and vector b = î - ĵ + k̂, then which of the following is correct?

(a) a ∥ b

(b) a ⊥ b <<

(c) |b| > |a|

(d) |a| = |b|

Explanation:

a·b = (3)(1) + (2)(-1) + (-1)(1) = 3 - 2 - 1 = 0.

Since the dot product is 0, a ⊥ b (option B).

|a| = √14, |b| = √3, so they are neither parallel nor equal.

Q.3 [1 mark]

$\int_{-1}^{1} (|x|/x)\, dx$, x ≠ 0 is equal to

(a) -1

(b) 0 <<

(c) 1

(d) 2

Explanation:

$|x|/x = -1$ for $x \in (-1, 0)$ and $+1$ for $x \in (0, 1)$.

Integral $= \int_{-1}^{0}(-1)dx + \int_{0}^{1}(1)dx = -1 + 1 = 0$.

(The integrand is an odd function.)

Q.4 [1 mark]

Which of the following is not a homogeneous function of x and y?

(a) $y^2 - xy$

(b) $x - 3y$

(c) $\sin^2(y/x) + y/x$

(d) $\tan x - \sec y$ <<

Explanation:

A function $f(x,y)$ is homogeneous if $f(tx,ty) = t^n f(x,y)$.

(A) y^2-xy is degree 2, (B) $x-3y$ is degree 1, (C) $\sin^2(y/x)+y/x$ is degree 0 — all homogeneous.

(D) $\tan x - \sec y$ cannot be written as $t^n f(x,y)$, so it is not homogeneous (option D).

Q.5 [1 mark]

If $f(x) = |x| + |x - 1|$, then which of the following is correct?

(a) f(x) is both continuous and differentiable, at x = 0 and x = 1.

(b) f(x) is differentiable but not continuous, at x = 0 and x = 1.

(c) f(x) is continuous but not differentiable, at x = 0 and x = 1. <<

(d) f(x) is neither continuous nor differentiable, at x = 0 and x = 1.

Explanation:

Sum of modulus functions is continuous everywhere, but has sharp corners at the break points x = 0 (from |x|) and x = 1 (from |x-1|), where the left- and right-hand derivatives differ.

So f is continuous but not differentiable at x = 0 and x = 1 (option C).

Q.6 [1 mark]

If A is a square matrix of order 2 such that det(A) = 4, then det(4 adj A) is equal to

(a) 16
(b) 64 <<
(c) 256
(d) 512

Explanation:
For order n=2: det(kM) = k^n det(M) and det(adj A) = det(A)^(n-1).
det(4 adj A) = 4^2 · det(adj A) = 16 · det(A)^(2-1) = 16 · 4 = 64 (option B).

Q.7 [1 mark]

If E and F are two independent events such that P(E) = 2/3, P(F) = 3/7, then P(E/F) is equal to

(a) 1/6
(b) 1/2
(c) 2/3 <<
(d) 7/9

Explanation:
For independent events E and F, E is also independent of F.
Hence P(E | F) = P(E) = 2/3 (option C).

Q.8 [1 mark]

The absolute maximum value of function $f(x) = x^3 - 3x + 2$ in [0, 2] is

(a) 0
(b) 2
(c) 4 <<
(d) 5

Explanation:
$f'(x) = 3x^2 - 3 = 0 \Rightarrow x = 1$ (in [0,2]).
f(0) = 2, f(1) = 0, f(2) = 8 - 6 + 2 = 4.

Absolute maximum = 4 at x = 2 (option C).

Q.9 [1 mark]

Let A = [[1, -2, -1], [0, 4, -1], [-3, 2, 1]], B = [[-2], [-5], [-7]], C = [9 8 7], which of the following is defined?

(a) Only AB <<

(b) Only AC

(c) Only BA

(d) All AB, AC and BA

Explanation:

A is 3×3, B is 3×1, C is 1×3.

AB: (3×3)(3×1) → 3×1 defined.

AC: (3×3)(1×3) columns 3 ≠ rows 1 → not defined.

BA: (3×1)(3×3) columns 1 ≠ rows 3 → not defined.

Only AB is defined (option A).

Q.10 [1 mark]

If $\int (2^{(1/x)}/x^2)\,dx = k \cdot 2^{(1/x)} + C$, then k is equal to

(a) -1/log 2 <<

(b) -log 2

(c) -1

(d) 1/2

Explanation:

Let u = 1/x, du = -1/x² dx.

$\int 2^{(1/x)}/x^2\,dx = -\int 2^u\,du = -2^u/\log 2 = -2^{(1/x)}/\log 2$.

Comparing, k = -1/log 2 (option A).

Q.11 [1 mark]

If a + b + c = 0, |a| = √37, |b| = 3 and |c| = 4, then angle between b and c is

(a) π/6

(b) π/4

(c) π/3 <<

(d) π/2

Explanation:

a = -(b+c) ⇒ $|a|^2 = |b|^2 + |c|^2 + 2|b||c|\cos\theta$.

$37 = 9 + 16 + 2(3)(4)\cos\theta \Rightarrow 37 = 25 + 24\cos\theta \Rightarrow \cos\theta = 12/24 = 1/2 \Rightarrow \theta = \pi/3$ (option C).

Q.12 [1 mark]

The integrating factor of differential equation $(x + 2y^3)\, dy/dx = 2y$ is

(a) e^(y^2/2)

(b) 1/√y <<

(c) $1/y^2$

(d) e^(-1/y^2)

Explanation:

Rewrite as dx/dy: $2y\,(dx/dy) = x + 2y^3 \Rightarrow dx/dy - x/(2y) = y^2$.

Linear in x, P(y) = -1/(2y).

IF = e^(∫-1/(2y)dy) = e^(-(1/2)log y) = y^(-1/2) = 1/√y (option B).

Q.13 [1 mark]

If A = [[7, 0, x], [0, 7, 0], [0, 0, y]] is a scalar matrix, then y^x is equal to

(a) 0

(b) 1 <<

(c) 7

(d) ± 7

Explanation:

A scalar matrix is diagonal with all diagonal entries equal, so off-diagonal x = 0 and y = 7.

Then $y^x = 7^0 = 1$ (option B).

Q.14 [1 mark]

The corner points of the feasible region in graphical representation of a

L.P.P. are (2, 72), (15, 20) and (40, 15). If Z = 18x + 9y be the objective function, then

(a) Z is maximum at (2, 72), minimum at (15, 20)

(b) Z is maximum at (15, 20) minimum at (40, 15)

(c) Z is maximum at (40, 15), minimum at (15, 20) <<

(d) Z is maximum at (40, 15), minimum at (2, 72)

Explanation:

Z(2,72) = 36 + 648 = 684; Z(15,20) = 270 + 180 = 450; Z(40,15) = 720 + 135 = 855.

Maximum = 855 at (40,15), minimum = 450 at (15,20) (option C).

Q.15 [1 mark]

If A and B are invertible matrices, then which of the following is not correct?

(a) $(A + B)^{-1} = B^{-1} + A^{-1}$ <<

(b) $(AB)^{-1} = B^{-1}A^{-1}$

(c) adj (A) = $|A|\ A^{-1}$

(d) $|A|^{-1} = |A^{-1}|$

Explanation:

$(AB)^{-1} = B^{-1}A^{-1}$, adj A = $|A|A^{-1}$, and $|A^{-1}| = |A|^{-1}$ are all valid identities.

But $(A+B)^{-1} = B^{-1} + A^{-1}$ is false in general — matrix inverse does not distribute over addition.

So option A is not correct.

Q.16 [1 mark]

If the feasible region of a linear programming problem with objective function Z = ax + by, is bounded, then which of the following is correct?

(a) It will only have a maximum value.

(b) It will only have a minimum value.

(c) It will have both maximum and minimum values. <<

(d) It will have neither maximum nor minimum value.

Explanation:

By the fundamental theorem of LPP, a bounded feasible region always yields both a maximum and a minimum value of the objective function, attained at corner points (option C).

Q.17 [1 mark]

The area of the shaded region bounded by the curves $y^2 = x$, $x = 4$ and the x-axis is given by

(a) $\int_0^4 x\,dx$

(b) $\int_0^2 y^2\,dy$

(c) $2\int_0^4 \sqrt{x}\,dx$

(d) $\int_0^4 \sqrt{x}\,dx$ <<

Explanation:

The shaded region is above the x-axis ($y \geq 0$), so $y = \sqrt{x}$.

Area = $\int_0^4 y\,dx = \int_0^4 \sqrt{x}\,dx$ (option D).

Q.18 [1 mark]

The graph of a trigonometric function is as shown (a curve with its peak on the Y-axis, crossing the x-axis at $-\pi/2$ and $\pi/2$, with its lowest point at π, drawn slightly beyond $-\pi/2$ and π). Which of the following will represent graph of its inverse?

(a) Graph: increasing curve through the origin, steepening away from it; x-axis marked $-\pi/2$ and $\pi/2$, y-axis marked -1 and 1

(b) Graph: increasing curve through the origin, flattening away from it; x-axis marked $-\pi/2$ and $\pi/2$, y-axis marked -1 and 1

(c) Graph: decreasing curve from a solid point at x = -1, level with π on the y-axis, through $\pi/2$ on the y-axis, to a solid point at (1, 0) <<

(d) Graph: decreasing curve crossing the y-axis at $\pi/2$, levelling off towards a dashed line at π on the left and the x-axis on the right; y-axis marked 0, $\pi/2$ and π

Explanation:

Figure: curve peaks on the y-axis, crosses zero at -π/2 and π/2, lowest at π: this is the cosine curve.

On its principal branch [0, π] it is invertible, and the inverse cos^-1 x has domain [-1, 1], range [0, π], decreasing from (-1, π) through (0, π/2) to (1, 0): printed option (C).

(A) and (B) are increasing, with x-axes marked in ±π/2.

(D) passes through π/2 but levels off towards π and 0 instead of ending at x = ±1.

Q.19 [1 mark]

Assertion (A): Let Z be the set of integers. A function f : Z → Z defined as f(x) = 3x - 5, ∀x ∈ Z is a bijective. Reason (R): A function is a bijective if it is both surjective and injective.

(a) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true. <<

Explanation:

f(x) = 3x - 5 on Z is injective but not surjective: for y = 0, x = 5/3 ∉ Z, so not every integer is an image.

Hence f is not bijective — Assertion is false.

The Reason (correct definition of bijective) is true.

So option D.

Q.20 [1 mark]

Assertion (A): f(x) = {3x - 8, x ≤ 5; 2k, x > 5} is continuous at x = 5 for k = 5/2. Reason (R): For a function f to be continuous at x = a, $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$.

(a) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the

correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true. <<

Explanation:

At x = 5: LHL = f(5) = 3(5) - 8 = 7; RHL = 2k.

For continuity, 2k = 7 ⇒ k = 7/2, not 5/2.

With k = 5/2, RHL = 5 ≠ 7, so f is not continuous.

Assertion is false; the Reason (definition of continuity) is true.

So option D.

Section B

Very Short Answer Questions (2 marks each)

Q.21 [2 marks]

(a) Differentiate 2^(cos²x) w.r.t. cos²x.

Answer: 2^(cos²x) · log 2

Explanation:

Let t = cos²x. d/dt (2^t) = 2^t · log 2 = 2^(cos²x) · log 2.

OR

Q.21 [2 marks]

(b) If $\tan^{-1}(x^2 + y^2) = a^2$, then find dy/dx.

Answer: dy/dx = -x/y

Explanation:

Differentiate $\tan^{-1}(x^2+y^2) = a^2$ (constant): $[1/(1+(x^2+y^2)^2)]\cdot(2x + 2y\cdot dy/dx) = 0 \Rightarrow 2x + 2y$ dy/dx = 0 ⇒ dy/dx = -x/y.

Q.22 [2 marks]

Evaluate: $\tan^{-1}[2 \sin(2 \cos^{-1}(\sqrt{3}/2))]$

Answer: π/3

Explanation:

$\cos^{-1}(\sqrt{3}/2) = \pi/6$.

Then $2\cdot(\pi/6) = \pi/3$, $\sin(\pi/3) = \sqrt{3}/2$, $2\cdot(\sqrt{3}/2) = \sqrt{3}$.

So $\tan^{-1}(\sqrt{3}) = \pi/3$.

Q.23 [2 marks]

The diagonals of a parallelogram are given by $a = 2\hat{i} - \hat{j} + \hat{k}$ and $b = \hat{i} + 3\hat{j} - \hat{k}$. Find the area of the parallelogram.

Answer: $\sqrt{62}$ / 2 square units

Explanation:

Area = $(1/2)|a \times b|$.

$a \times b = |\hat{i}\ \hat{j}\ \hat{k};\ 2\ -1\ 1;\ 1\ 3\ -1| = \hat{i}(1-3) - \hat{j}(-2-1) + \hat{k}(6+1) = -2\hat{i} + 3\hat{j} + 7\hat{k}$.

$|a\times b| = \sqrt{(4+9+49)} = \sqrt{62}$.

Area = $\sqrt{62}/2$.

Q.24 [2 marks]

Find the intervals in which function f(x) = 5x^(3/2) - 3x^(5/2) is (i) increasing (ii) decreasing.

Answer: Increasing on (0, 1); decreasing on (1, ∞)

Explanation:

Domain $x \geq 0$.

$f'(x) = (15/2)x^{1/2} - (15/2)x^{3/2} = (15/2)\sqrt{x}(1 - x)$.

For $x \in (0,1)$, $1-x > 0 \Rightarrow f' > 0$ (increasing).

For $x > 1$, $f' < 0$ (decreasing).

Q.25 [2 marks]

(a) Two friends while flying kites from different locations, find the strings of their kites crossing each other. The strings can be represented by vectors $a = 3\hat{i} + \hat{j} + 2\hat{k}$ and $b = 2\hat{i} - 2\hat{j} + 4\hat{k}$. Determine the angle formed between the kite strings. Assume there is no slack in the strings.

Answer: $\cos^{-1}(3/\sqrt{21}) \approx 49.1°$

Explanation:

$\cos\theta = (a\cdot b)/(|a||b|)$.

$a\cdot b = 6 - 2 + 8 = 12$.

$|a| = \sqrt{14}$, $|b| = \sqrt{24} = 2\sqrt{6}$.

$\cos\theta = 12/(\sqrt{14}\cdot 2\sqrt{6}) = 12/(2\sqrt{84}) = 6/(2\sqrt{21}) = 3/\sqrt{21}$.

$\theta = \cos^{-1}(3/\sqrt{21})$.

OR

Q.25 [2 marks]

(b) Find a vector of magnitude 21 units in the direction opposite to that of AB where A and B are the points A(2, 1, 3) and B(8, -1, 0) respectively.

Answer: $-18\hat{i} + 6\hat{j} + 9\hat{k}$

Explanation:

$AB = B - A = (6, -2, -3)$, $|AB| = \sqrt{(36+4+9)} = \sqrt{49} = 7$.

Unit vector $= (6,-2,-3)/7$.

Opposite direction, magnitude 21: $-21\cdot(6,-2,-3)/7 = -3(6,-2,-3) = -18\hat{i} + 6\hat{j} + 9\hat{k}$.

Q.26 [2 marks]

The side of an equilateral triangle is increasing at the rate of 3 cm/s. At what rate its area increasing when the side of the triangle is 15 cm?

Answer: $(45\sqrt{3})/2$ cm²/s

Explanation:

Area $A = (\sqrt{3}/4)s^2$.

$dA/dt = (\sqrt{3}/2)s\cdot(ds/dt) = (\sqrt{3}/2)(15)(3) = (45\sqrt{3})/2$ cm²/s ≈ 38.97 cm²/s.

Section C

Short Answer Questions (3 marks each)

Q.27 [3 marks]

Solve the following linear programming problem graphically: Maximise $Z = x + 2y$ subject to the constraints: $x - y \geq 0$, $x - 2y \geq -2$, $x \geq 0$, $y \geq 0$.

Answer: Z has no maximum (feasible region is unbounded)

Explanation:

The feasible region ($x \geq y$, $x \geq 2y - 2$, $x,y \geq 0$) is unbounded, extending indefinitely for large x.

Along that direction $Z = x + 2y$ increases without bound, so Z has no maximum value.

(Corner (0,0) gives Z = 0;

corner (2,2) of the two lines gives Z = 6, but larger values are attainable, confirming unboundedness.)

Q.28 [3 marks]

(a) Find: $\int (x + \sin x)/(1 + \cos x)\, dx$

Answer: $x \cdot \tan(x/2) + C$

Explanation:

Split: $x/(1+\cos x) + \sin x/(1+\cos x)$.

Using $1+\cos x = 2\cos^2(x/2)$ and $\sin x = 2\sin(x/2)\cos(x/2)$: $\sin x/(1+\cos x) = \tan(x/2)$, and $x/(1+\cos x) = (x/2)\sec^2(x/2)$.

Integrating $(x/2)\sec^2(x/2)$ by parts gives $x \cdot \tan(x/2) - \int \tan(x/2)dx$, and the $-\int \tan(x/2)dx$ cancels $\int \tan(x/2)dx$ from the second term.

Result: $x \cdot \tan(x/2) + C$.

(Verified: $d/dx[x \tan(x/2)] = (x+\sin x)/(1+\cos x)$.)

OR

Q.28 [3 marks]

(b) Evaluate: $\int_0^{(\pi/4)} dx / (\cos^3 x \sqrt{(2 \sin 2x)})$

Answer: 6/5

Explanation:

$2 \sin 2x = 4 \sin x \cos x$, so $\sqrt{(2 \sin 2x)} = 2\sqrt{(\sin x \cos x)}$.

Integrand $= 1/(2 \cos^3 x \sqrt{(\sin x \cos x)}) = (1/2)\tan^{(-1/2)}x \cdot \sec^4 x$.

Put $t = \tan x$, $dt = \sec^2 x\, dx$: $(1/2)\int_0^1 t^{(-1/2)}(1+t^2)dt = (1/2)[2\sqrt{t} + (2/5)t^{(5/2)}]_0^1 = (1/2)(2 + 2/5) = (1/2)(12/5) = 6/5$.

Q.29 [3 marks]

(a) Verify that lines given by $r = (1 - \lambda)\hat{i} + (\lambda - 2)\hat{j} + (3 - 2\lambda)\hat{k}$ and $r = (\mu + 1)\hat{i} +$

(2μ - 1)ĵ - (2μ + 1)k̂ are skew lines. Hence, find shortest distance between the lines.

Answer: Skew; shortest distance = 8/√29 units

Explanation:

Line 1: point (1,-2,3), direction d_1 = (-1,1,-2).

Line 2: point (1,-1,-1), direction d_2 = (1,2,-2).

$d_1 \times d_2$ = (2,-4,-3), $|d_1 \times d_2| = \sqrt{29}$.

$(p_2-p_1)\cdot(d_1\times d_2) = (0,1,-4)\cdot(2,-4,-3) = 0 - 4 + 12 = 8 \neq 0$, so lines are non-coplanar (skew).

Shortest distance = $|8|/\sqrt{29} = 8/\sqrt{29}$ units.

OR

Q.29 [3 marks]

(b) During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by B = 2î + 8ĵ, W = 6î + 12ĵ and F = 12î + 18ĵ respectively. Calculate the ratio in which the wicketkeeper divides the line segment joining the bowler and the leg slip fielder.

Answer: 2 : 3

Explanation:

Let W divide BF in ratio k:1.

W = (kF + B)/(k+1).

For î: 6 = (12k + 2)/(k+1) ⇒ 6k + 6 = 12k + 2 ⇒ 4 = 6k ⇒ k = 2/3.

Check ĵ: (18·(2/3) + 8)/((2/3)+1) = 20/(5/3) = 12 ✓.

So ratio = 2 : 3.

Q.30 [3 marks]

(a) The probability distribution for the number of students being absent in a class on a Saturday is as follows: X = 0, 2, 4, 5 with P(X) = p, 2p, 3p, p. Where X is the number of students absent. (i) Calculate p. (ii) Calculate the mean of the number of absent students on Saturday.

Answer: p = 1/7; mean = 3

Explanation:

(i) $\Sigma P(X) = p + 2p + 3p + p = 7p = 1 \Rightarrow p = 1/7$.

(ii) Mean = $\Sigma x \cdot P(x) = 0 \cdot p + 2 \cdot 2p + 4 \cdot 3p + 5 \cdot p = 4p + 12p + 5p = 21p = 21 \cdot (1/7) = 3$.

OR

Q.30 [3 marks]

(b) For the vacancy advertised in the newspaper, 3000 candidates submitted their applications. From the data it was revealed that two third of the total applicants were females and other were males. The selection for the job was done through a written test. The performance of the applicants indicates that the probability of a male getting a distinction in written test is 0.4 and that a female getting a distinction is 0.35. Find the probability that the candidate chosen at random will have a distinction in the written test.

Answer: $11/30 \approx 0.3667$

Explanation:

$P(male) = 1/3$, $P(female) = 2/3$.

$P(distinction) = P(M) \cdot 0.4 + P(F) \cdot 0.35 = (1/3)(0.4) + (2/3)(0.35) = 0.1333 + 0.2333 = 0.3667 = 11/30$.

Q.31 [3 marks]

Sketch the graph of $y = |x + 3|$ and find the area of the region enclosed by the curve, x-axis, between $x = -6$ and $x = 0$, using integration.

Answer: 9 square units

Explanation:

$y = |x+3|$ is V-shaped with vertex at (-3, 0).

Area = $\int_{-6}^{0} |x+3| dx = \int_{-6}^{-3} -(x+3) dx + \int_{-3}^{0} (x+3) dx$.

Each triangular piece has base 3, height 3, area = $(1/2)(3)(3) = 4.5$.

Total = $4.5 + 4.5 = 9$ square units.

Q.32 [3 marks]

(a) If $\sqrt{(1 - x^2)} + \sqrt{(1 - y^2)} = a(x - y)$, then prove that $dy/dx = \sqrt{((1 - y^2)/(1 - x^2))}$.

Answer: Proved: $dy/dx = \sqrt{((1-y^2)/(1-x^2))}$

Explanation:

Put $x = \sin A$, $y = \sin B$.

LHS = $\cos A + \cos B$, RHS = $a(\sin A - \sin B)$.

Then $\cos A + \cos B = a(\sin A - \sin B) \Rightarrow 2\cos((A+B)/2)\cos((A-B)/2) = a \cdot 2\cos((A+B)/2)\sin((A-B)/2) \Rightarrow \cot((A-B)/2) = a \Rightarrow (A-B)/2 = \cot^{-1}a \Rightarrow A - B = 2\cot^{-1}a$ (constant).

So $\sin^{-1}x - \sin^{-1}y$ = constant.

Differentiate: $1/\sqrt{(1-x^2)} - (1/\sqrt{(1-y^2)})dy/dx = 0 \Rightarrow dy/dx = \sqrt{(1-y^2)}/\sqrt{(1-x^2)} = \sqrt{((1-y^2)/(1-x^2))}$.

Proved.

OR

Q.32 [3 marks]

(b) If $x = a(\cos \theta + \log \tan(\theta/2))$ and $y = \sin \theta$, then find d^2y/dx^2 at $\theta = \pi/4$.

Answer: $2\sqrt{2} / a^2$

Explanation:

$dx/d\theta = a(-\sin \theta + 1/\sin \theta) = a \cos^2\theta / \sin \theta$.

$dy/d\theta = \cos \theta$.

$dy/dx = \cos \theta \cdot \sin \theta / (a \cos^2\theta) = \tan \theta / a$.

$d^2y/dx^2 = (\sec^2\theta / a) \div (a \cos^2\theta / \sin \theta) = \sin \theta / (a^2 \cos^4\theta)$.

At $\theta = \pi/4$: $\sin \theta = 1/\sqrt{2}$, $\cos^4\theta = 1/4$, so $d^2y/dx^2 = 4/(\sqrt{2}\, a^2) = 2\sqrt{2} / a^2$.

Section D

Long Answer Questions (5 marks each)

Q.33 [5 marks]

Find the absolute maximum and absolute minimum of function $f(x) = 2x^3 - 15x^2 + 36x + 1$ on $[1, 5]$.

Answer: Absolute maximum = 56 at $x = 5$; absolute minimum = 24 at $x = 1$

Explanation:

$f'(x) = 6x^2 - 30x + 36 = 6(x-2)(x-3) = 0 \Rightarrow x = 2, 3$ (both in $[1,5]$).

f(1) = 2-15+36+1 = 24;

f(2) = 16-60+72+1 = 29;

f(3) = 54-135+108+1 = 28;

f(5) = 250-375+180+1 = 56.

Absolute maximum = 56 at x = 5; absolute minimum = 24 at x = 1.

Q.34 [5 marks]

(a) Find the image A' of the point A(1, 6, 3) in the line x/1 = (y - 1)/2 = (z - 2)/3. Also, find the equation of the line joining A and A'.

Answer: A' = (1, 0, 7); line: (x-1)/0 = (y-6)/(-3) = (z-3)/2

Explanation:

Point on line: (t, 1+2t, 2+3t).

Foot of perpendicular: (P-A)·d = 0 where P-A = (t-1, 2t-5, 3t-1), d = (1,2,3).

(t-1) + 2(2t-5) + 3(3t-1) = 14t - 14 = 0 ⇒ t = 1.

Foot M = (1, 3, 5).

Image A' = 2M - A = (2-1, 6-6, 10-3) = (1, 0, 7).

Line AA': direction A'-A = (0, -6, 4) ∝ (0, -3, 2), through A(1,6,3): (x-1)/0 = (y-6)/(-3) = (z-3)/2.

OR

Q.34 [5 marks]

(b) Find a point P on the line (x + 5)/1 = (y + 3)/4 = (z - 6)/(-9) such that its distance from point Q(2, 4, -1) is 7 units. Also, find the equation of line joining P and Q.

Answer: P = (-4, 1, -3); line PQ: (x-2)/(-6) = (y-4)/(-3) = (z+1)/(-2)

Explanation:

P = (-5+r, -3+4r, 6-9r).

PQ² = (r-7)² + (4r-7)² + (7-9r)² = 98r² - 196r + 147 = 49 ⇒ 98r² - 196r + 98 = 0 ⇒ r² - 2r + 1 = 0 ⇒ (r-1)² = 0 ⇒ r = 1.

So P = (-4, 1, -3).

Direction PQ = Q-P = (6, 3, 2) (or -6,-3,-2).

Line PQ: (x-2)/(-6) = (y-4)/(-3) = (z+1)/(-2), equivalently (x+4)/6 = (y-1)/3 = (z+3)/2.

Q.35 [5 marks]

A school wants to allocate students into three clubs: Sports, Music and Drama, under following conditions: The number of students in Sports club should be equal to the sum of the number of students in Music and Drama club. The number of students in Music club should be 20 more than half the number of students in Sports club. The total number of students to be allocated in all three clubs are 180. Find the number of students allocated to different clubs, using matrix method.

Answer: Sports = 90, Music = 65, Drama = 25

Explanation:

Let S, M, D be the counts.

Equations: $S = M + D$; $M = 20 + S/2 \Rightarrow S - 2M = -40$; $S + M + D = 180$.

From $S = M+D$ and $S+M+D = 180$: $S + S = 180 \Rightarrow S = 90$.

Then $M = 20 + 45 = 65$, $D = S - M = 90 - 65 = 25$.

(Solving the 3×3 system by matrix method $AX = B$ gives $X = (90, 65, 25)$.)

Q.36 [5 marks]

A technical company is designing a rectangular solar panel installation on a roof using 300 metres of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections. Let the length of the side perpendicular to the partition be x metres and with parallel to the partition be y metres. (i) Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of x and y. (ii) Write the area of the solar panel as a function of x. (iii)(a) Find the critical points of the area function. Use second derivative test to determine critical points at the maximum area. Also, find the maximum area.

Answer: (i) $2x + 3y = 300$; (ii) $A(x) = x(300 - 2x)/3$; (iii) critical point $x = 75$; $A''(x) = -4/3 < 0$ (maximum); maximum area = 3750 m² (at $x = 75$, $y = 50$)

Explanation:

The partition is parallel to the y-side (length y), so material = perimeter + one partition = 2x + 2y + y = 2x + 3y = 300.

(ii) y = (300 - 2x)/3, so A(x) = x·y = x(300 - 2x)/3.

(iii) A'(x) = (300 - 4x)/3 = 0 ⇒ x = 75.

A''(x) = -4/3 < 0 ⇒ maximum.

y = (300 - 150)/3 = 50.

Maximum area = 75 × 50 = 3750 m².

OR

Q.36 [5 marks]

(iii)(b) Using first derivative test, calculate the maximum area the company can enclose with the 300 metres of boundary material, considering the parallel partition.

Answer: Maximum area = 3750 m²

Explanation:

A(x) = x(300-2x)/3 = 100x - (2/3)x².

A'(x) = 100 - (4/3)x = 0 ⇒ x = 75.

A' > 0 for x < 75 and A' < 0 for x > 75, so by the first derivative test A is maximum at x = 75.

y = (300 - 150)/3 = 50.

Maximum area = 75 × 50 = 3750 m².

Section E

Case Study Based Questions (4 marks each)

Q.37 [4 marks]

A class-room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set A = {1, 2, 3}: R_1 = {(2,3),(3,2)}; R_2 = {(1,2),(1,3),(3,2)}; R_3 = {(1,2),(2,1),(1,1)}; R_4 = {(1,1),(1,2),(3,3),(2,2)}; R_5 = {(1,1),(1,2),(3,3),(2,2),(2,1),(2,3),(3,2)}. The students are asked to answer the

following questions about the above relations: (i) Identify the relation which is reflexive, transitive but not symmetric. (ii) Identify the relation which is reflexive and symmetric but not transitive. (iii)(a) Identify the relations which are symmetric but neither reflexive nor transitive.

Answer: (i) R_4; (ii) R_5; (iii)(a) R_1 and R_3

Explanation:

(i) R_4 has (1,1),(2,2),(3,3) →
reflexive; extra pair (1,2) with (2,2) gives (1,2) →
transitive; but (1,2) present without (2,1) →
not symmetric.
So R_4.
(ii) R_5 has all (i,i) →
reflexive; every pair has its reverse →
symmetric; but (1,2)&(2,3) present without (1,3) →
not transitive.
So R_5.
(iii)(a) R_1 = {(2,3),(3,2)} is symmetric, not reflexive (missing (1,1) etc.), not transitive ((2,3)&(3,2) without (2,2)).
R_3 = {(1,2),(2,1),(1,1)} is symmetric, not reflexive (missing (2,2),(3,3)), not transitive ((2,1)&(1,2) without (2,2)).
So R_1 and R_3.

OR

Q.37 [4 marks]

(iii)(b) What pairs should be added to the relation R_2 = {(1,2),(1,3),(3,2)} to make it an equivalence relation?

Answer: Add (1,1),(2,2),(3,3),(2,1),(3,1),(2,3) — the equivalence relation becomes the universal relation A×A (all 9 ordered pairs)

Explanation:

For an equivalence relation on A = {1,2,3} it must be reflexive, symmetric and transitive.

Reflexive requires (1,1),(2,2),(3,3).

Symmetric requires the reverses of (1,2),(1,3),(3,2): i.e. (2,1),(3,1),(2,3).

Adding these and taking transitive closure links all elements, giving the universal relation A×A.

Pairs to add: (1,1),(2,2),(3,3),(2,1),(3,1),(2,3).

Q.38 [4 marks]

A bank offers loan to its customers on different types of interest namely, fixed rate, floating rate and variable rate. From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate or variable rate with probabilities 10%, 20% and 70% respectively. A customer after availing loan can pay the loan or default on loan repayment. The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate and variable rate is 5%, 3% and 1% respectively. (i) What is the probability that a customer after availing the loan will default on the loan repayment? (ii) A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest?

Answer: (i) 0.018; (ii) 7/18 ≈ 0.3889

Explanation:

Let F, Fl, V be fixed/floating/variable with P = 0.1, 0.2, 0.7 and default probs 0.05, 0.03, 0.01.

(i) By total probability, P(default) = 0.1(0.05) + 0.2(0.03) + 0.7(0.01) = 0.005 + 0.006 + 0.007 = 0.018.

(ii) By Bayes: P(V|default) = 0.7(0.01)/0.018 = 0.007/0.018 = 7/18 ≈ 0.3889.