CBSE Class 12 Mathematics 2025 — Set 2 — Answer Key

English Medium

Time Allowed: 3 Hours | Maximum Marks: 80

General Instructions:

1. This question paper contains 38 questions. All questions are compulsory.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 6 questions carrying 2 marks each.
4. Section C has 6 questions carrying 3 marks each.
5. Section D has 4 questions carrying 5 marks each.
6. Section E has 3 case-study based questions carrying 4 marks each.
7. Draw neat figures wherever required.

Section A

Multiple Choice Questions (1 mark each)

Q.1 [1 mark]

The projection vector of vector $\vec{a}$ on vector $\vec{b}$ is

(a) $((\vec{a}\cdot\vec{b})/|\vec{b}|^2)\,\vec{b}$ <<

(b) $(\vec{a}\cdot\vec{b})/|\vec{b}|$

(c) $(\vec{a}\cdot\vec{b})/|\vec{a}|$

(d) $((\vec{a}\cdot\vec{b})/|\vec{a}|^2)\,\vec{b}$

Explanation:

Projection vector of $\vec{a}$ on $\vec{b}$ = $((\vec{a}\cdot\vec{b})/|\vec{b}|^2)\,\vec{b}$. Option A.

Q.2 [1 mark]

The function $f(x) = x^2 - 4x + 6$ is increasing in the interval

(a) $(0, 2)$

(b) $(-\infty, 2]$

(c)

(d) $[2, \infty)$ <<

Explanation:

$f'(x)=2x-4\geq 0 \Rightarrow x\geq 2 \Rightarrow$ increasing on $[2, \infty)$. Option D.

Q.3 [1 mark]

If $f(2a - x) = f(x)$, then $\int_0^{2}a\ f(x)\,dx$ is

(a) $\int_0^{2}a\ f(x/2)\,dx$

(b) $\int_0^{}a\ f(x)\,dx$

(c) $2\int_a^{0}\ f(x)\,dx$

(d) $2\int_0^{}a\ f(x)\,dx$ <<

Explanation:

Property: if $f(2a-x)=f(x)$ then $\int_0^{2}a\ f(x)dx = 2\int_0^{}a\ f(x)dx$. Option D.

Q.4 [1 mark]

If A = [[1, 12, 4y], [6x, 5, 2x], [8x, 4, 6]] is a symmetric matrix, then (2x + y) is

(a) −8

(b) 0

(c) 6

(d) 8 <<

Explanation:

Symmetric ⇒ aij=aji: 12=6x ⇒ x=2; 4y=8x ⇒ y=4; 2x=4 ⇒ x=2 (consistent).

2x+y=4+4=8.

Option D.

Q.5 [1 mark]

If $y = \sin^{-1}x$, $-1 \le x \le 0$, then the range of y is

(a) $(-\pi/2, 0)$

(b) <<

(c) $[-\pi/2, 0)$

(d) $(-\pi/2, 0]$

Explanation:

$\sin^{-1}(-1)=-\pi/2$ and $\sin^{-1}(0)=0$, both attained, so $y \in [-\pi/2, 0]$, a closed interval.

Option B.

Q.6 [1 mark]

If a line makes angles of $3\pi/4$, $\pi/3$ and θ with the positive directions of x, y and z-axis respectively, then θ is

(a) $-\pi/3$ only

(b) $\pi/3$ only <<

(c) $\pi/6$

(d) $\pm\pi/3$

Explanation:

$\cos^2(3\pi/4)+\cos^2(\pi/3)+\cos^2\theta=1 \Rightarrow 1/2+1/4+\cos^2\theta=1 \Rightarrow \cos^2\theta=1/4 \Rightarrow \cos\theta=\pm1/2$.

As a valid direction angle $\theta \in [0, \pi]$, the answer taken is $\theta = \pi/3$.

Option B.

Note: θ can be $\pi/3$ or $2\pi/3$, and no printed option lists both.

$\pi/3$ is the closest printed option;

this could not be confirmed against an official CBSE marking scheme.

Q.7 [1 mark]

If E and F are two events such that $P(E) > 0$ and $P(F) \neq 1$, then $P(\bar{E}/F)$ is

(a) $P(\bar{E})/P(F)$

(b) $1 - P(\bar{E}/F)$

(c) $1 - P(E/F)$

(d) $(1 - P(E \cup F))/P(F)$ <<

Explanation:

$P(\bar{E}|F) = P(\bar{E} \cap F)/P(F) = P((E \cup F)^c)/P(F) = (1 - P(E \cup F))/P(F)$. Option D.

Q.8 [1 mark]

Which of the following can be both a symmetric and skew-symmetric matrix?

(a) Unit Matrix

(b) Diagonal Matrix

(c) Null Matrix <<

(d) Row Matrix

Explanation:

Only the null (zero) matrix satisfies $A = A^T$ and $A = -A^T$ simultaneously.

Option C.

Q.9 [1 mark]

The equation of a line parallel to the vector $3\hat{i} + \hat{j} + 2\hat{k}$ and passing through the point $(4, -3, 7)$ is :

(a) $x = 4t + 3, y = -3t + 1, z = 7t + 2$

(b) $x = 3t + 4, y = t + 3, z = 2t + 7$

(c) $x = 3t + 4, y = t - 3, z = 2t + 7$ <<

(d) $x = 3t + 4, y = -t + 3, z = 2t + 7$

Explanation:

Parametric form: $x=4+3t$, $y=-3+t$, $z=7+2t$. Option C.

Q.10 [1 mark]

Four friends Abhay, Bina, Chhaya and Devesh were asked to simplify $4\,AB + 3(AB + BA) - 4\,BA$, where A and B are both matrices of order 2×2. It is known that $A \neq B \neq I$ and $A^{-1} \neq B$. Their answers are given as : Abhay : $6\,AB$, Bina : $7\,AB - BA$, Chhaya : $8\,AB$, Devesh : $7\,BA - AB$. Who answered it correctly?

(a) Abhay

(b) Bina <<

(c) Chhaya

(d) Devesh

Explanation:

$4AB+3AB+3BA-4BA = 7AB - BA$. This matches Bina. Option B.

Q.11 [1 mark]

A cylindrical tank of radius 10 cm is being filled with sugar at the rate of 100π cm³/s. The rate, at which the height of the sugar inside the tank is increasing, is :

(a) 0.1 cm/s

(b) 0.5 cm/s

(c) 1 cm/s <<

(d) 1.1 cm/s

Explanation:

$V=\pi r^2h=100\pi h \Rightarrow dV/dt=100\pi \cdot dh/dt=100\pi \Rightarrow dh/dt=1$ cm/s. Option C.

Q.12 [1 mark]

Let $\vec{p}$ and $\vec{q}$ be two unit vectors and α be the angle between them. Then $(\vec{p} + \vec{q})$ will be a unit vector for what value of α?

(a) π/4

(b) π/3

(c) π/2

(d) 2π/3 <<

Explanation:

$|\vec{p}+\vec{q}|^2=1 \Rightarrow 1+1+2\cos\alpha=1 \Rightarrow \cos\alpha=-1/2 \Rightarrow \alpha=2\pi/3$. Option D.

Q.13 [1 mark]

The line x = 1 + 5μ, y = −5 + μ, z = −6 −3μ passes through which of the following point?

(a) (1, −5, 6)

(b) (1, 5, 6)

(c) (1, −5, −6) <<

(d) (−1, −5, 6)

Explanation:

μ=0 gives (1, −5, −6). Option C.

Q.14 [1 mark]

If A denotes the set of continuous functions and B denotes set of differentiable functions, then which of the following depicts the correct relation between set A and B? (Venn diagrams:

(a) A is inside B

(b) B is inside A <<

(c) A and B intersect (overlapping)

(d) A and B are disjoint

Explanation:

Every differentiable function is continuous, so B ⊆ A (B lies inside A).

Printed option (B).

Q.15 [1 mark]

The area of the shaded region (figure) represented by the curves $y = x^2$, 0 ≤

$x \leq 2$ and y-axis is given by

(a) $\int_0^2 x^2\,dx$

(b) $\int_0^2 \sqrt{y}\,dy$

(c) $\int_0^4 x^2\,dx$

(d) $\int_0^4 \sqrt{y}\,dy$ <<

Explanation:

Region is between the curve and the y-axis (horizontal strips), y from 0 to 4, $x=\sqrt{y} \Rightarrow$ Area$=\int_0^4 \sqrt{y}\,dy$.

Option D.

Q.16 [1 mark]

A factory produces two products X and Y. The profit earned by selling X and Y is represented by the objective function Z = 5x + 7y, where x and y are the number of units of X and Y respectively sold. Which of the following statement is correct?

(a) The objective function maximizes the difference of the profit earned from products X and Y.

(b) The objective function measures the total production of products X and Y.

(c) The objective function maximizes the combined profit earned from selling X and Y. <<

(d) The objective function ensures the company produces more of product X than product Y.

Explanation:

Z=5x+7y is the total (combined) profit to be maximized. Option C.

Q.17 [1 mark]

If A and B are square matrices of order m such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following is always correct?

(a) A = B

(b) AB = BA <<

(c) A = 0 or B = 0

(d) A = I or B = I

Explanation:

(A−B)(A+B)=A²+AB−BA−B². Equating to A²−B² gives AB−BA=0 ⇒ AB=BA. Option B.

Q.18 [1 mark]

If p and q are respectively the order and degree of the differential equation d/dx (dy/dx)³ = 0, then (p − q) is

(a) 0

(b) 1 <<

(c) 2

(d) 3

Explanation:

d/dx[(dy/dx)³]=3(y')²y''=0.

Highest order derivative y'' ⇒ order p=2; its power is 1 ⇒ degree q=1.

p−q=1.

Option B.

Q.19 [1 mark]

Assertion (A): A = diag[3 5 2] is a scalar matrix of order 3 × 3. Reason (R): If a diagonal matrix has all non-zero elements equal, it is known as a scalar matrix. Select:

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true <<

Explanation:

A=diag[3 5 2] has unequal diagonal entries, so it is NOT scalar ⇒ Assertion false.

The Reason (definition of scalar matrix) is true.

Option D.

Q.20 [1 mark]

Assertion (A): Every point of the feasible region of a Linear Programming Problem is an optimal solution. Reason (R): The optimal solution for a Linear Programming Problem exists only at one or more corner point(s) of the feasible region. Select:

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true <<

Explanation:

Not every point is optimal (only corner points can be) ⇒ Assertion false.

Reason (corner-point theorem) is true.

Option D.

Section B

Very Short Answer Questions (2 marks each)

Q.21 [2 marks]

(a) A vector $\vec{a}$ makes equal angles with all the three axes. If the magnitude of the vector is $5\sqrt{3}$ units, then find $\vec{a}$.

Answer: $\vec{a} = \pm 5(\hat{i} + \hat{j} + \hat{k})$ i.e. $5\hat{i} + 5\hat{j} + 5\hat{k}$ or $-5\hat{i} - 5\hat{j} - 5\hat{k}$

Explanation:

Equal angles ⇒ $l=m=n$ with $l^2+m^2+n^2=1 \Rightarrow 3l^2=1 \Rightarrow l=\pm 1/\sqrt{3}$.

$\vec{a}=|\vec{a}|(l,m,n)=5\sqrt{3}\cdot(\pm 1/\sqrt{3})(\hat{i}+\hat{j}+\hat{k})=\pm 5(\hat{i}+\hat{j}+\hat{k})$.

OR

Q.21 [2 marks]

(b) If $\vec{\alpha}$ and $\vec{\beta}$ are position vectors of two points P and Q respectively, then find the position vector of a point R in QP produced such that QR = (3/2) QP.

Answer: Position vector of R = $(3/2)\vec{\alpha} - (1/2)\vec{\beta}$

Explanation:

$\overrightarrow{QP} = \vec{\alpha} - \vec{\beta}$ (P − Q). $\overrightarrow{R} = \overrightarrow{Q} + \overrightarrow{QR} = \vec{\beta} + (3/2)(\vec{\alpha} - \vec{\beta}) = (3/2)\vec{\alpha} - (1/2)\vec{\beta}$.

Q.22 [2 marks]

Evaluate : $\int_0^{\pi/4} \sqrt{(1 + \sin 2x)}\, dx$

Answer: 1

Explanation:

$1+\sin 2x=(\sin x+\cos x)^2$; $\sqrt{}=\sin x+\cos x$ on $[0,\pi/4]$.

$\int_0^{\pi/4}(\sin x+\cos x)dx=[-\cos x+\sin x]_0^{\pi/4}=(0)-(-1)=1$.

Q.23 [2 marks]

Find the values of 'a' for which f(x) = sin x − ax + b is increasing on R.

Answer: $a \le -1$

Explanation:

$f'(x)=\cos x-a\ge 0$ for all $x \Rightarrow a\le\cos x$ for all $x \Rightarrow a\le\min(\cos x)=-1$. Hence $a\le -1$.

Q.24 [2 marks]

If $\vec{a}$ and $\vec{b}$ are two non-collinear vectors, then find x, such that $\vec{\alpha} = (x - 2)\vec{a} + \vec{b}$ and $\vec{\beta} = (3 + 2x)\vec{a} - 2\vec{b}$ are collinear.

Answer: x = 1/4

Explanation:

Collinear $\Rightarrow \vec{\beta}=k\,\vec{\alpha}$.

Matching $\vec{a}$ and $\vec{b}$ coefficients: $3+2x=k(x-2)$ and $-2=k(1) \Rightarrow k=-2 \Rightarrow 3+2x=-2(x-2)=-2x+4 \Rightarrow 4x=1 \Rightarrow x=1/4$.

Q.25 [2 marks]

(a) If $x = e^{x/y}$, then prove that $dy/dx = (x - y)/(x \log x)$.

Answer: Proved

Explanation:

Take ln: ln x = x/y ⇒ y ln x = x.

Differentiate: $y' \cdot \ln x + y \cdot (1/x) = 1 \Rightarrow y' = (1 - y/x)/\ln x = (x - y)/(x \log x)$.

Proved.

OR

Q.25 [2 marks]

(b) If f(x) = { 2x − 3, −3 ≤ x ≤ −2 ; x + 1, −2 < x ≤ 0 }, check the differentiability of f(x) at x = −2.

Answer: Not differentiable at x = −2 (f is discontinuous there)

Explanation:

Left value at $x=-2$: $2(-2)-3=-7$.

Right limit as $x \to -2^+$: $x+1=-1$.

Since $-7 \neq -1$, f is discontinuous at $x=-2$, hence not differentiable there.

Q.26 [2 marks]

(a) Solve the differential equation 2(y + 3) − xy dy/dx = 0; given y(1) = −2.

Answer: $y - 3 \ln|y + 3| = 2 \ln|x| - 2$

Explanation:

Separate: $y/(y+3)\, dy = 2/x\, dx$.

Since $y/(y+3) = 1 - 3/(y+3)$: $y - 3\ln|y+3| = 2\ln|x| + C$.

$y(1) = -2 \Rightarrow -2 - 0 = 0 + C \Rightarrow C = -2$.

So $y - 3\ln|y+3| = 2\ln|x| - 2$.

OR

Q.26 [2 marks]

(b) Solve the following differential equation : $(1 + x^2)\, dy/dx + 2xy = 4x^2$.

Answer: $y = (4x^3/3 + C)/(1 + x^2)$

Explanation:

LHS = $d/dx[(1+x^2)y]$. So $(1+x^2)y = \int 4x^2\, dx = 4x^3/3 + C \Rightarrow y = (4x^3/3 + C)/(1+x^2)$.

Section C

Short Answer Questions (3 marks each)

Q.27 [3 marks]

Let R be a relation defined over N, where N is set of natural numbers, defined as "mRn if and only if m is a multiple of n, m, n ∈ N." Find whether R is reflexive, symmetric and transitive or not.

Answer: Reflexive: Yes; Symmetric: No; Transitive: Yes

Explanation:

Reflexive: every m is a multiple of itself ⇒ mRm ⇒ reflexive.

Symmetric: 6 is a multiple of 3 but 3 is not a multiple of 6 ⇒ not symmetric.

Transitive: if m mult of n and n mult of p, then m mult of p ⇒ transitive.

Q.28 [3 marks]

Solve the following linear programming problem graphically : Minimise $Z = x - 5y$ subject to the constraints : $x - y \geq 0$, $-x + 2y \geq 2$, $x \geq 3$, $y \leq 4$, $y \geq 0$.

Answer: Minimum $Z = -16$ at (4, 4)

Explanation:

Corner points of feasible region: (3, 2.5)→Z=−9.5, (3,3)→Z=−12, (6,4)→Z=−14, (4,4)→Z=4−20=−16.

Minimum Z=−16 at (4, 4).

Q.29 [3 marks]

(a) If $y = \log((\sqrt{x} + 1/\sqrt{x})^2)$, then show that $x(x + 1)^2 y_2 + (x + 1)^2 y_1 = 2$.

Answer: Proved (expression = 2)

Explanation:

$y = \log((x+1)^2/x) = 2\log(x+1) - \log x$.

$y_1 = 2/(x+1) - 1/x$ and $y_2 = -2/(x+1)^2 + 1/x^2$.

Then $x(x+1)^2y_2 = -2x + (x+1)^2/x$ and $(x+1)^2y_1 = 2(x+1) - (x+1)^2/x$.

Adding gives $-2x + 2x + 2 = 2$.

OR

Q.29 [3 marks]

(b) If $x\sqrt{(1 + y)} + y\sqrt{(1 + x)} = 0$, $-1 < x < 1$, $x \neq y$, then prove that $dy/dx = -1/(1 + x)^2$.

Answer: Proved

Explanation:

The relation gives $y=-x/(1+x)$.

Then $dy/dx=d/dx[-x/(1+x)]=-[(1+x)-x]/(1+x)^2=-1/(1+x)^2$.

Proved.

Q.30 [3 marks]

(a) A die with number 1 to 6 is biased such that P(2) = 3/10 and probability of other numbers is equal. Find the mean of the number of times number 2 appears on the dice, if the dice is thrown twice.

Answer: 3/5 (= 0.6)

Explanation:

Let X = number of times 2 appears in 2 throws, $p=P(2)=3/10$, $n=2$.

X is binomial $\Rightarrow$ mean $=np=2\times3/10=3/5$.

OR

Q.30 [3 marks]

(b) Two dice are thrown. Defined are the following two events A and B : A = {(x, y) : x + y = 9}, B = {(x, y) : x ≠ 3}, where (x, y) denote a point in the sample space. Check if events A and B are independent or mutually exclusive.

Answer: Neither independent nor mutually exclusive

Explanation:

$P(A)=4/36=1/9$, $P(B)=30/36=5/6$, $P(A\cap B)=3/36=1/12$.

$P(A)\cdot P(B)=5/54 \neq 1/12 \Rightarrow$ not independent.

$A\cap B \neq \varnothing \Rightarrow$ not mutually exclusive.

Q.31 [3 marks]

Find : $\int (1/x)\sqrt{((x+a)/(x-a))}\,dx$.

Answer: $\ln|x+\sqrt{(x^2-a^2)}| + \sec^{-1}(x/a) + C$

Explanation:

Write $(1/x)\sqrt{((x+a)/(x-a))} = (x+a)/(x\sqrt{(x^2-a^2)}) = 1/\sqrt{(x^2-a^2)} + a/(x\sqrt{(x^2-a^2)})$.

Integrate: $\int dx/\sqrt{(x^2-a^2)} = \ln|x+\sqrt{(x^2-a^2)}|$; $a\int dx/(x\sqrt{(x^2-a^2)}) = \sec^{-1}(x/a)$.

Sum = $\ln|x+\sqrt{(x^2-a^2)}|+\sec^{-1}(x/a)+C$.

(Verified numerically.)

Q.32 [3 marks]

Using integration, find the area of the region bounded by the line $y = 5x + 2$, the x-axis and the ordinates $x = -2$ and $x = 2$.

Answer: 104/5 sq units (= 20.8)

Explanation:

$y=5x+2=0$ at $x=-2/5$.

Area=$\int_{-2}^{-2/5} -(5x+2)dx + \int_{-2/5}^{2}(5x+2)dx$.

Computing: first part + second part = 104/5 = 20.8 sq units.

Section D

Long Answer Questions (5 marks each)

Q.33 [5 marks]

Find : $\int (x^2 + x + 1)/((x + 2)(x^2 + 1))\,dx$.

Answer: $(3/5)\ln|x + 2| + (1/5)\ln(x^2 + 1) + (1/5)\tan^{-1}x + C$

Explanation:

Partial fractions: $(x^2+x+1)/((x+2)(x^2+1)) = (3/5)/(x+2) + ((2/5)x + 1/5)/(x^2+1)$.

Integrate: $(3/5)\ln|x+2| + (1/5)\ln(x^2+1) + (1/5)\tan^{-1}x + C$.

Q.34 [5 marks]

(a) Find the shortest distance between the lines : $(x + 1)/2 = (y - 1)/1 = (z - 9)/(-3)$ and $(x - 3)/2 = (y + 15)/(-7) = (z - 9)/5$.

Answer: $4\sqrt{3}$ units

Explanation:

$a_1=(-1,1,9)$, $d_1=(2,1,-3)$; $a_2=(3,-15,9)$, $d_2=(2,-7,5)$.

$d_1\times d_2=(-16,-16,-16)$, $|d_1\times d_2|=16\sqrt{3}$.

$(a_2-a_1)=(4,-16,0)$, dot with $d_1\times d_2$ = 192.

SD=$192/(16\sqrt{3})=12/\sqrt{3}=4\sqrt{3}$.

OR

Q.34 [5 marks]

(b) Find the image A′ of the point A(2, 1, 2) in the line l : $\vec{r} = 4\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k})$. Also, find the equation of line joining AA′. Find the foot of perpendicular from point A on the line l.

Answer: Foot = (11/3, 7/3, 7/3); Image A′ = (16/3, 11/3, 8/3); line AA′ $\propto$ direction (5, 4, 1) through A(2,1,2)

Explanation:

Foot = P_0+td, $t=(A-P_0)\cdot d/|d|^2=((-2,-1,0)\cdot(1,-1,-1))/3=(-2+1+0)/3=-1/3 \Rightarrow$ foot=(4−1/3, 2+1/3, 2+1/3)=(11/3, 7/3, 7/3).

A′=2·foot−A=(16/3, 11/3, 8/3).

Direction AA′=A′−A=(10/3, 8/3, 2/3)$\propto$(5,4,1); line: $\vec{r}$=(2,1,2)+μ(5,4,1).

Q.35 [5 marks]

(a) Given A = [[−4, 4, 4], [−7, 1, 3], [5, −3, −1]] and B = [[1, −1, 1], [1, −2, −2], [2, 1, 3]], find AB. Hence, solve the system of linear equations : x − y + z = 4, x − 2y − 2z = 9, 2x + y + 3z = 1.

Answer: AB = 8I; x = 3, y = −2, z = −1

Explanation:

AB = 8I (diagonal 8).

So $B^{-1} = A/8$.

System has coefficient matrix B: $X=B^{-1}\cdot[4,9,1]^T=(1/8)A\cdot[4,9,1]^T \Rightarrow (x,y,z)=(3, -2, -1)$.

OR

Q.35 [5 marks]

(b) If A = [[1, 2, 0], [−2, −1, −2], [0, −1, 1]], then find A^{-1}. Hence, solve the system of linear equations : x − 2y = 10, 2x − y − z = 8, −2y + z = 7.

Answer: A^{-1} = [[−3, −2, −4], [2, 1, 2], [2, 1, 3]]; x = 0, y = −5, z = −3

Explanation:

A^{-1}=[[−3,−2,−4],[2,1,2],[2,1,3]].

The system's coefficient matrix is A^T;

X=(A^T)$^{-1}$·[10,8,7]^T=(A^{-1})^T·[10,8,7]^T ⇒ (x,y,z)=(0, −5, −3).

Q.36 [5 marks]

A school is organizing a debate competition with participants as speakers S = {S_1, S_2, S_3, S_4} and these are judged by judges J = {J_1, J_2, J_3}. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as R = {(x, y) : speaker x is judged by judge y, x ∈ S, y ∈ J}. Based on the above, answer the following : (i) How many relations can be there from S to J?

Answer: 4096 (= 2^{12})

Explanation:

A relation from S to J is any subset of S×J.

|S×J|=4×3=12, so number of relations = 2^{12} = 4096.

Q.0

A school is organizing a debate competition (S = {S_1, S_2, S_3, S_4}, J = {J_1, J_2, J_3}). (ii) A student identifies a function from S to J as f = {(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)}. Check if it is bijective.

Answer: Not bijective

Explanation:

f is a valid function but S_2 and S_3 both map to J_2, so f is not one-one (not injective).

Since |S|=4>|J|=3 a bijection is impossible.

Hence f is not bijective.

Q.0

A school is organizing a debate competition (S = {S_1, S_2, S_3, S_4}, J = {J_1, J_2, J_3}). (iii)(a) How many one-one functions can be there from set S to set J?

Answer: 0

Explanation:

A one-one (injective) function from S to J requires |S| ≤ |J|.

Here |S|=4 > |J|=3, so no injective function exists ⇒ 0.

Q.0

A school is organizing a debate competition (S = {S_1, S_2, S_3, S_4}). (iii)(b) Another student considers a relation R_1 = {(S_1, S_2), (S_2, S_4)} in set S. Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric.

Answer: Add {(S_1, S_1), (S_2, S_2), (S_3, S_3), (S_4, S_4)} — 4 pairs

Explanation:

For reflexivity all (S_i, S_i) must be present: add (S_1,S_1),(S_2,S_2),(S_3,S_3),(S_4,S_4) = 4 pairs.

This keeps it non-symmetric since e.g.

(S_2,S_1) is still absent while (S_1,S_2) is present.

Section E

Case Study Based Questions (4 marks each)

Q.37 [4 marks]

Three persons viz. Amber, Bonzi and Comet are manufacturing cars which run on petrol and on battery as well. Their production share in the market is 60%, 30% and 10% respectively. Of their respective production capacities, 20%, 10% and 5% cars respectively are electric (or battery operated). Based on the above, answer the following : (i)(a) What is the probability that a

randomly selected car is an electric car?

Answer: 31/200 (= 0.155)

Explanation:

P(E)=0.60×0.20 + 0.30×0.10 + 0.10×0.05 = 0.12 + 0.03 + 0.005 = 0.155 = 31/200.

OR

Q.37 [4 marks]

Three persons Amber, Bonzi and Comet manufacture cars (market share 60%, 30%, 10%; electric fractions 20%, 10%, 5%). (i)(b) What is the probability that a randomly selected car is a petrol car?

Answer: 169/200 (= 0.845)

Explanation:

P(petrol)=1 − P(electric)=1 − 31/200 = 169/200 = 0.845.

Q.0

Three persons Amber, Bonzi and Comet manufacture cars (market share 60%, 30%, 10%; electric fractions 20%, 10%, 5%). (ii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Comet?

Answer: 1/31

Explanation:

By Bayes: P(Comet|E)=(0.10×0.05)/0.155 = 0.005/0.155 = 1/31.

Q.0

Three persons Amber, Bonzi and Comet manufacture cars (market share 60%, 30%, 10%; electric fractions 20%, 10%, 5%). (iii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Amber or Bonzi?

Answer: 30/31

Explanation:

P(Amber or Bonzi|E)=(0.12+0.03)/0.155 = 0.15/0.155 = 30/31.

(Equivalently 1 − 1/31.)

Q.38 [4 marks]

A small town is analyzing the pattern of a new street light installation. The lights are set up in such a way that the intensity of light at any point x metres from the start of the street can be modelled by $f(x) = e^x \sin x$, where x is in metres. Based on the above, answer the following : (i) Find the intervals on which the f(x) is increasing or decreasing, $x \in [0, \pi]$.

Answer: Increasing on [0, 3π/4], decreasing on [3π/4, π]

Explanation:

$f'(x)=e^x(\sin x+\cos x)=\sqrt{2}\, e^x \sin(x+\pi/4)$.

On [0,π], f'>0 for x<3π/4 and f'<0 for x>3π/4.

So increasing on [0, 3π/4], decreasing on [3π/4, π].

Q.0

A small town street light intensity $f(x) = e^x \sin x$, $x \in [0, \pi]$. (ii) Verify whether each critical point when $x \in [0, \pi]$ is a point of local maximum or local minimum or a point of inflexion.

Answer: x = 3π/4 is a point of local maximum

Explanation:

Critical point in [0,π]: x=3π/4 (where sinx+cosx=0).

f''(3π/4)=−√2 e^{3π/4}<0, so x=3π/4 is a point of local maximum.