CBSE Class 12 Mathematics 2026 — Set 1 — Answer Key

English Medium

Time Allowed: 3 Hours | Maximum Marks: 80

General Instructions:

1. This question paper contains 38 questions. All questions are compulsory.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 6 questions carrying 2 marks each.
4. Section C has 6 questions carrying 3 marks each.
5. Section D has 4 questions carrying 5 marks each.
6. Section E has 3 case-study based questions carrying 4 marks each.
7. Draw neat figures wherever required.

Section A

Multiple Choice Questions (1 mark each)

Q.1 [1 mark]

If $2\cos^{-1}x = y$, then y belongs to which of the following intervals?

(a) $0 \le y \le \pi$

(b) $-\pi \le y \le \pi$

(c) $0 \le y \le 2\pi$ <<

(d) $-\pi \le y \le 0$

Explanation:

The range of $\cos^{-1}x$ is $[0, \pi]$, so $\cos^{-1}x \in [0, \pi]$.

Therefore $y = 2\cos^{-1}x \in [0, 2\pi]$.

Answer: (C) $0 \le y \le 2\pi$.

Q.2 [1 mark]

Which of the following cannot be the order of a row-matrix?

(a) 2×1 <<

(b) 1×2

(c) 1×1

(d) $1 \times n$

Explanation:

A row-matrix has exactly one row, so its order must be $1 \times n$ for some $n \ge 1$.

Options (B) 1×2, (C) 1×1, and (D) 1×n are valid row-matrix orders.

Option (A) 2×1 is a column-matrix (two rows, one column), NOT a row-matrix.

Answer: (A).

Q.3 [1 mark]

Which of the following properties is/are true for two matrices of suitable orders? (i) $(A + B)' = A' + B'$ (ii) $(A - B)' = B' - A'$ (iii) $(AB)' = A'B'$ (iv) $(kAB)' = kB'A'$ (k is a scalar)

(a) (i) only

(b) (i), (ii) and (iii)

(c) (i) and (ii)

(d) (i) and (iv) <<

Explanation:

Check each: (i) (A+B)' = A'+B' — TRUE (standard transpose property).

(ii) (A–B)' = A'–B', NOT B'–A' — FALSE (the correct identity is (A–B)'=A'–B').

(iii) (AB)' = B'A' (reversal rule), NOT A'B' — FALSE.

(iv) (kAB)' = k(AB)' = k·B'A' — TRUE (scalar k factors out, then reversal rule applies).

So (i) and (iv) are true.

Answer: (D).

Q.4 [1 mark]

If A_1 = [[1,0,0],[0,2,0],[0,0,3]] and A_2 = [[0,2,0],[1,0,0],[0,0,6]], which of the following is correct?

(a) $|A_1| = 2|A_2|$

(b) $|A_2| = -2|A_1|$ <<

(c) $|A_1| = 2$

(d) $|A_2| = -1$

Explanation:

$|A_1|$: diagonal matrix, det = 1×2×3 = 6.

$|A_2|$: expand along row 1: 0·(minor) – 2·(1×6–0×0) + 0·(minor) = –2·6 = –12.

Check options: (A) 6 = 2(–12)? No.

(B) –12 = –2×6 = –12.

YES.

(C) $|A_1|=6\neq 2$.

(D) $|A_2|=-12\neq -1$.

Answer: (B) $|A_2| = -2|A_1|$.

Q.5 [1 mark]

One of the values of x for which the determinant |[cos x, sin x],[–cos x, sin x]| = 1 is

(a) 0

(b) π/4 <<

(c) π/3

(d) π/2

Explanation:

det = cos x · sin x – (sin x)(–cos x) = 2 sin x cos x = sin 2x.

sin 2x = 1 gives 2x = π/2, so x = π/4.

Answer: (B) π/4.

Q.6 [1 mark]

If A and B are skew-symmetric matrices of the same order, which of the following is also skew-symmetric?

(a) AB

(b) AB + BA

(c) $(A + B)^2$

(d) A – B <<

Explanation:

For a matrix M to be skew-symmetric: M' = –M.

(A) (AB)' = B'A' = (–B)(–A) = BA ≠ –AB in general.

(B) (AB+BA)' = B'A'+A'B' = (–B)(–A)+(–A)(–B) = BA+AB = AB+BA, so (AB+BA)' = AB+BA ≠ –(AB+BA) unless AB+BA=0.

Not always skew-symmetric.

(C) $(A+B)^2$=(A+B)(A+B); not necessarily skew-symmetric.

(D) (A–B)' = A'–B' = –A–(–B) = –A+B = –(A–B).

So A–B is skew-symmetric.

Answer: (D).

Q.7 [1 mark]

The least value of $f(x) = x^3 – 12x$ on [0, 3] is:

(a) –16 <<

(b) –9

(c) 0

(d) 16

Explanation:

f'(x) = $3x^2 - 12 = 3(x^2-4) = 0 \Rightarrow x = \pm 2$.

On [0,3], only x=2 is a critical point.

f(0)=0, f(2)=8–24=–16, f(3)=27–36=–9.

Values at endpoints and critical point: f(0)=0, f(2)=–16, f(3)=–9.

The least (minimum) value = –16 at x=2.

Answer: (A) –16.

Q.8 [1 mark]

If $\int 3ax/(b^2 + c^2x^2)\,dx = A\cdot\log|b^2 + c^2x^2| + K$, then A =

(a) 3a

(b) $3a/(2b^2)$

(c) $3a/(b^2c^2)$

(d) $3a/(2c^2)$ <<

Explanation:

Let $u = b^2 + c^2x^2$, then $du = 2c^2x\,dx$, so $x\,dx = du/(2c^2)$.

The integral becomes: $\int 3a/(u) \cdot du/(2c^2) = (3a/(2c^2)) \cdot \log|u| + K = (3a/(2c^2)) \cdot \log|b^2 + c^2x^2| + K$.

Comparing with $A\cdot\log|b^2+c^2x^2|+K$: $A = 3a/(2c^2)$.

Answer: (D).

Q.9 [1 mark]

$\int_{-1}^{1} x^3/(x^2 + 2|x| + 1)\,dx$ equals:

(a) 0 <<

(b) log 2

(c) 2 log 2

(d) (1/2) log 2

Explanation:

Let $f(x) = x^3/(x^2 + 2|x| + 1)$.

Check if f is odd: $f(-x) = (-x)^3/((-x)^2 + 2|-x| + 1) = -x^3/(x^2 + 2|x| + 1) = -f(x)$.

Since f is an odd function and the interval [–1, 1] is symmetric about 0, $\int_{-1}^{1} f(x)\,dx = 0$.

Answer: (A) 0.

Q.10 [1 mark]

The area bounded by y = x|x|, the x-axis, x = –1 and x = 1 is:

(a) 0

(b) 1/3

(c) 2/3 <<

(d) 4/3

Explanation:

y = x|x| = x^2 for x≥0 and $-x^2$ for x<0.

Note y is an odd function.

Area = $\int_{-1}^{0} |-x^2|\,dx + \int_{0}^{1} |x^2|\,dx = \int_{-1}^{0} x^2\,dx + \int_{0}^{1} x^2\,dx$.

By symmetry = $2\int_{0}^{1} x^2\,dx = 2[x^3/3]_0^1 = 2/3$.

Answer: (C) 2/3.

Q.11 [1 mark]

The integrating factor of the differential equation R(dx/dy) + Px = Q, where P, Q, R are functions of y, is:

(a) e^∫(P/Q)dy

(b) e^∫P dy

(c) e^∫(P/R)dy <<

(d) e^∫(P/R)dx

Explanation:

Divide the equation R(dx/dy)+Px=Q by R: dx/dy + (P/R)x = Q/R.

This is now in standard linear form dx/dy + p(y)x = q(y) where p(y)=P/R.

The integrating factor is e^∫p(y)dy = e^∫(P/R)dy.

Answer: (C).

Q.12 [1 mark]

The order and degree of the differential equation $d/dx(e^y) = 0$ are, respectively:

(a) 0, 1

(b) 1, 1 <<

(c) 2, 1

(d) 1, not defined

Explanation:

$d/dx(e^y) = e^y \cdot (dy/dx) = 0$.

Since $e^y \neq 0$, this gives $dy/dx = 0$.

The highest derivative is dy/dx (first order), and it appears to the first power.

Order = 1, Degree = 1.

Answer: (B).

Q.13 [1 mark]

The value of p for which the vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $2\hat{i} - p\hat{j} + \hat{k}$ are perpendicular to each other is:

(a) 0

(b) 1

(c) 5/2 <<

(d) –5/2

Explanation:

For perpendicularity, dot product = 0.

$(\hat{i}+2\hat{j}+3\hat{k})\cdot(2\hat{i}-p\hat{j}+\hat{k}) = 1(2)+2(-p)+3(1) = 2-2p+3 = 5-2p = 0$.

So $2p=5$, $p=5/2$.

Answer: (C) 5/2.

Q.14 [1 mark]

The value of m for which the points with position vectors $-\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{i} + m\hat{j} + 5\hat{k}$ and $3\hat{i} + 11\hat{j} + 6\hat{k}$ are collinear is:

(a) 8 <<

(b) –8

(c) 2

(d) 4

Explanation:

Let A=(–1,–1,2), B=(2,m,5), C=(3,11,6).

AB = B–A = (3, m+1, 3).

AC = C–A = (4, 12, 4).

For collinearity, AB = t·AC for some scalar t.

From the x-components: 3 = 4t => t = 3/4.

From z-components: 3 = 4(3/4) = 3 ✓.

From y-components: m+1 = 12t = 12(3/4) = 9, so m = 8.

Answer: (A) 8.

Q.15 [1 mark]

If |a| = 8, |b| = 3 and |a × b| = 12, then |a · b| is:

(a) 6√3

(b) 8√3

(c) 12√3 <<

(d) 3√12

Explanation:

|a × b| = |a||b| sin θ ⇒ 12 = 24 sin θ ⇒ sin θ = 1/2 ⇒ |cos θ| = √3/2.

|a · b| = |a||b||cos θ| = 24 × √3/2 = 12√3 (C).

Q.16 [1 mark]

The length of the perpendicular drawn from the point (2, 5, 7) to the line x/1 = y/0 = z/0 is:

(a) √53

(b) √65

(c) √74 <<

(d) √78

Explanation:

The line x/1 = y/0 = z/0 is the x-axis (parametrically: x=t, y=0, z=0).

The foot of perpendicular from P=(2,5,7) to the x-axis is F=(2,0,0) (project P onto the x-axis by taking x-coordinate only).

Distance = |PF| = √((5–0)² + (7–0)²) = √(25+49) = √74.

Answer: (C) √74.

Q.17 [1 mark]

The feasible region of an LPP with objective function Z = 5x + 7y is shown in the figure (corner points (0, 0), (0, 2), (3, 4) and (7, 0)). The maximum value of Z – minimum value of Z is:

(a) 8

(b) 29

(c) 35

(d) 43 <<

Explanation:

Corner points from the figure: (0, 0), (0, 2), (3, 4), (7, 0).

Z = 5x + 7y: Z(0, 0) = 0, Z(0, 2) = 14, Z(3, 4) = 15 + 28 = 43, Z(7, 0) = 35.

Max Z – Min Z = 43 – 0 = 43 (D).

Q.18 [1 mark]

The degree of the objective function of a Linear Programming Problem (LPP) is:

(a) 0

(b) 1 <<

(c) 2

(d) Any natural number

Explanation:

In a Linear Programming Problem, the objective function is linear (e.g., Z = ax + by + c).

A linear expression has degree 1.

The constraints are also linear.

Answer: (B) 1.

Q.19 [1 mark]

(Assertion-Reason) Assertion (A): In throwing an unbiased die, P(prime | odd) = 2/3. Reason (R): P(A|B) = P(A∩B)/P(B), provided P(B) ≠ 0.

(a) Both A and R are true, and R is the correct explanation of A <<

(b) Both A and R are true, but R is NOT the correct explanation of A

(c) A is true, but R is false

(d) A is false, but R is true

Explanation:

Assertion (A): When an unbiased die is thrown, the sample space = {1,2,3,4,5,6}.

Odd outcomes B = {1,3,5}.

Prime numbers among odd: {3,5} (1 is not prime).

So P(prime|odd) = n(prime∩odd)/n(odd) = 2/3.

A is TRUE.

Reason (R): P(A|B) = P(A∩B)/P(B) is the standard definition of conditional probability.

R is TRUE.

R directly explains how P(prime|odd) is calculated: P(prime∩odd)/P(odd) = (2/6)/(3/6) = 2/3.

R is the correct explanation.

Answer: (A).

Q.20 [1 mark]

(Assertion-Reason) Assertion (A): Lines x = py + q, z = ry + s and x = p'y + q', z = r'y + s' are perpendicular when pp' + rr' = 1. Reason (R): Two lines r = $a_1 + \lambda b_1$ and r = $a_2 + \mu b_2$ are perpendicular if $b_1 \cdot b_2 = 0$.

(a) Both A and R are true, and R is the correct explanation of A

(b) Both A and R are true, but R is NOT the correct explanation of A

(c) A is true, but R is false

(d) A is false, but R is true <<

Explanation:

Reason (R): The condition $b_1 \cdot b_2 = 0$ for perpendicularity is a standard theorem — TRUE.

Assertion (A): Rewrite x=py+q, z=ry+s as (x–q)/p = y/1 = (z–s)/r, direction vector d_1=(p,1,r).

Similarly d_2=(p',1,r').

Perpendicularity requires $d_1 \cdot d_2 = 0$: pp'+(1)(1)+rr'=0 => pp'+rr'=–1.

The assertion states pp'+rr'=1, which is WRONG (should be –1).

Assertion A is FALSE.

Answer: (D) A is false, but R is true.

Section B

Very Short Answer Questions (2 marks each)

Q.21 [2 marks]

Check whether the function f(x) defined as f(x) = |x – 3| / (2(x – 3)) for x < 3, and f(x) = (x – 6)/6 for x ≥ 3, is continuous at x = 3 or not.

Answer: f is continuous at x = 3

Explanation:

For x < 3, |x – 3| = –(x – 3), so f(x) = –1/2 and the left-hand limit is –1/2.

At x = 3, f(3) = (3 – 6)/6 = –1/2, and the right-hand limit is also (3 – 6)/6 = –1/2.

Since LHL = RHL = f(3) = –1/2, f is continuous at x = 3.

OR

Q.21 [2 marks]

OR: If $\sqrt{3}(x^2 + y^2) = 4xy$, find dy/dx at the point (1/2, √3/2).

Answer: √3

Explanation:

First verify (1/2, √3/2) is on the curve: √3(1/4 + 3/4) = √3·1 = √3;

4·(1/2)·(√3/2) = 4·√3/4 = √3.

✓ Differentiate $\sqrt{3}(x^2+y^2)=4xy$ implicitly with respect to x: $\sqrt{3}(2x+2y\cdot dy/dx) = 4y + 4x\cdot dy/dx$.

=> $2\sqrt{3}x + 2\sqrt{3}y\cdot dy/dx = 4y + 4x\cdot dy/dx$.

=> $(2\sqrt{3}y – 4x)\cdot dy/dx = 4y – 2\sqrt{3}x$.

=> $dy/dx = (4y – 2\sqrt{3}x)/(2\sqrt{3}y – 4x) = (2y – \sqrt{3}x)/(\sqrt{3}y – 2x)$.

At (1/2, √3/2): $dy/dx = (2\cdot\sqrt{3}/2 – \sqrt{3}\cdot 1/2)/(\sqrt{3}\cdot\sqrt{3}/2 – 2\cdot 1/2) = (\sqrt{3} – \sqrt{3}/2)/(3/2 – 1) = (\sqrt{3}/2)/(1/2) = \sqrt{3}$.

Q.22 [2 marks]

A room freshener bottle in the shape of an inverted cone sprays perfume at regular intervals such that the volume of perfume in the bottle decreases at the steady rate of 1 mm³/min. Find the rate at which the level of perfume is dropping at the instant when the level of perfume in the bottle is 10 mm, if the semi-vertical angle of the conical bottle is π/6.

Answer: –3/(100π) mm/min

Explanation:

Semi-vertical angle π/6, so r = h tan(π/6) = h/√3.

V = (1/3)πr²h = πh³/9, so dV/dt = (πh²/3)(dh/dt).

With dV/dt = –1 and h = 10: –1 = (100π/3)(dh/dt), so dh/dt = –3/(100π) mm/min.

The level drops at 3/(100π) mm/min.

Q.23 [2 marks]

Find the vector of magnitude 14 in the direction of QP, where P = (1, 3, 2) and Q = (–1, 0, 8).

Answer: 4î + 6ĵ – 12k̂

Explanation:

QP = P – Q = (1–(–1), 3–0, 2–8) = (2, 3, –6).

|QP| = √(4+9+36) = √49 = 7.

Unit vector in direction QP: (2/7, 3/7, –6/7).

Vector of magnitude 14: 14·(2/7)î + 14·(3/7)ĵ + 14·(–6/7)k̂ = 4î + 6ĵ – 12k̂.

Q.24 [2 marks]

If a = 3î – 2ĵ + 2k̂ and b = î + 2k̂ are adjacent sides of a parallelogram, find the diagonals and their lengths.

Answer: d_1 = 4î – 2ĵ + 4k̂ with $|d_1|$ = 6; d_2 = 2î – 2ĵ with $|d_2|$ = 2√2

Explanation:

The diagonals of a parallelogram with adjacent sides a and b are d_1 = a + b and d_2 = a – b.

$d_1 = (3+1)\hat{i} + (-2+0)\hat{j} + (2+2)\hat{k} = 4\hat{i} - 2\hat{j} + 4\hat{k}$.

$|d_1| = \sqrt{(16+4+16)} = \sqrt{36} = 6$.

$d_2 = (3-1)\hat{i} + (-2-0)\hat{j} + (2-2)\hat{k} = 2\hat{i} - 2\hat{j} + 0\hat{k}$.

$|d_2| = \sqrt{(4+4+0)} = \sqrt{8} = 2\sqrt{2}$.

Q.25 [2 marks]

Simplify: $\tan^{-1}[(\cos 2x - \sin 2x)/(\cos 2x + \sin 2x)]$, where $0 < x < \pi/4$.

Answer: $\pi/4 - 2x$

Explanation:

Divide numerator and denominator by cos 2x: $\tan^{-1}[(1 - \tan 2x)/(1 + \tan 2x)]$.

Use the identity $\tan(A-B) = (\tan A - \tan B)/(1+\tan A \cdot \tan B)$ with $A = \pi/4$ ($\tan(\pi/4)=1$) and $B = 2x$: $= \tan^{-1}[\tan(\pi/4 - 2x)]$.

For $0 < x < \pi/4$: $0 < 2x < \pi/2$, so $-\pi/4 < \pi/4 - 2x < \pi/4$, which lies in $(-\pi/2, \pi/2)$, the range of $\tan^{-1}$.

Therefore the expression = $\pi/4 - 2x$.

OR

Q.25 [2 marks]

OR: Evaluate: $\tan(\sin^{-1} 1 - \cos^{-1}(-1/2))$.

Answer: $-1/\sqrt{3}$

Explanation:

$\sin^{-1}(1) = \pi/2$.

$\cos^{-1}(-1/2) = \pi - \pi/3 = 2\pi/3$.

So the angle = $\pi/2 - 2\pi/3 = -\pi/6$.

$\tan(-\pi/6) = -\tan(\pi/6) = -1/\sqrt{3}$.

Q.26 [2 marks]

Evaluate: $\int_0^1 x \cdot \tan^{-1}x \, dx$.

Answer: $\pi/4 - 1/2$

Explanation:

Use integration by parts: $u = \tan^{-1}x$, $dv = x\,dx$.

Then du = 1/(1+x²)dx, v = x²/2.

= [x²/2 · tan⁻¹x]₀¹ – ∫₀¹ x²/(2(1+x²)) dx = (1/2)(π/4) – 0 – (1/2)∫₀¹ x²/(1+x²) dx = π/8 – (1/2)∫₀¹ [1 – 1/(1+x²)] dx = π/8 – (1/2)[x – tan⁻¹x]₀¹ = π/8 – (1/2)(1 – π/4) = π/8 – 1/2 + π/8 = π/4 – 1/2.

Section C

Short Answer Questions (3 marks each)

Q.27 [3 marks]

Find: ∫ √((x + 2)/(x – 2)) dx.

Answer: √(x² – 4) + 2·log|x + √(x² – 4)| + C

Explanation:

Rewrite: √((x+2)/(x–2)) = √(x+2)/√(x–2).

Multiply by √(x+2)/√(x+2): = (x+2)/√((x+2)(x–2)) = (x+2)/√(x²–4).

Split: ∫(x+2)/√(x²–4) dx = ∫x/√(x²–4) dx + 2∫1/√(x²–4) dx.

First integral: let u=x²–4, du=2x dx => ∫x/√u · du/(2x) = (1/2)∫u^(–1/2)du = √u = √(x²–4).

Second integral: 2∫1/√(x²–4) dx = 2·log|x+√(x²–4)| (standard form).

Answer: √(x²–4) + 2·log|x+√(x²–4)| + C.

OR

Q.27 [3 marks]

OR: Find: ∫ x²/((x² + 9)(x² + 16)) dx.

Answer: –(3/7)·tan⁻¹(x/3) + (4/7)·tan⁻¹(x/4) + C

Explanation:

Use partial fractions: x²/((x²+9)(x²+16)) = A/(x²+9) + B/(x²+16).

Multiply both sides by (x²+9)(x²+16): x² = A(x²+16) + B(x²+9).

Set x²=–9 (i.e., let t=x²): –9 = 7A => A = –9/7.

Set x²=–16: –16 = –7B => B = 16/7.

Verify: (–9/7)(x²+16)+(16/7)(x²+9) = (–9x²–144+16x²+144)/7 = 7x²/7 = x² ✓.

Integrate: (–9/7)∫1/(x²+9)dx + (16/7)∫1/(x²+16)dx = (–9/7)·(1/3)tan⁻¹(x/3) + (16/7)·(1/4)tan⁻¹(x/4) + C = –(3/7)tan⁻¹(x/3) + (4/7)tan⁻¹(x/4) + C.

Q.28 [3 marks]

If $I_1 = \int_{-\pi/4}^{\pi/4} dx/(1 + \cos 2x)$ and $I_2 = \int_{-(1/2)}^{1/2} |x|\, dx$, show that $I_1 - 4I_2 = 0$.

Answer: $I_1 = 1$, $I_2 = 1/4$, so $I_1 - 4I_2 = 0$

Explanation:

I_1: Use $1 + \cos 2x = 2\cos^2 x$.

$I_1 = \int_{-\pi/4}^{\pi/4} dx/(2\cos^2 x) = (1/2)\int_{-\pi/4}^{\pi/4} \sec^2 x\, dx = (1/2)[\tan x]_{-\pi/4}^{\pi/4} = (1/2)(\tan(\pi/4) - \tan(-\pi/4)) = (1/2)(1-(-1)) = (1/2)(2) = 1$.

I_2: Since $|x|$ is even, $I_2 = 2\int_0^{1/2} x\, dx = 2[x^2/2]_0^{1/2} = [x^2]_0^{1/2} = (1/2)^2 = 1/4$.

Therefore $I_1 - 4I_2 = 1 - 4\cdot(1/4) = 1 - 1 = 0$.

✓

Q.29 [3 marks]

Find the general solution of the differential equation $x^2(dy/dx) = x^2 + xy + y^2$.

Answer: $\tan^{-1}(y/x) = \log|x| + C$

Explanation:

The equation is homogeneous.

Let $y = vx$, so $dy/dx = v + x(dv/dx)$.

Substitute: $x^2(v + x\cdot dv/dx) = x^2 + x(vx) + v^2x^2 = x^2(1 + v + v^2)$.

Divide by x^2: $v + x\cdot dv/dx = 1 + v + v^2$.

$\Rightarrow x\cdot dv/dx = 1 + v^2$.

Separate variables: $dv/(1+v^2) = dx/x$.

Integrate both sides: $\tan^{-1}v = \log|x| + C$.

Replace $v = y/x$: $\tan^{-1}(y/x) = \log|x| + C$.

OR

Q.29 [3 marks]

OR: Find the particular solution of the differential equation $xy\cdot(dy/dx) = (x + 2)(y + 2)$, given that $y = -1$ when $x = 1$.

Answer: $y - 2\log|y + 2| = x + 2\log|x| - 2$

Explanation:

Separate variables: y/(y+2) dy = (x+2)/x dx.

Rewrite: [1 – 2/(y+2)]dy = [1 + 2/x]dx.

Integrate: y – 2log|y+2| = x + 2log|x| + C.

Apply initial condition y(1) = –1: (–1) – 2log|–1+2| = 1 + 2log|1| + C => –1 – 2log(1) = 1 + 0 + C => –1 – 0 = 1 + C => C = –2.

Particular solution: y – 2log|y+2| = x + 2log|x| – 2.

Q.30 [3 marks]

Minimize Z = 13x – 15y subject to: x + y ≤ 7, 2x – 3y + 6 ≥ 0, x ≥ 0, y ≥ 0.

Answer: Minimum Z = –30 at (0, 2)

Explanation:

Constraints: x+y≤7, 2x–3y≥–6 (i.e.

y≤(2x+6)/3), x≥0, y≥0.

Find corner points of feasible region: (0,0): x+y=0≤7✓, 2(0)–3(0)+6=6≥0✓.

Z=0.

(7,0): Z=91.

(0,2): 0+2=2≤7✓; 2(0)–3(2)+6=0≥0✓.

Z=13(0)–15(2)=–30.

(3,4): intersection of x+y=7 and 2x–3y=–6: from x=7–y, 2(7–y)–3y=–6, 14–5y=–6, y=4, x=3.

Z=13(3)–15(4)=39–60=–21.

Corner points and Z-values: (0,0)→0, (7,0)→91, (3,4)→–21, (0,2)→–30.

Minimum Z = –30 at (0, 2).

Q.31 [3 marks]

Bag I contains 3 red and 4 white balls. Bag II contains 8 red and 6 white balls. An unbiased die is rolled; if number < 3, Bag I is selected, otherwise Bag II is selected. A ball is drawn at random. Find the probability that the ball drawn is red.

Answer: 11/21

Explanation:

P(Bag I selected) = P(die shows 1 or 2) = 2/6 = 1/3.

P(Bag II selected) = 4/6 = 2/3.

P(Red|Bag I) = 3/7.

P(Red|Bag II) = 8/14 = 4/7.

By total probability: P(Red) = P(Bag I)·P(Red|Bag I) + P(Bag II)·P(Red|Bag II) = (1/3)(3/7) + (2/3)(4/7) = 1/7 + 8/21 = 3/21 + 8/21 = 11/21.

OR

Q.31 [3 marks]

OR: The probability of simultaneous occurrence of at least one of the two events X and Y is a. If the probability that exactly one of the events X, Y occurs is b, prove that P(X′) + P(Y′) = 2 – 2a + b.

Answer: Proof: P(X′) + P(Y′) = 2 – 2a + b

Explanation:

Here a = P(X ∪ Y) and b = P(X) + P(Y) – 2P(X ∩ Y).

Since a = P(X) + P(Y) – P(X ∩ Y), subtracting gives a – b = P(X ∩ Y), so P(X) + P(Y) = a + (a – b) = 2a – b.

Therefore P(X′) + P(Y′) = 2 – [P(X) + P(Y)] = 2 – 2a + b.

Q.32 [3 marks]

A relation R is defined on Z, the set of integers, as R = {(x, y) : |x – y| is divisible by a prime number p, x, y ∈ Z}. Check whether R is an equivalence relation or not.

Answer: R is an equivalence relation (reflexive, symmetric, transitive)

Explanation:

Reflexive: For any x ∈ Z, |x–x| = 0 = 0·p, so p divides 0.

Thus (x,x) ∈ R for all x.

R is reflexive.

Symmetric: If (x,y) ∈ R, then p||x–y|.

Since |y–x| = |x–y|, p||y–x|.

So $(y,x) \in R$.

R is symmetric.

Transitive: If $(x,y) \in R$ and $(y,z) \in R$, then $p|(x-y)$ and $p|(y-z)$.

So $x-y = kp$ and $y-z = lp$ for integers k,l.

Then $x-z = (x-y)+(y-z) = (k+l)p$.

So $p||x-z|$, giving $(x,z) \in R$.

R is transitive.

Since R is reflexive, symmetric, and transitive, R is an equivalence relation.

OR

Q.32 [3 marks]

OR: Let $f: R-\{3/5\} \to R-\{3/5\}$ be defined by $f(x) = (3x + 2)/(5x - 3)$. Show that f is one-one and onto.

Answer: f is bijective (one-one and onto)

Explanation:

One-one: Suppose $f(x_1) = f(x_2)$.

Then $(3x_1+2)/(5x_1-3) = (3x_2+2)/(5x_2-3)$.

Cross-multiply: $(3x_1+2)(5x_2-3) = (3x_2+2)(5x_1-3)$.

Expand: $15x_1x_2-9x_1+10x_2-6 = 15x_1x_2-9x_2+10x_1-6$.

Cancel $15x_1x_2$ and -6: $-9x_1+10x_2 = -9x_2+10x_1 \Rightarrow 19x_2 = 19x_1 \Rightarrow x_1 = x_2$.

So f is one-one.

Onto: For any $y \in R-\{3/5\}$, solve $y = (3x+2)/(5x-3)$: $y(5x-3) = 3x+2 \Rightarrow 5xy-3y = 3x+2 \Rightarrow x(5y-3) = 3y+2 \Rightarrow x = (3y+2)/(5y-3)$.

Since $y \neq 3/5$, $(5y-3) \neq 0$, so x is defined.

Also $x \neq 3/5$ (verify: if $x=3/5$, then $5(3/5)-3 = 0$, contradiction).

So every y in the codomain has a pre-image x in the domain.

f is onto.

Section D

Long Answer Questions (5 marks each)

Q.33 [5 marks]

If $A = [[0, 2, 1], [-2, -1, -2], [1, -1, 0]]$, find A^{-1} and use it to solve the

following system of equations: –2y + z = 7, 2x – y – z = 8, x – 2y = 10.

Answer: x = 0, y = –5, z = –3

Explanation:

|A| = 0(0 – 2) – 2(0 + 2) + 1(2 + 1) = –1 ≠ 0.

adj A = [[–2, –1, –3], [–2, –1, –2], [3, 2, 4]], so A^{-1} = adj A / (–1) = [[2, 1, 3], [2, 1, 2], [–3, –2, –4]].

The system is A′X = B, where A′ = [[0, –2, 1], [2, –1, –1], [1, –2, 0]], X = (x, y, z)′, B = (7, 8, 10)′.

So $X = (A')^{-1}B = (A^{-1})'B$ = [[2, 2, –3], [1, 1, –2], [3, 2, –4]](7, 8, 10)′ = (14 + 16 – 30, 7 + 8 – 20, 21 + 16 – 40)′ = (0, –5, –3)′.

So x = 0, y = –5, z = –3.

OR

Q.33 [5 marks]

(b) If the matrix [[3, −1, sin 3x], [−7, 4, cos 2x], [−11, 7, 2]] is a singular matrix, then find all values of x, where x ∈ [0, π/2].

Answer: x = 0 or x = π/6

Explanation:

For a singular matrix, det = 0.

Expanding: det = 3(4·2 − 7·cos2x) − (−1)((−7)·2 − (−11)·cos2x) + sin3x((−7)·7 − 4·(−11)) = 3(8 − 7cos2x) + (−14 + 11cos2x) + sin3x(−49 + 44) = 24 − 21cos2x − 14 + 11cos2x − 5sin3x = 10 − 10cos2x − 5sin3x.

Using cos2x = 1 − 2sin²x: = 20sin²x − 5sin3x = 0.

With sin3x = 3sinx − 4sin³x: 20sin²x − 5(3sinx − 4sin³x) = 5sinx(4sinx + 4sin²x − 3) = 0.

So sinx = 0 (x = 0), or 4sin²x + 4sinx − 3 = 0 ⇒ sinx = 1/2 (x = π/6).

Both lie in [0, π/2].

Answer: x = 0 or x = π/6.

Q.34 [5 marks]

If x = cos t and y = cos mt, prove that $(1 – x^2)y_2 – xy_1 + m^2y = 0$, where $y_1 = dy/dx$ and $y_2 = d^2y/dx^2$.

Answer: Proved: $(1 – x^2)y_2 – xy_1 + m^2y = 0$

Explanation:

$x = \cos t$, $y = \cos mt$.

$y_1 = dy/dx = (dy/dt)/(dx/dt) = (–m \sin mt)/(–\sin t) = m \cdot \sin mt/\sin t$.

Note: $1–x^2 = 1–\cos^2 t = \sin^2 t$.

Differentiate y_1 with respect to t: $d(y_1)/dt = m \cdot [m \cos mt \cdot \sin t – \sin mt \cdot \cos t]/\sin^2 t = m(m \cos mt \cdot \sin t – \sin mt \cdot \cos t)/\sin^2 t$.

$y_2 = d^2y/dx^2 = (d(y_1)/dt)/(dx/dt) = [m(m \cos mt \cdot \sin t – \sin mt \cdot \cos t)/\sin^2 t]/(–\sin t) = –m(m \cos mt \cdot \sin t – \sin mt \cdot \cos t)/\sin^3 t$.

Now compute $(1–x^2)y_2$: $= \sin^2 t \cdot [–m(m \cos mt \cdot \sin t – \sin mt \cdot \cos t)/\sin^3 t] = –m(m \cos mt \cdot \sin t – \sin mt \cdot \cos t)/\sin t = –m^2 \cos mt + m \cdot \sin mt \cdot \cos t/\sin t$.

And $xy_1 = \cos t \cdot m \cdot \sin mt/\sin t = m \cdot \sin mt \cdot \cos t/\sin t$.

Therefore: $(1–x^2)y_2 – xy_1 = (–m^2 \cos mt + m \cdot \sin mt \cdot \cos t/\sin t) – m \cdot \sin mt \cdot \cos t/\sin t = –m^2 \cos mt = –m^2y$.

So $(1–x^2)y_2 – xy_1 + m^2y = –m^2y + m^2y = 0$.

✓

Q.35 [5 marks]

Check whether the lines $(x–1)/2 = (y–2)/3 = (z–3)/4$ and $(x–4)/5 = (y–1)/2 = z$ are parallel or not. If parallel, find the distance between them, otherwise find their point of intersection, if the lines are intersecting.

Answer: The lines intersect at (–1, –1, –1)

Explanation:

Direction vectors: $d_1=(2,3,4)$, $d_2=(5,2,1)$.

$d_1 \times d_2 = |\hat{i},\hat{j},\hat{k};2,3,4;5,2,1| = \hat{i}(3–8)–\hat{j}(2–20)+\hat{k}(4–15) = –5\hat{i}+18\hat{j}–11\hat{k} \neq 0$, so lines are not parallel.

Parametric forms: L_1: (1+2s, 2+3s, 3+4s); L_2: (4+5t, 1+2t, t).

For intersection, equate components: (i) $1+2s = 4+5t \Rightarrow 2s–5t=3$.

(ii) $2+3s = 1+2t \Rightarrow 3s–2t=–1$.

(iii) $3+4s = t$.

From (iii): $t=3+4s$.

Sub into (i): $2s–5(3+4s)=3 \Rightarrow 2s–15–20s=3 \Rightarrow –18s=18 \Rightarrow s=–1$.

Then $t=3+4(-1)=-1$.

Check (ii): $3(-1)-2(-1)=-3+2=-1$ ✓.

Point of intersection on L_1: $(1+2(-1), 2+3(-1), 3+4(-1)) = (-1, -1, -1)$.

Verify on L_2: $(4+5(-1), 1+2(-1), -1) = (-1, -1, -1)$ ✓.

The lines intersect at $(-1, -1, -1)$.

Q.36(a) [5 marks]

(Case Study) An online delivery company has 5000 subscribers at Rs. 300/year. For every Rs. 1 increase in fee, 10 subscribers leave. The company increases the fee by Rs. x. (i) How many subscribers discontinue their subscriptions after an increase of Rs. x?

Answer: 10x subscribers

Explanation:

For every Rs.

1 increase in fee, 10 subscribers leave.

If the fee increases by Rs.

x, the number of subscribers who discontinue = $10 \times x = 10x$.

Q.36(b) [5 marks]

(Case Study — continued) (ii) Express R(x), the total revenue of the company, as a function of x.

Answer: $R(x) = -10x^2 + 2000x + 1500000$

Explanation:

After increasing fee by Rs.

x: New fee = Rs.

(300+x).

Remaining subscribers = 5000–10x.

Revenue $R(x) = (300+x)(5000-10x) = 1500000-3000x+5000x-10x^2 = 1500000+2000x-10x^2$.

So $R(x) = -10x^2 + 2000x + 1500000$.

Q.36(c) [5 marks]

(Case Study — continued) (iii)(a) Find the value of x for which R(x) is maximum.

Answer: x = 100 (i.e., fee increase of Rs. 100 maximises revenue)

Explanation:

R(x) = $-10x^2+2000x+1500000$.

R'(x) = $-20x+2000$.

Set R'(x)=0: $-20x+2000=0 \Rightarrow x=100$.

R''(x)=$-20<0$, so x=100 gives a maximum.

The fee should be increased by Rs.

100 to maximise revenue.

Maximum revenue = $-10(100)^2+2000(100)+1500000 = -100000+200000+1500000$ = Rs.

1,600,000.

OR

Q.36(c) [5 marks]

(Case Study — continued) (iii)(b) OR: Find the sub-intervals of (0, 5000) in which R(x) is strictly increasing or strictly decreasing.

Answer: R(x) is increasing on (0, 100) and decreasing on (100, 5000)

Explanation:

R'(x) = $-20x + 2000 = 20(100-x)$.

R'(x) > 0 when x < 100 (i.e., $100-x > 0$).

R'(x) < 0 when x > 100.

Therefore: R(x) is strictly increasing on (0, 100) and strictly decreasing on (100, 5000).

Q.37(a) [4 marks]

(Case Study) A jackpot lottery has 1,00,000 tickets sold at Rs. 100 each. Prizes: 1 first prize of Rs. 3,00,000; 2 second prizes of Rs. 2,00,000 each; 3 third prizes of Rs. 50,000 each. Rohan bought 1 ticket. (i) List the possible prize amounts Rohan can win.

Answer: Rs. 0, Rs. 50,000, Rs. 2,00,000, Rs. 3,00,000

Explanation:
Rohan can win one of the following: No prize: Rs.
0 (probability highest).
Third prize: Rs.
50,000.
Second prize: Rs.
2,00,000.
First prize: Rs.
3,00,000.
These are the four possible prize amounts (not accounting for the ticket cost of Rs.
100 paid).
The possible prize values are Rs.
0, Rs.
50,000, Rs.
2,00,000, and Rs.
3,00,000.

Q.37(b) [4 marks]

(Case Study — continued) (ii)(a) Find the probability that Rohan wins at least Rs. 2,00,000.

Answer: 3/1,00,000

Explanation:
Winning at least Rs.
2,00,000 means winning either first prize (1 ticket) or second prize (2 tickets).
Favourable outcomes = 1 + 2 = 3.
Total tickets = 1,00,000.
P(winning at least Rs.
2,00,000) = 3/1,00,000.

OR

Q.37(b) [4 marks]

(Case Study — continued) (ii)(b) OR: Find the probability that Rohan wins no prize.

Answer: 99994/1,00,000 (= 49997/50000)

Explanation:

Total prize-winning tickets = 1 (first) + 2 (second) + 3 (third) = 6 tickets out of 1,00,000.

P(winning some prize) = 6/1,00,000.

P(no prize) = 1 – 6/1,00,000 = 99994/1,00,000 = 49997/50000.

Q.37(c) [4 marks]

(Case Study — continued) (iii) Rohan also bought a ticket in another jackpot where the prize is Rs. 5,00,000 and the probability of winning is 1 in 1,00,000. Find the probability that he wins exactly one jackpot (consider 'winning' as winning the top prize in each lottery).

Answer: 199998/10,00,00,00,000 (approximately 2/1,00,000)

Explanation:

Let p_1 = P(winning first jackpot top prize) = 1/1,00,000.

Let p_2 = P(winning second jackpot) = 1/1,00,000.

The two events are independent.

P(exactly one wins) = $p_1(1-p_2) + (1-p_1)p_2$ = (1/1,00,000)(99999/1,00,000) + (99999/1,00,000)(1/1,00,000) = $2\times99999/(1,00,000)^2$ = 199998/10,00,00,00,000 = 99999/5,00,00,00,000.

Approximately 2/1,00,000 = 0.00002.

Note: the paper does not say which jackpot counts, so this answer depends on that reading;

it could not be confirmed against an official CBSE marking scheme.

Q.38(a) [4 marks]

(Case Study) A roundabout traffic island is bounded by a circle C_1: $x^2 + y^2 = 64$. In the centre is a circular pond C_2: $x^2 + y^2 = 4$. (i) Represent the curves C_1 and C_2 with the help of a diagram.

Answer: diagram

Explanation:

The diagram shows two concentric circles centred at the origin O(0,0): C_2 (inner circle): $x^2+y^2=4$, radius = 2 units.

C_1 (outer circle): $x^2+y^2=64$, radius = 8 units.

The region between the two circles (the annular ring, radius 2 to 8) represents the roundabout road area.

The inner circle C_2 represents the pond.

Both circles are centred at the origin with C_2 entirely inside C_1.

Q.38(b) [4 marks]

(Case Study — continued) (ii) Express y as a function of x for each curve.

Answer: $y = \sqrt{(64 - x^2)}$ for C_1 and $y = \sqrt{(4 - x^2)}$ for C_2 (upper semicircles; the full circles are $y = \pm\sqrt{(64 - x^2)}$ and $y = \pm\sqrt{(4 - x^2)}$)

Explanation:

From $x^2+y^2=64$: $y^2 = 64–x^2$, so $y = \pm\sqrt{(64–x^2)}$, where $–8 \le x \le 8$ (+ gives the upper semicircle, – the lower).

From $x^2+y^2=4$: $y = \pm\sqrt{(4–x^2)}$, where $–2 \le x \le 2$.

For the areas in part (iii), the upper semicircles $y = \sqrt{(64–x^2)}$ and $y = \sqrt{(4–x^2)}$ are integrated and doubled.

Q.38(c) [4 marks]

(Case Study — continued) (iii)(a) Find the area covered by the roundabout (the region bounded by C_1) using integration.

Answer: 64π square units

Explanation:

Area of circle C_1 ($x^2+y^2=64$, radius=8) using integration: Area = $\int_{-8}^{8} 2\sqrt{(64–x^2)}\,dx = 2\cdot[x/2\cdot\sqrt{(64–x^2)} + 32\cdot\sin^{-1}(x/8)]_{-8}^{8} = 2\cdot[(0+32\cdot\pi/2)–(0+32\cdot(–\pi/2))] = 2\cdot[16\pi+16\pi] = 2\cdot32\pi = 64\pi$.

Alternatively, area = $\pi r^2 = \pi(8^2) = 64\pi$ square units.

OR

Q.38(c) [4 marks]

(Case Study — continued) (iii)(b) OR: Find the area covered by the circular pond C_2 using integration.

Answer: 4π square units

Explanation:

Area of circle C_2 ($x^2+y^2=4$, radius=2) using integration: Area = $\int_{-2}^{2} 2\sqrt{(4-x^2)}\,dx$ = $2\cdot[x/2\cdot\sqrt{(4-x^2)} + 2\cdot\sin^{-1}(x/2)]_{-2}^{2}$ = $2\cdot[(0+2\cdot\pi/2)-(0+2\cdot(-\pi/2))]$ = $2\cdot[\pi+\pi]$ = $2\cdot2\pi$ = 4π.

Alternatively, area = πr^2 = $\pi(2^2)$ = 4π square units.