CBSE Class 12 Mathematics 2025 — Set 3 — Answer Key

English Medium

Time Allowed: 3 Hours | Maximum Marks: 80

General Instructions:

1. This question paper contains 38 questions. All questions are compulsory.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 6 questions carrying 2 marks each.
4. Section C has 6 questions carrying 3 marks each.
5. Section D has 4 questions carrying 5 marks each.
6. Section E has 3 case-study based questions carrying 4 marks each.
7. Draw neat figures wherever required.

Section A

Multiple Choice Questions (1 mark each)

Q.1 [1 mark]

The principal value of $\sin^{-1}(\sin(-10\pi/3))$ is :

(a) $-2\pi/3$

(b) $-\pi/3$

(c) $\pi/3$ <<

(d) $2\pi/3$

Explanation:

$\sin(-10\pi/3)$: add $4\pi \rightarrow -10\pi/3 + 12\pi/3 = 2\pi/3$, so $\sin(-10\pi/3) = \sin(2\pi/3) = \sqrt{3}/2$.

The principal value of $\sin^{-1}$ lies in $[-\pi/2, \pi/2]$.

The angle there with sine $\sqrt{3}/2$ is $\pi/3$.

So answer = $\pi/3$.

Q.2 [1 mark]

If A and B are square matrices of same order such that AB = A and BA = B, then $A^2 + B^2$ is equal to :

(a) A + B <<

(b) BA

(c) 2 (A + B)

(d) 2BA

Explanation:

AB = A and BA = B imply A and B are idempotent: $A^2 = A \cdot A = (AB)A = A(BA) = A \cdot B = A$, and similarly $B^2 = B$.

Hence $A^2 + B^2 = A + B$.

Q.3 [1 mark]

For real x, let $f(x) = x^3 + 5x + 1$. Then :

(a) f is one-one but not onto on R

(b) f is onto on R but not one-one

(c) f is one-one and onto on R <<

(d) f is neither one-one nor onto on R

Explanation:

$f'(x) = 3x^2 + 5 > 0$ for all x, so f is strictly increasing ⇒ one-one.

As $x \to \pm\infty$, $f \to \pm\infty$, and f is continuous ⇒ range = R ⇒ onto.

So f is one-one and onto on R.

Q.4 [1 mark]

If $y = \sin^{-1} x$, then $(1 - x^2)\, d^2y/dx^2$ is equal to :

(a) $x\, dy/dx$ <<

(b) $-x\, dy/dx$

(c) $x^2\, dy/dx$

(d) $-x^2\, dy/dx$

Explanation:

$y = \sin^{-1}x \Rightarrow y' = 1/\sqrt{(1-x^2)} \Rightarrow (1-x^2)(y')^2 = 1$.

Differentiating: $(1-x^2)\cdot 2y'y'' + (-2x)(y')^2 = 0 \Rightarrow (1-x^2)y'' = x\, y'$.

So $(1 - x^2)\, d^2y/dx^2 = x\, dy/dx$.

Q.5 [1 mark]

The values of λ so that $f(x) = \sin x - \cos x - \lambda x + C$ decreases for all real values of x are :

(a) $1 < \lambda < \sqrt{2}$

(b) $\lambda \geq 1$

(c) $\lambda \geq \sqrt{2}$ <<

(d) $\lambda < 1$

Explanation:

$f'(x) = \cos x + \sin x - \lambda$.

For f decreasing for all x, need $f'(x) \leq 0 \Rightarrow \lambda \geq \cos x + \sin x$ for all x.

The maximum of $\cos x + \sin x$ is $\sqrt{2}$, so $\lambda \geq \sqrt{2}$.

Q.6 [1 mark]

If P is a point on the line segment joining (3, 6, −1) and (6, 2, −2) and y-coordinate of P is 4, then its z-coordinate is :

(a) −3/2 <<
(b) 0
(c) 1
(d) 3/2

Explanation:
Parametrize: P = (3+3t, 6−4t, −1−t).
y = 6 − 4t = 4 ⇒ t = 1/2.
Then z = −1 − t = −1 − 1/2 = −3/2.

Q.7 [1 mark]

If M and N are square matrices of order 3 such that det (M) = m and MN = mI, then det (N) is equal to :

(a) −1
(b) 1
(c) $-m^2$
(d) m^2 <<

Explanation:
det(MN) = det(mI) = m^3 (order 3).
Also det(MN) = det(M)det(N) = m·det(N).
So m·det(N) = m^3 ⇒ det(N) = m^2.

Q.8 [1 mark]

If f(x) = { 3x − 2, 0 < x ≤ 1 ; $2x^2$ + ax, 1 < x < 2 } is continuous for x ∈ (0, 2), then a is equal to :

(a) −4
(b) −7/2
(c) −2
(d) −1 <<

Explanation:

Continuity at x = 1: left value = 3(1) − 2 = 1; right value = 2(1)² + a = 2 + a.

Equate: 1 = 2 + a ⇒ a = −1.

Q.9 [1 mark]

If f : N → W is defined as f(n) = { n/2, if n is even ; 0, if n is odd }, then f is :

(a) injective only

(b) surjective only <<

(c) a bijection

(d) neither surjective nor injective

Explanation:

Not injective (all odd n map to 0).

Surjective: every w ∈ W is attained — w = 0 by n = 1, and any w ≥ 1 by n = 2w (even).

So f is surjective only.

Q.10 [1 mark]

The matrix [[0, 1, −2], [−1, 0, −7], [2, 7, 0]] is a :

(a) diagonal matrix

(b) symmetric matrix

(c) skew symmetric matrix <<

(d) scalar matrix

Explanation:

a_i_j = −a_j_i for all off-diagonal entries (a_{12}=1, a_{21}=−1;

a_{13}=−2, a_{31}=2;

a_{23}=−7, a_{32}=7) and diagonal entries are 0.

So A^T = −A ⇒ skew symmetric matrix.

Q.11 [1 mark]

If the sides AB and AC of Δ ABC are represented by vectors ĵ + k̂ and 3î − ĵ + 4k̂ respectively, then the length of the median through A on BC is :

(a) 2√2 units

(b) √18 units

(c) √34/2 units <<

(d) √48/2 units

Explanation:

Median from A to midpoint of BC: AM = (AB + AC)/2 = ((0,1,1)+(3,−1,4))/2 = (3, 0, 5)/2.

|AM| = (1/2)√(9+0+25) = √34/2 units.

Q.12 [1 mark]

The function f defined by f(x) = { x, if x ≤ 1 ; 5, if x > 1 } is not continuous at :

(a) x = 0

(b) x = 1 <<

(c) x = 2

(d) x = 5

Explanation:

At x = 1: left limit = 1, f(1) = 1, but right limit = 5.

Since left ≠ right, f is not continuous at x = 1.

Q.13 [1 mark]

If f(x) = 2x + cos x, then f(x) :

(a) has a maxima at x = π

(b) has a minima at x = π

(c) is an increasing function <<

(d) is a decreasing function

Explanation:

f′(x) = 2 − sin x.

Since −1 ≤ sin x ≤ 1, f′(x) ≥ 1 > 0 for all x.

So f is an increasing function.

Q.14 [1 mark]

$\int (\cos 2x - \cos 2\alpha)/(\cos x - \cos \alpha)\, dx$ is equal to :

(a) 2(sin x + x cos α) + C <<

(b) $2(\sin x - x\cos\alpha) + C$

(c) $2(\sin x + 2x\cos\alpha) + C$

(d) $2(\sin x + \sin\alpha) + C$

Explanation:

$\cos 2x - \cos 2\alpha = (2\cos^2 x - 1) - (2\cos^2\alpha - 1) = 2(\cos^2 x - \cos^2\alpha) = 2(\cos x - \cos\alpha)(\cos x + \cos\alpha)$.

Dividing gives $2(\cos x + \cos\alpha)$.

$\int 2(\cos x + \cos\alpha)dx = 2(\sin x + x\cos\alpha) + C$.

Q.15 [1 mark]

The value of $\int_0^1 dx/(e^x + e^{-x})$ is :

(a) $-\pi/4$

(b) $\pi/4$

(c) $\tan^{-1} e - \pi/4$ <<

(d) $\tan^{-1} e$

Explanation:

Multiply by e^x/e^x: $\int_0^1 e^x\, dx/(e^{2x}+1)$.

Let $u = e^x$, $du = e^x dx \rightarrow \int_1^e du/(u^2+1) = \tan^{-1}u\,|_1^e = \tan^{-1}e - \tan^{-1}1 = \tan^{-1}e - \pi/4$.

Q.16 [1 mark]

The order and degree of the differential equation $(d^2y/dx^2)^2 + (dy/dx)^2 = x\sin(dy/dx)$ are :

(a) order 2, degree 2

(b) order 2, degree 1

(c) order 2, degree not defined <<

(d) order 1, degree not defined

Explanation:

Highest derivative is $d^2y/dx^2 \Rightarrow$ order 2.

The term $\sin(dy/dx)$ makes the equation non-polynomial in the derivatives, so the degree is not defined.

Q.17 [1 mark]

The area of the region enclosed by the curve y = √x and the lines x = 0 and x = 4 and x-axis is :

(a) 16/9 sq. units
(b) 32/9 sq. units
(c) 16/3 sq. units <<
(d) 32/3 sq. units

Explanation:
Area = $\int_0^4 \sqrt{x}\,dx = (2/3)x^{3/2}\,|_0^4 = (2/3)(8) = 16/3$ sq. units.

Q.18 [1 mark]

The corner points of the feasible region of a Linear Programming Problem are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). If Z = ax + by; (a, b > 0) be the objective function, and maximum value of Z is obtained at (0, 2) and (3, 0), then the relation between a and b is :

(a) a = b
(b) a = 3b
(c) b = 6a
(d) 3a = 2b <<

Explanation:
Max occurs at both (0,2) and (3,0), so Z is equal there: Z(0,2) = 2b and Z(3,0) = 3a.
Equate: 2b = 3a ⇒ 3a = 2b.

Q.19 [1 mark]

Assertion (A): If A and B are two events such that P(A ∩ B) = 0, then A and B are independent events. Reason (R): Two events are independent if the occurrence of one does not effect the occurrence of the other. Select the correct answer from the codes given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true. <<

Explanation:

Independence requires P(A ∩ B) = P(A)P(B), not P(A ∩ B) = 0.

P(A∩B)=0 means mutually exclusive (not independent unless one has probability 0).

So A is false.

R is the correct definition of independence, so R is true.

Answer: A false, R true.

Q.20 [1 mark]

Assertion (A): In a Linear Programming Problem, if the feasible region is empty, then the Linear Programming Problem has no solution. Reason (R): A feasible region is defined as the region that satisfies all the constraints. Select the correct answer from the codes given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). <<

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

Explanation:

A is true: an empty feasible region means no point satisfies all constraints, so the LPP has no solution.

R is a true definition of the feasible region.

But R merely defines the feasible region;

it does not by itself explain why an empty region gives no solution.

So both true, R not the correct explanation.

Section B

Very Short Answer Questions (2 marks each)

Q.21 [2 marks]

Let A and B be two square matrices of order 3 such that det (A) = 3 and det (B) = − 4. Find the value of det (− 6AB).

Answer: 2592

Explanation:

For order 3, $\det(-6AB) = (-6)^3 \cdot \det(A) \cdot \det(B) = -216 \cdot 3 \cdot (-4) = -216 \cdot (-12) = 2592$.

Q.22 [2 marks]

Find the least value of 'a' so that $f(x) = 2x^2 - ax + 3$ is an increasing function on [2, 4].

Answer: $a \leq 8$

Explanation:

$f'(x) = 4x - a$.

For f increasing on [2,4], need $f'(x) \geq 0$ for all x in [2,4], i.e.

$a \leq 4x$.

The minimum of 4x on [2,4] is at x = 2, giving 4(2) = 8.

So f is increasing on [2,4] for $a \leq 8$; the boundary value is a = 8.

Note: the paper asks for the least value of a, but the condition gives $a \leq 8$, so 8 is the greatest value and no least value exists.

OR

Q.22 [2 marks]

If $f(x) = x + 1/x$, $x \geq 1$, show that f is an increasing function.

Answer: Proved

Explanation:

$f'(x) = 1 - 1/x^2$.

For $x \geq 1$, $x^2 \geq 1 \Rightarrow 1/x^2 \leq 1 \Rightarrow f'(x) = 1 - 1/x^2 \geq 0$.

Hence f is increasing on $[1, \infty)$.

Proved.

Q.23 [2 marks]

Simplify $\sin^{-1}(x/\sqrt{(1 + x^2)})$.

Answer: $\tan^{-1} x$

Explanation:

Put $x = \tan\theta$, $\theta \in (-\pi/2, \pi/2)$.

Then $x/\sqrt{(1+x^2)} = \tan\theta/\sqrt{(1+\tan^2\theta)} = \tan\theta/\sec\theta = \sin\theta$.

So $\sin^{-1}(\sin\theta) = \theta = \tan^{-1} x$.

OR

Q.23 [2 marks]

Find domain of $\sin^{-1}\sqrt{(x - 1)}$.

Answer: [1, 2]

Explanation:

Need $x - 1 \geq 0$ (for the square root) and $0 \leq \sqrt{(x-1)} \leq 1$ (for $\sin^{-1}$).

$\sqrt{(x-1)} \leq 1 \Rightarrow x - 1 \leq 1 \Rightarrow x \leq 2$.

Combined with $x \geq 1$: domain = [1, 2].

Q.24 [2 marks]

Calculate the area of the region bounded by the curve $x^2/9 + y^2/4 = 1$ and the x-axis using integration.

Answer: 3π sq. units

Explanation:

The region bounded by the ellipse and the x-axis is the upper half of the ellipse ($y \geq 0$).

Area = $\int_{-3}^{3} 2\sqrt{(1 - x^2/9)}\, dx = (2/3)\int_{-3}^{3} \sqrt{(9 - x^2)}\, dx = (2/3)\cdot(\pi\cdot 3^2/2) = (2/3)(9\pi/2) = 3\pi$ sq.

units.

(Equivalently half of the full ellipse area 6π.)

Q.25 [2 marks]

For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/s, then how fast is the slope of the curve changing when $x = 2$?

Answer: −48 units/s

Explanation:

Slope $m = dy/dx = 5 - 6x^2$.

Then $dm/dt = -12x \cdot dx/dt$.

With $dx/dt = 2$ and $x = 2$: $dm/dt = -12(2)(2) = -48$.

The slope is decreasing at 48 units/s.

Q.26 [2 marks]

If $f : R^+ \to R$ is defined as $f(x) = \log_a x$ ($a > 0$ and $a \neq 1$), prove that f is a bijection. (R^+ is a set of all positive real numbers.)

Answer: Proved (bijection)

Explanation:

One-one: if $\log_a x_1 = \log_a x_2$ then $x_1 = x_2$ (log is injective for $a > 0$, $a \neq 1$).

Onto: for any $y \in R$, take $x = a^y \in R^+$; then $f(x) = \log_a(a^y) = y$.

So f is both one-one and onto $\Rightarrow$ a bijection.

Proved.

OR

Q.26 [2 marks]

Let A = {1, 2, 3} and B = {4, 5, 6}. A relation R from A to B is defined as R = {(x, y) : x + y = 6, $x \in A$, $y \in B$}. (i) Write all elements of R. (ii) Is R a function? Justify. (iii) Determine domain and range of R.

Answer: R = {(1,5),(2,4)}; not a function; domain {1,2}, range {4,5}

Explanation:

(i) Pairs with x + y = 6, $x \in \{1,2,3\}$, $y \in \{4,5,6\}$: (1,5), (2,4).

(3 needs $y=3 \notin B$).

So R = {(1,5),(2,4)}.

(ii) R is a function only if every element of A has exactly one image;

but $3 \in A$ has no image, so R is not a function on A.

(iii) Domain = {1, 2}, Range = {4, 5}.

Section C

Short Answer Questions (3 marks each)

Q.27 [3 marks]

Find k so that f(x) = { $(x^2 - 2x - 3)/(x + 1)$, $x \neq -1$; k, $x = -1$ } is continuous at $x = -1$.

Answer: $k = -4$

Explanation:

For $x \neq -1$: $(x^2 - 2x - 3)/(x+1) = (x-3)(x+1)/(x+1) = x - 3$.

Limit as $x \to -1$ is $-1 - 3 = -4$.

For continuity, $k = -4$.

OR

Q.27 [3 marks]

Check the differentiability of function $f(x) = x|x|$ at $x = 0$.

Answer: Differentiable at $x = 0$ ($f'(0) = 0$)

Explanation:

$f(x) = x|x| = x^2$ for $x \geq 0$ and $-x^2$ for $x < 0$.

Right derivative at 0: $\lim (h^2-0)/h = 0$.

Left derivative: $\lim (-h^2-0)/h = 0$.

Both equal 0, so f is differentiable at $x = 0$ with $f'(0) = 0$.

Q.28 [3 marks]

Evaluate : $\int_{\pi/2}^{\pi} e^x ((1 - \sin x)/(1 - \cos x))\, dx$

Answer: $e^{\pi/2}$

Explanation:

Write $(1 - \sin x)/(1 - \cos x)$.

Using $1 - \cos x = 2\sin^2(x/2)$ and $1 - \sin x = 1 - 2\sin(x/2)\cos(x/2)$: the expression = $(1/2)\text{cosec}^2(x/2) - \cot(x/2)$.

Take $f(x) = -\cot(x/2)$ so $f'(x) = (1/2)\text{cosec}^2(x/2)$.

Then integrand = $e^x(f(x) + f'(x))$, whose integral is $e^x f(x) = -e^x \cot(x/2)$.

Evaluate from $\pi/2$ to π: at π, $-e^{\pi} \cot(\pi/2) = 0$; at $\pi/2$, $-e^{\pi/2}\cot(\pi/4) = -e^{\pi/2}$.

Result = 0 − (−e^{π/2}) = e^{π/2}.

Q.29 [3 marks]

Find the probability distribution of the number of boys in families having three children, assuming equal probability for a boy and a girl.

Answer: P(0)=1/8, P(1)=3/8, P(2)=3/8, P(3)=1/8

Explanation:

X = number of boys ~ Binomial(n=3, p=1/2).

P(X=r) = C(3,r)(1/2)³.

P(0)=1/8, P(1)=3/8, P(2)=3/8, P(3)=1/8.

OR

Q.29 [3 marks]

A coin is tossed twice. Let X be a random variable defined as number of heads minus number of tails. Obtain the probability distribution of X and also find its mean.

Answer: P(−2)=1/4, P(0)=1/2, P(2)=1/4; mean = 0

Explanation:

Outcomes: HH → X=2, HT → 0, TH → 0, TT → −2, each prob 1/4.

So P(X=−2)=1/4, P(X=0)=1/2, P(X=2)=1/4.

Mean = (−2)(1/4)+0(1/2)+2(1/4) = 0.

Q.30 [3 marks]

Find the distance of the point (−1, −5, −10) from the point of intersection of the lines (x−1)/2 = (y−2)/3 = (z−3)/4 and (x−4)/5 = (y−1)/2 = z.

Answer: √97 units

Explanation:

L1: (1+2t, 2+3t, 3+4t); L2: (4+5s, 1+2s, s).

Equate: from z: 3+4t = s.

Sub into x: 1+2t = 4+5(3+4t) ⇒ 1+2t = 19+20t ⇒ −18 = 18t ⇒ t = −1.

Then point = (−1, −1, −1) (and s = −1 satisfies all).

Distance from (−1,−5,−10): $\sqrt{(0^2 + (-5-(-1))^2 + (-10-(-1))^2)} = \sqrt{(0+16+81)} = \sqrt{97}$ units.

Q.31 [3 marks]

Solve the following Linear Programming Problem using graphical method : Maximise Z = 100x + 50y subject to the constraints $3x + y \leq 600$, $x + y \leq 300$, $y \leq x + 200$, $x \geq 0$, $y \geq 0$.

Answer: Max Z = 22500 at (150, 150)

Explanation:

Corner points of the feasible region: (0,0), (200,0), (150,150), (50,250), (0,200).

Z values: (0,0)→0, (200,0)→20000, (150,150)→100(150)+50(150)=22500, (50,250)→17500, (0,200)→10000.

Maximum Z = 22500 at (150, 150).

Q.32 [3 marks]

If A is a 3 × 3 invertible matrix, show that for any scalar $k \neq 0$, $(kA)^{-1} = (1/k)A^{-1}$. Hence calculate $(3A)^{-1}$, where A = [[2, −1, 1], [−1, 2, −1], [1, −1, 2]].

Answer: $(3A)^{-1} = (1/12)[[3,1,-1],[1,3,1],[-1,1,3]]$

Explanation:

Proof: $(kA)((1/k)A^{-1}) = (k \cdot 1/k)(A\,A^{-1}) = I$, so $(kA)^{-1} = (1/k)A^{-1}$.

For A: det A = 2(4−1) − (−1)(−2+1) + 1(1−2) = 6 − 1 − 1 = 4.

adj A gives $A^{-1} = (1/4)[[3,1,-1],[1,3,1],[-1,1,3]]$.

Then $(3A)^{-1} = (1/3)A^{-1} = (1/12)[[3,1,-1],[1,3,1],[-1,1,3]]$.

Section D

Long Answer Questions (5 marks each)

Q.33 [5 marks]

The relation between the height of the plant (y cm) with respect to exposure to sunlight is governed by the equation $y = 4x - (1/2)x^2$, where x is the number of days exposed to sunlight. (i) Find the rate of growth of the plant with respect to sunlight. (ii) In how many days will the plant attain its

maximum height? What is the maximum height?

Answer: (i) $dy/dx = 4 - x$; (ii) 4 days, maximum height 8 cm

Explanation:

(i) Rate of growth = $dy/dx = 4 - x$.

(ii) Maximum when $dy/dx = 0 \Rightarrow x = 4$ days.

$d^2y/dx^2 = -1 < 0$ confirms a maximum.

$y_{max} = 4(4) - (1/2)(16) = 16 - 8 = 8$ cm.

Q.34 [5 marks]

Find : $\int \cos x / ((4 + \sin^2 x)(5 - 4\cos^2 x))\, dx$

Answer: $(2/15)\tan^{-1}(2 \sin x) - (1/30)\tan^{-1}((\sin x)/2) + C$

Explanation:

$5 - 4\cos^2 x = 5 - 4(1 - \sin^2 x) = 1 + 4\sin^2 x$.

Put $t = \sin x$, $dt = \cos x\, dx$: $\int dt/((4+t^2)(1+4t^2))$.

Partial fractions: $1/((t^2+4)(4t^2+1)) = a/(t^2+4) + b/(4t^2+1)$ with $4a+b=0$, $a+4b=1 \Rightarrow a=-1/15$, $b=4/15$.

So integral $= -1/15 \cdot (1/2)\tan^{-1}(t/2) + 4/15 \cdot (1/2)\tan^{-1}(2t) = (2/15)\tan^{-1}(2 \sin x) - (1/30)\tan^{-1}((\sin x)/2) + C$.

OR

Q.34 [5 marks]

Evaluate : $\int_0^{\pi} dx / (a^2\cos^2 x + b^2\sin^2 x)$

Answer: $\pi/(ab)$

Explanation:

By symmetry $\int_0^{\pi} = 2\int_0^{\pi/2}$.

Divide numerator and denominator by $\cos^2 x$: $2\int_0^{\pi/2} \sec^2 x\, dx/(a^2 + b^2\tan^2 x)$.

Put $u = \tan x$, $du = \sec^2 x\, dx$: $2\int_0^{\infty} du/(a^2 + b^2u^2) = (2/b^2) \cdot (1/(a/b)) \cdot [\tan^{-1}(bu/a)]_0^{\infty} = (2/(ab)) \cdot (\pi/2) = \pi/(ab)$.

Q.35 [5 marks]

Show that the area of a parallelogram whose diagonals are represented by

$\vec{a}$ and $\vec{b}$ is given by $(1/2)|\vec{a} \times \vec{b}|$. Also find the area of a parallelogram whose diagonals are $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} - \hat{k}$.

Answer: $\sqrt{62}/2$ sq. units

Explanation:

If diagonals are d_1, d_2, the sides are $(d_1+d_2)/2$ and $(d_1-d_2)/2$;

area = $|(d_1+d_2)/2 \times (d_1-d_2)/2| = (1/4)|-2(d_1 \times d_2)| = (1/2)|d_1 \times d_2|$.

Here $d_1 = (2,-1,1)$, $d_2 = (1,3,-1)$: $d_1 \times d_2 = (-2, 3, 7)$, $|d_1 \times d_2| = \sqrt{(4+9+49)} = \sqrt{62}$.

Area = $(1/2)\sqrt{62}$ sq.

units.

OR

Q.35 [5 marks]

Find the equation of a line in vector and cartesian form which passes through the point (1, 2, − 4) and is perpendicular to the lines $(x-8)/3 = (y+19)/-16 = (z-10)/7$, and $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

Answer: $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$; $(x-1)/2 = (y-2)/3 = (z+4)/6$

Explanation:

Direction perpendicular to both = $b_1 \times b_2$ where $b_1 = (3,-16,7)$, $b_2 = (3,8,-5)$.

$b_1 \times b_2 = (24, 36, 72) = 12(2, 3, 6)$.

Take direction (2,3,6).

Vector form: $\vec{r} = (\hat{i}+2\hat{j}-4\hat{k}) + \lambda(2\hat{i}+3\hat{j}+6\hat{k})$.

Cartesian: $(x-1)/2 = (y-2)/3 = (z+4)/6$.

Q.36 [5 marks]

Case Study: Students may have a misconception while comparing decimals (e.g. writing 78·56 > 78·9). A decimal comparison test was given to class VI students. In a Sports Day, 5 students threw a javelin with distances — Ajay 47·7, Bijoy 47·07, Kartik 43·09, Dinesh 43·9, Devesh 45·2 (meters) — and were asked who threw farthest. The teacher concludes 40% of students have the misconception and the rest do not. 80% of students having misconception answered Bijoy as the correct answer. 90% of students not

having the misconception did NOT answer Bijoy. (i) What is the probability of a student not having misconception but still answers Bijoy in the test? (ii) What is the probability that a randomly selected student answers Bijoy as his answer in the test?

Answer: (i) 0.06; (ii) 0.38

Explanation:

Let M = has misconception, P(M)=0.4, P(M')=0.6.

P(Bijoy|M)=0.8.

Since 90% of M' did NOT answer Bijoy, P(Bijoy|M')=0.10.

(i) P(M' and Bijoy) = P(M')·P(Bijoy|M') = 0.6×0.10 = 0.06.

(ii) P(Bijoy) = P(M)P(Bijoy|M) + P(M')P(Bijoy|M') = 0.4×0.8 + 0.6×0.10 = 0.32 + 0.06 = 0.38.

OR

Q.36 [5 marks]

Case Study (continued): (iii)(a) What is the probability that a student who answered as Bijoy is having misconception?

Answer: 16/19

Explanation:

By Bayes' theorem: P(M|Bijoy) = P(M)P(Bijoy|M)/P(Bijoy) = 0.32/0.38 = 32/38 = 16/19.

Q.0

Case Study (continued): (iii)(b) What is the probability that a student who answered as Bijoy is amongst students who do not have the misconception?

Answer: 3/19

Explanation:

By Bayes' theorem: P(M'|Bijoy) = P(M')P(Bijoy|M')/P(Bijoy) = 0.06/0.38 = 6/38 = 3/19.

Section E

Case Study Based Questions (4 marks each)

Q.37 [4 marks]

Case Study: An engineer is designing a new metro rail network. Line A's track is l_1: $(x-2)/3 = (y+1)/-2 = (z-3)/4$, and Line B's track is l_2: $(x-1)/2 = (y-3)/1 = (z+2)/-3$. (i) Find whether the two metro tracks are parallel. (ii) Solar panels are to be installed on the rooftops of Line A's stations, positioned parallel to Line A's track (l_1) and passing through the point (1, −2, −3). Determine the equation of that line.

Answer: (i) Not parallel; (ii) $(x-1)/3 = (y+2)/-2 = (z+3)/4$

Explanation:

(i) Direction of $l_1 = (3,-2,4)$, of $l_2 = (2,1,-3)$.

These are not scalar multiples ($3/2 \neq -2/1$), so the tracks are not parallel.

(ii) A line parallel to l_1 has direction $(3,-2,4)$;

through $(1,-2,-3)$: $(x-1)/3 = (y+2)/-2 = (z+3)/4$ (vector form $\vec{r} = (\hat{i}-2\hat{j}-3\hat{k}) + \lambda(3\hat{i}-2\hat{j}+4\hat{k})$).

OR

Q.37 [4 marks]

Case Study (continued): (iii)(a) To connect the stations, a pedestrian pathway perpendicular to the two metro lines is to be constructed which passes through point (3, 2, 1). Determine the equation of the pedestrian walkway.

Answer: $(x-3)/2 = (y-2)/17 = (z-1)/7$

Explanation:

The walkway is perpendicular to both l_1 (dir $(3,-2,4)$) and l_2 (dir $(2,1,-3)$), so its direction = $(3,-2,4) \times (2,1,-3) = (2, 17, 7)$.

Through $(3,2,1)$: $(x-3)/2 = (y-2)/17 = (z-1)/7$ (vector form $\vec{r} = (3\hat{i}+2\hat{j}+\hat{k}) + \lambda(2\hat{i}+17\hat{j}+7\hat{k})$).

Q.0

Case Study (continued): (iii)(b) Find the shortest distance between Line A and Line B.

Answer: 31/√342 units

Explanation:

Points $a_1=(2,-1,3)$, $a_2=(1,3,-2)$; directions $b_1=(3,-2,4)$, $b_2=(2,1,-3)$.

$b_1\times b_2 = (2,17,7)$, $|b_1\times b_2| = \sqrt{(4+289+49)} = \sqrt{342}$.

$a_2-a_1 = (-1,4,-5)$; $(a_2-a_1)\cdot(b_1\times b_2) = -2+68-35 = 31$.

Shortest distance = $|31|/\sqrt{342} = 31/\sqrt{342} \approx 1.676$ units.

Q.38 [4 marks]

Case Study: During a heavy gaming session, a laptop processor's temperature rises. After the session it cools, with the rate of cooling proportional to the difference between processor temperature and room temperature (25°C). Initially the processor's temperature is 85°C. The rate of cooling is $(d/dt)(T(t)) = -k(T(t) - 25)$, where T(t) is temperature at time t (minutes) and k is a constant. (i) Find the expression for temperature of processor, T(t), given that T(0) = 85°C.

Answer: T(t) = 25 + 60 e^{−kt}

Explanation:

$dT/dt = -k(T - 25)$.

Separate: $dT/(T-25) = -k\,dt \Rightarrow \ln|T-25| = -kt + C \Rightarrow T - 25 = A\,e^{-kt}$.

At t=0, T=85 ⇒ A = 60.

So $T(t) = 25 + 60\,e^{-kt}$.

Section A

Multiple Choice Questions (1 mark each)

Q.2 [1 mark]

Case Study (continued): (ii) How long will it take for the processor's temperature to reach 40°C? Given that k = 0·03, log_e 4 = 1·3863.

Answer: ≈ 46.21 minutes

Explanation:

Set T = 40: 40 = 25 + 60 e^{−kt} ⇒ 15 = 60 e^{−kt} ⇒ e^{−kt} = 1/4 ⇒ −kt = −ln 4 ⇒ t = (ln 4)/k = 1·3863/0·03 = 46·21 minutes.