CBSE Class 12 Mathematics 2026 — Set 2 — Answer Key

English Medium

Time Allowed: 3 Hours | Maximum Marks: 80

General Instructions:

1. This question paper contains 38 questions. All questions are compulsory.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 6 questions carrying 2 marks each.
4. Section C has 6 questions carrying 3 marks each.
5. Section D has 4 questions carrying 5 marks each.
6. Section E has 3 case-study based questions carrying 4 marks each.
7. Draw neat figures wherever required.

Section A

Multiple Choice Questions (1 mark each)

Q.1 [1 mark]

A relation R on set A = {1, 2, 3} defined as R = {(1,1),(2,2),(1,2)} is

(a) Reflexive only

(b) Reflexive and Transitive

(c) Symmetric and Transitive

(d) Transitive only <<

Explanation:

A = {1,2,3}.

(3,3) is not in R so R is NOT reflexive.

(1,2) is in R but (2,1) is not, so NOT symmetric.

Check transitivity: all pairs (a,b),(b,c) in R imply (a,c) in R — verified for (1,1),(1,2)=>(1,2)✓ and all other cases.

So R is Transitive only → (D).

Q.2 [1 mark]

If A and B are square matrices of same order, then which of the following statements is/are always true?

(i) (A+B)(A-B) = A^2 - B^2

(ii) AB = BA

(iii) (A+B)^2 = A^2 + AB + BA + B^2

(iv) AB = 0 => A = 0 or B = 0

(a) Only (i) and (iii)

(b) Only (ii) and (iii)

(c) Only (iii) <<

(d) Only (iii) and (iv)

Explanation:

(A+B)^2 = A^2 + AB + BA + B^2 is always true by direct expansion.

(i) requires AB=BA (not always true).

(ii) AB=BA is not always true (matrices don't generally commute).

(iv) is false — zero divisors exist in matrix algebra.

Only (iii) is always true → (C).

Q.3 [1 mark]

If A = [[1, a, b], [-1, 2, c], [0, 5, 3]] is a symmetric matrix, then the value of 3a + b + c is

(a) 2 <<

(b) 6

(c) 4

(d) 0

Explanation:

For symmetric matrix, a_ij = a_ji.

So a_12 = a_21 => a = -1; a_13 = a_31 => b = 0; a_23 = a_32 => c = 5.

Therefore 3a + b + c = 3(-1) + 0 + 5 = -3 + 5 = 2 → (A).

Q.4 [1 mark]

If A = [[cos x, -sin x],[sin x, cos x]] and A + A' = I, then the value of x in (0, pi/2) is

(a) 0

(b) pi/4

(c) pi/3 <<

(d) pi/2

Explanation:

A' = [[cos x, sin x],[-sin x, cos x]].

So A + A' = [[2cos x, 0],[0, 2cos x]] = I = [[1,0],[0,1]].

This gives 2cos x = 1 => cos x = 1/2 => x = pi/3 → (C).

Q.5 [1 mark]

For a square matrix A, (3A)^(-1) =

(a) 3A^(-1)

(b) 9A^(-1)

(c) (1/3)A^(-1) <<

(d) (1/9)A^(-1)

Explanation:

For a scalar c and invertible matrix A, (cA)^(-1) = (1/c)A^(-1).

Therefore (3A)^(-1) = (1/3)A^(-1) → (C).

Q.6 [1 mark]

If det[[-1,-2,5],[-2,a,-1],[0,4,2a]] = -86, then the sum of all possible values of a is

(a) 4

(b) 5

(c) -4 <<

(d) 9

Explanation:

Expanding: det = -1*(2a^2 + 4) - (-2)*(-4a - 0) + 5*(-8) = -2a^2 - 4 - 8a - 40 = -2a^2 - 8a - 44 = -86.

So 2a^2 + 8a - 42 = 0 => a^2 + 4a - 21 = 0 => (a+7)(a-3) = 0 => a = -7 or a = 3.

Sum = -7 + 3 = -4 → (C).

Q.7 [1 mark]

If e^(-x) + e^(-y) = 2, then dy/dx is

(a) e^(x-y)

(b) e^(y-x)

(c) -e^(x-y)

(d) -e^(y-x) <<

Explanation:

Differentiating e^(-x) + e^(-y) = 2 w.r.t.

x: -e^(-x) - e^(-y)(dy/dx) = 0 => dy/dx = -e^(-x)/e^(-y) = -e^(y-x) → (D).

Q.8 [1 mark]

For f(x) = x + 1/x (x != 0)

(a) local maximum value is 2

(b) local minimum value is -2

(c) local maximum value is -2 <<

(d) local minimum value < local maximum value

Explanation:

f'(x) = 1 - 1/x^2 = 0 => x = ±1.

f''(x) = 2/x^3.

At x = -1: f''(-1) = -2 < 0 => local maximum; f(-1) = -1 - 1 = -2.

At x = 1: f''(1) = 2 > 0 => local minimum; f(1) = 2.

So local maximum value = -2 → (C).

Q.9 [1 mark]

If integral from 0 to 2a of 1/(1+4x^2) dx = pi/6, then the value of a is

(a) sqrt(3)/4 <<

(b) sqrt(3)/2

(c) sqrt(3)

(d) 2*sqrt(3)

Explanation:

integral of 1/(1+4x^2) dx = (1/2)arctan(2x) + C.

Evaluating from 0 to 2a: (1/2)arctan(4a) = pi/6 => arctan(4a) = pi/3 => 4a = tan(pi/3) = sqrt(3) => a = sqrt(3)/4 →

(A).

Q.10 [1 mark]

Which of the following expressions will give the area of region bounded by the curve y = x^2 and line y = 16?

(a) integral from 0 to 4 of x^2 dx

(b) 2 * integral from 0 to 4 of x^2 dx

(c) integral from 0 to 16 of sqrt(y) dy

(d) 2 * integral from 0 to 16 of sqrt(y) dy <<

Explanation:

y = x^2 meets y = 16 at x = ±4.

Using horizontal strips: width at height y is 2*sqrt(y) (from x = -sqrt(y) to x = sqrt(y)).

Area = integral from 0 to 16 of 2*sqrt(y) dy = 2 * integral from 0 to 16 of sqrt(y) dy → (D).

Q.11 [1 mark]

The general solution of the differential equation dy/dx = √y/√x is

(a) log √y = log √x + C

(b) √y + √x = C

(c) √y – √x = C <<

(d) log √y + log √x = C

Explanation:

Separate variables: dy/√y = dx/√x.

Integrating, 2√y = 2√x + C′, so √y – √x = C.

Answer: (C).

Q.12 [1 mark]

The integrating factor of the differential equation 2x(dy/dx) - y = 3 is

(a) sqrt(x)

(b) 1/sqrt(x) <<

(c) e^x

(d) e^(-x)

Explanation:

Rewrite: dy/dx - y/(2x) = 3/(2x).

P(x) = -1/(2x).

Integrating factor = e^(integral of P dx) = e^(-1/2 * ln|x|) = x^(-1/2) = 1/sqrt(x) → (B).

Q.13 [1 mark]

If |a| = 5 and -2 <= lambda <= 1, then the sum of greatest and smallest value

of |lambda*a| is

(a) -5

(b) 5

(c) 10 <<

(d) 15

Explanation:

|lambda*a| = |lambda| * |a| = 5|lambda|.

For lambda in [-2,1], |lambda| ranges from 0 to 2.

Greatest value = 5*2 = 10 (at lambda = -2); smallest value = 5*0 = 0 (at lambda = 0).

Sum = 10 + 0 = 10 → (C).

Q.14 [1 mark]

Vector of magnitude 3 making equal angles with x and y axes and perpendicular to z axis is

(a) i_hat + 2*sqrt(2)*j_hat

(b) 3*k_hat

(c) (3*sqrt(2)/2)*i_hat + (3*sqrt(2)/2)*j_hat <<

(d) sqrt(3)*i_hat + sqrt(3)*j_hat + sqrt(3)*k_hat

Explanation:

Perpendicular to z-axis => z-component = 0.

Equal angles with x and y axes => direction cosines l = m.

l^2 + m^2 = 1 => 2l^2 = 1 => l = 1/sqrt(2).

Unit vector = (1/sqrt(2))*i_hat + (1/sqrt(2))*j_hat.

Scaling to magnitude 3: (3/sqrt(2))*i_hat + (3/sqrt(2))*j_hat = (3*sqrt(2)/2)*i_hat + (3*sqrt(2)/2)*j_hat →

(C).

Q.15 [1 mark]

Direction cosines of the line given by equations: (2x-1)/4 = (1-y)/3 = -z/6 are

(a) 2, -3, -6

(b) 2/7, -3/7, -6/7 <<

(c) 2/7, -3/7, 6/7

(d) 4/sqrt(61), -3/sqrt(61), -6/sqrt(61)

Explanation:

Rewrite: (x - 1/2)/2 = (y - 1)/(-3) = z/(-6).

Direction ratios are 2, -3, -6.

Magnitude = sqrt(4+9+36) = sqrt(49) = 7.

Direction cosines: 2/7, -3/7, -6/7 → (B).

Q.16 [1 mark]

In a linear programming problem, the linear function which has to be maximized or minimized is called

(a) a feasible function

(b) an objective function <<

(c) an optimal function

(d) a constraint

Explanation:

The function to be maximized or minimized in a linear programming problem is the objective function →

(B).

Q.17 [1 mark]

[Graph-based question] For the feasible region shown in the graph, the non-trivial constraints of the linear programming problem are (graph: one line through (0, 5) and (5, 0), another through (0, 3) and (9, 0); shaded triangle bounded by the two lines and the x-axis from x = 5 to x = 9)

(a) $x + y \leq 5, x + 3y \leq 9$

(b) $x + y \leq 5, x + 3y \geq 9$

(c) $x + y \geq 5, x + 3y \leq 9$ <<

(d) $x + y \geq 5, 3x + y \leq 9$

Explanation:

The two boundary lines are x + y = 5 (through (5,0),(0,5)) and x + 3y = 9 (through (9,0),(0,3)), meeting at (3,2).

The shaded feasible region is the triangle (3,2),(5,0),(9,0): it lies to the right of x + y = 5 (so x + y ≥ 5) and below x + 3y = 9 (so x + 3y ≤ 9), with x,y ≥ 0.

Hence the non-trivial constraints are x + y ≥ 5, x + 3y ≤ 9 — option (C).

Q.18 [1 mark]

For two events A and B such that P(A) != 0 and P(B) != 1, P(A_complement / B_complement) =

(a) 1 – P(A/B)

(b) 1 – P(A′/B)

(c) / P(B′)

(d) / P(B′) <<

Explanation:

P(A′/B′) = P(A′ ∩ B′)/P(B′).

By De Morgan, A′ ∩ B′ = (A ∪ B)′, so P(A′ ∩ B′) = 1 – P(A ∪ B).

Hence P(A′/B′) = [1 – P(A ∪ B)]/P(B′).

Answer: (D).

Q.19 [1 mark]

(Assertion-Reason) For two vectors a and b:

Assertion (A): |a x b|^2 + (a . b)^2 = |a|^2 * |b|^2

Reason (R): |a x b| = (a . b) * tan(theta), where theta is the angle between them.

(a) Both A and R are true, and R is the correct explanation of A

(b) Both A and R are true, and R is not the correct explanation of A <<

(c) A is true but R is false

(d) A is false but R is true

Explanation:

Assertion A: |a x b|^2 + (a .

b)^2 = |a|^2|b|^2 sin^2(theta) + |a|^2|b|^2 cos^2(theta) = |a|^2|b|^2.
TRUE (Lagrange identity).
Reason R: |a x b| = |a||b|sin(theta) and a.b = |a||b|cos(theta), so |a x b| = (a.b)tan(theta).
TRUE.
However R gives a derived relationship, not the fundamental reason why A holds (A follows directly from sin^2+cos^2=1).
R is not the correct explanation of A → (B).

Q.20 [1 mark]

(Assertion-Reason)
Assertion (A): A line can have direction cosines <1, 1, 1>
Reason (R): cos(theta) = 1 is possible for theta = 0.

(a) Both A and R are true, and R is the correct explanation of A
(b) Both A and R are true, and R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true <<

Explanation:
Assertion A: For <1,1,1> to be direction cosines, we need 1^2 + 1^2 + 1^2 = 1, but 3 ≠ 1.
So A is FALSE.
Reason R: cos(0) = 1, so cos(theta) = 1 is possible.
R is TRUE.
→ (D).

Section B

Very Short Answer Questions (2 marks each)

Q.21 [2 marks]

Check whether f : R – {3} → R defined as f(x) = (x – 2)/(x – 3) is onto or not.

Answer: f is NOT onto. The range of f is R – {1}, so 1 has no pre-image in R – {3}.

Explanation:

Put y = (x – 2)/(x – 3).

Then y(x – 3) = x – 2, so x(y – 1) = 3y – 2 and x = (3y – 2)/(y – 1), undefined for y = 1.

Also (x – 2)/(x – 3) = 1 gives –2 = –3, impossible.

So 1 is not in the range and f is not onto.

OR

Q.21 [2 marks]

(OR) Check whether f: Z×Z -> Z×Z (where Z is the set of integers) defined as f(x,y) = (2y, 3x) is injective or not.

Answer: f IS injective. If f(x1,y1) = f(x2,y2) then (2y1,3x1) = (2y2,3x2), giving y1 = y2 and x1 = x2, so (x1,y1) = (x2,y2). Hence f is injective (though not surjective).

Explanation:

If f(x1,y1) = f(x2,y2), then (2y1, 3x1) = (2y2, 3x2).

This gives 2y1 = 2y2 => y1 = y2 and 3x1 = 3x2 => x1 = x2.

Since (x1,y1) = (x2,y2), f is injective.

However, f is not onto since, e.g., (1,1) has no pre-image (we'd need 2y=1 which has no integer solution).

So f is injective but not surjective.

Q.22 [2 marks]

If x = a*sin^3(t), y = b*cos^3(t), then find dy/dx at t = pi/4.

Answer: -b/a

Explanation:

dx/dt = 3a*sin^2(t)*cos(t).

dy/dt = -3b*cos^2(t)*sin(t).

dy/dx = dy/dt / dx/dt = [-3b*cos^2(t)*sin(t)] / [3a*sin^2(t)*cos(t)] = -(b/a)*cot(t).

At t = pi/4: cot(pi/4) = 1.

Therefore dy/dx = -b/a.

Q.23 [2 marks]

Find the absolute maximum value of f(x) = cos x + sin^2(x), x in [0, pi].

Answer: 5/4

Explanation:

f(x) = cos x + 1 - cos^2(x).

Let u = cos x, u in [-1,1].

h(u) = -u^2 + u + 1.

h'(u) = -2u + 1 = 0 => u = 1/2 => cos x = 1/2 => x = pi/3.

h(1/2) = -1/4 + 1/2 + 1 = 5/4.

Check endpoints: f(0) = 1, f(pi) = -1.

Absolute maximum = 5/4 at x = pi/3.

OR

Q.23 [2 marks]

(OR) If the volume of a solid hemisphere increases at a uniform rate, prove that its surface area varies inversely as its radius.

Answer: dS/dt is proportional to 1/r, hence surface area varies inversely as radius.

Explanation:

Volume V = (2/3)*pi*r^3 => dV/dt = 2*pi*r^2*(dr/dt) = k (constant).

So dr/dt = k/(2*pi*r^2).

Total surface area S = 3*pi*r^2 => dS/dt = 6*pi*r*(dr/dt) = 6*pi*r * k/(2*pi*r^2) = 3k/r.

Since k is constant, dS/dt = 3k/r ∝ 1/r.

Hence surface area varies inversely as radius.

Q.24 [2 marks]

If AB_vec = j_hat + k_hat and AC_vec = 3*i_hat - j_hat + 4*k_hat represent the two vectors along the sides AB and AC of triangle ABC, prove that the median AD_vec = (AB_vec + AC_vec)/2, where D is the midpoint of BC. Hence, find the length of median AD.

Answer: AD = sqrt(34)/2

Explanation:

D is midpoint of BC, so BD_vec = (1/2)BC_vec = (1/2)(AC_vec - AB_vec).

AD_vec = AB_vec + BD_vec = AB_vec + (1/2)(AC_vec - AB_vec) = (1/2)(AB_vec +

AC_vec).

AD_vec = (1/2)((j+k) + (3i-j+4k)) = (1/2)(3i+5k) = (3/2)i_hat + (5/2)k_hat.

|AD| = (1/2)*sqrt(9+25) = (1/2)*sqrt(34) = sqrt(34)/2.

Q.25 [2 marks]

Find the co-ordinates of the point on the line r_vec = -j_hat + 3*k_hat + lambda*(2*i_hat - 2*j_hat + k_hat) such that the sum of co-ordinates is 3.

Answer: (2, -3, 4)

Explanation:

General point on line: (2*lambda, -1-2*lambda, 3+lambda).

Sum of coordinates: 2*lambda + (-1-2*lambda) + (3+lambda) = lambda + 2 = 3 => lambda = 1.

Point: (2*1, -1-2, 3+1) = (2, -3, 4).

Verification: 2 + (-3) + 4 = 3 ✓.

Q.26 [2 marks]

Find: integral of (x+2)/sqrt(9x - x^2) dx

Answer: -sqrt(9x - x^2) + (13/2)*sin^(-1)((2x-9)/9) + C

Explanation:

Write 9x - x^2 = -(x^2 - 9x) = 81/4 - (x - 9/2)^2.

Write x+2 = A*(d/dx)(9x-x^2) + B = A*(9-2x) + B.

Comparing: -2A = 1 => A = -1/2; 9A + B = 2 => B = 13/2.

Integral = (-1/2)*integral[(9-2x)/sqrt(9x-x^2)]dx + (13/2)*integral[1/sqrt(81/4-(x-9/2)^2)]dx = -sqrt(9x-x^2) + (13/2)*sin^(-1)((2x-9)/9) + C.

Section C

Short Answer Questions (3 marks each)

Q.27 [3 marks]

Evaluate: $\int$ from π/12 to 5π/12 of dx/(1 + √cot x)

Answer: pi/6

Explanation:

$I = \int \sqrt{\sin x}/(\sqrt{\sin x} + \sqrt{\cos x})\,dx$ over $[\pi/12, 5\pi/12]$.

Using $\int f(x)dx = \int f(a + b - x)dx$ with $a + b = \pi/2$, $I = \int \sqrt{\cos x}/(\sqrt{\sin x} + \sqrt{\cos x})\,dx$.

Adding, $2I = \int 1\,dx = 5\pi/12 - \pi/12 = \pi/3$, so $I = \pi/6$.

OR

Q.27 [3 marks]

(OR) Evaluate: integral from -pi/6 to pi/2 of (sin|x| + cos|x|) dx

Answer: (7 - sqrt(3))/2

Explanation:

Split at 0: integral from -pi/6 to 0 of (-sin x + cos x) dx + integral from 0 to pi/2 of (sin x + cos x) dx.

First part: [cos x + sin x] from -pi/6 to 0 = (1+0) - (sqrt(3)/2 - 1/2) = 1 - sqrt(3)/2 + 1/2 = 3/2 - sqrt(3)/2.

Second part: [-cos x + sin x] from 0 to pi/2 = (0+1) - (-1+0) = 2.

Total = 3/2 - sqrt(3)/2 + 2 = 7/2 - sqrt(3)/2 = (7 - sqrt(3))/2.

Q.28 [3 marks]

If d/dx(F(x)) = 1/(1 + e^x), then find F(x) given that F(0) = log(1/2).

Answer: F(x) = x - log(1 + e^x)

Explanation:

F'(x) = 1/(1+e^x).

Integrate: integral of 1/(1+e^x) dx.

Multiply numerator and denominator: = integral of e^(-x)/(e^(-x)+1) dx ...

alternatively note that 1/(1+e^x) = 1 - e^x/(1+e^x).

integral = x - log(1+e^x) + C.

F(0) = 0 - log(1+1) + C = -log 2 + C = log(1/2) = -log 2 => C = 0.

Therefore F(x) = x - log(1+e^x).

Q.29 [3 marks]

Solve the differential equation: x*(dy/dx) = y - x*sin^2(y/x), given that y(1) = pi/6.

Answer: cot(y/x) = log|x| + sqrt(3)

Explanation:

Homogeneous: let v = y/x, y = vx.

dy/dx = v + x*dv/dx.

Substituting: x*(v + x*dv/dx) = vx - x*sin^2(v) => x^2*(dv/dx) = -x*sin^2(v) => dv/sin^2(v) = -dx/x.

Integrating: -cot(v) = -log|x| + C => cot(y/x) = log|x| + C.

At x=1, y=pi/6: cot(pi/6) = log(1) + C => sqrt(3) = 0 + C => C = sqrt(3).

Solution: cot(y/x) = log|x| + sqrt(3).

OR

Q.29 [3 marks]

(OR) Find the general solution of the differential equation: y*log(y)*(dx/dy) + x = 2/y.

Answer: x*log(y) = -2/y + C

Explanation:

Rearrange: dx/dy + x/(y*log y) = 2/(y^2*log y).

This is linear in x.

Integrating factor = e^(integral dy/(y*log y)) = e^(log(log y)) = log y.

Multiply: d/dy(x*log y) = 2/(y^2*log y) * log y = 2/y^2.

Integrate: x*log y = integral(2/y^2)dy = -2/y + C.

Q.30 [3 marks]

Solve the following linear programming problem graphically:

Maximize Z = 10500x + 9000y

Subject to: x + y <= 50, 2x + y <= 80, x >= 0, y >= 0.

Answer: Maximum Z = 495000 at (x, y) = (30, 20)

Explanation:

Corner points: (0,0): Z=0;

(40,0): Z=420000;

(0,50): Z=450000;

intersection of x+y=50 and 2x+y=80: subtracting gives x=30, y=20 => check: 30+20=50✓, 60+20=80✓;
Z=10500(30)+9000(20)=315000+180000=495000.
Maximum Z = 495000 at (30,20).

Q.31 [3 marks]

The probability of hitting the target by a trained sniper is three times the probability of not hitting the target on a stormy day. The sniper fired two shots. Find the probability that (i) target is hit, (ii) at least one shot misses the target.

Answer: (i) 15/16, (ii) 7/16

Explanation:
Let p = P(hit).
p = 3*(1-p) => p = 3/4, so P(miss) = 1/4.
(i) P(target is hit in 2 shots) = 1 - P(both miss) = 1 - (1/4)^2 = 1 - 1/16 = 15/16.
(ii) P(at least one misses) = 1 - P(both hit) = 1 - (3/4)^2 = 1 - 9/16 = 7/16.

OR

Q.31 [3 marks]

(OR) Mother, Father and Son line up at random for a family picture. Let events E: Son on one end and F: Father in the middle. Find P(E/F).

Answer: 1

Explanation:
Total arrangements = 3! = 6.
F: Father in middle — arrangements: (Mother, Father, Son) and (Son, Father, Mother) = 2 arrangements.
P(F) = 2/6 = 1/3.
E intersection F: Son on one end AND Father in middle — this forces the other end to be Mother.
So same 2 arrangements count: Son-Father-Mother and Mother-Father-Son.
P(E intersection F) = 2/6 = 1/3.
P(E/F) = P(E intersection F)/P(F) = (1/3)/(1/3) = 1.

Q.32 [3 marks]

If P = [[1,-1,0],[2,3,4],[0,1,2]] and Q = [[2,2,-4],[-4,2,-4],[2,-1,5]], find the product QP and hence solve the system of equations: x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7.

Answer: QP = 6I; x = 2, y = -1, z = 4

Explanation:

Compute QP: Row1 of Q times cols of P: (2+4+0, -2+6-4, 0+8-8)=(6,0,0);

Row2: (-4+4+0, 4+6-4, 0+8-8)=(0,6,0);

Row3: (2-2+0, -2-3+5, 0-4+10)=(0,0,6).

So QP = 6I => P^(-1) = Q/6.

System in matrix form Px = b where b=(3,17,7)^T.

x = P^(-1)b = (1/6)Qb.

Qb: (6+34-28, -12+34-28, 6-17+35) = (12,-6,24).

x = (1/6)(12,-6,24) = (2,-1,4).

Verification: 2-(-1)=3✓; 4-3+16=17✓; -1+8=7✓.

OR

Q.32 [3 marks]

(OR) Obtain the value of Delta = |[[1+x, 1, 1],[1, 1+y, 1],[1, 1, 1+z]]| in terms of x, y, z. Further, if Delta = 0 and x, y, z are non-zero real numbers, prove that x^(-1) + y^(-1) + z^(-1) = -1.

Answer: Delta = xyz + xy + yz + zx

Explanation:

Apply R1 -> R1-R2, R2 -> R2-R3: Delta = |[[x,-y,0],[0,y,-z],[1,1,1+z]]|.

Expand along R1: x*(y(1+z)+z) + y*(z) = x(y+yz+z) + yz = xy+xyz+xz+yz = xyz+xy+yz+zx.

If Delta=0: xyz+xy+yz+zx=0.

Dividing by xyz: 1 + 1/z + 1/x + 1/y = 0 => x^(-1) + y^(-1) + z^(-1) = -1.

Section D

Long Answer Questions (5 marks each)

Q.33 [5 marks]

Find the sub-intervals in which f(x) = cot^(-1)(sin x + cos x), x in (0, pi) is increasing and decreasing.

Answer: Increasing on (pi/4, pi); Decreasing on (0, pi/4)

Explanation:

f'(x) = -1/(1+(sin x+cos x)^2) * (cos x - sin x).

Denominator = 1+(1+sin 2x) = 2+sin 2x > 0 always.

So sign of f'(x) depends on -(cos x - sin x) = sin x - cos x.

f'(x) > 0 when sin x > cos x, i.e., tan x > 1, which in (0,pi) gives x in (pi/4, pi).

f'(x) < 0 when cos x > sin x, i.e., x in (0, pi/4).

So f is decreasing on (0, pi/4) and increasing on (pi/4, pi).

OR

Q.33 [5 marks]

(OR) A rectangle of perimeter 36 cm is revolved around one of its sides to sweep out a cylinder of maximum volume. Find the dimensions of the rectangle.

Answer: 6 cm x 12 cm (height 6 cm, radius 12 cm)

Explanation:

Let sides be x (height, the side revolved around) and y (radius).

2x + 2y = 36 => x + y = 18 => y = 18-x.

Volume V = pi*y^2*x = pi*(18-x)^2*x.

dV/dx = pi*(18-x)^2 + pi*x*2*(18-x)*(-1) = pi*(18-x)*[(18-x)-2x] = pi*(18-x)*(18-3x).

Setting dV/dx = 0: x = 18 (gives y=0, rejected) or x = 6.

At x=6: y=12.

V is maximum (verify d^2V/dx^2 < 0).

Rectangle dimensions: 6 cm x 12 cm.

Q.34 [5 marks]

Find the domain of g(x) = cos^(-1)(x^2 - 1). Hence, find the value of x for which g(x) = pi/3. Also, write the range of cos^(-1)(x) other than its principal

branch.

Answer: Domain: [-sqrt(2), sqrt(2)]; x = ±sqrt(6)/2 for g(x) = pi/3; Other ranges: [pi, 2*pi], [-pi, 0], etc.

Explanation:

Domain: need -1 <= x^2-1 <= 1 => 0 <= x^2 <= 2 => x in [-sqrt(2), sqrt(2)].

For g(x) = pi/3: cos^(-1)(x^2-1) = pi/3 => x^2-1 = cos(pi/3) = 1/2 => x^2 = 3/2 => x = ±sqrt(3/2) = ±sqrt(6)/2.

Other ranges of cos^(-1)(x) beyond principal branch [0,pi]: [pi, 2*pi], [-pi, 0], [2*pi, 3*pi], etc.

(i.e., [2n*pi, (2n+1)*pi] for any integer n).

Q.35 [5 marks]

A line passing through the points A(1,2,3) and B(5,8,11) intersects the line r_vec = 4*i_hat + j_hat + lambda*(5*i_hat + 2*j_hat + k_hat). Find the co-ordinates of the point of intersection. Hence, write the equation of a line passing through the point of intersection and perpendicular to both the lines.

Answer: Point of intersection: (-1, -1, -1); Perpendicular line: r_vec = (-i_hat - j_hat - k_hat) + mu*(-5*i_hat + 18*j_hat - 11*k_hat), or (x+1)/(-5) = (y+1)/18 = (z+1)/(-11)

Explanation:

Direction of AB: (4,6,8) ~ (2,3,4).

Line AB: (1+2t, 2+3t, 3+4t).

Line L2: (4+5*lambda, 1+2*lambda, lambda).

Equating: 1+2t=4+5*lambda, 2+3t=1+2*lambda, 3+4t=lambda.

From eq3: lambda=3+4t.

Sub in eq2: 2+3t=1+2(3+4t)=7+8t => -5t=5 => t=-1.

lambda=3-4=-1.

Point: (1-2, 2-3, 3-4)=(-1,-1,-1).

Verify eq1: -1=4-5=-1✓.

Perpendicular direction = d1 x d2 = (2,3,4)x(5,2,1) = (3-8, -(2-20), 4-15) = (-5,18,-11).

Line: (x+1)/(-5) = (y+1)/18 = (z+1)/(-11).

Q.36(a) [5 marks]

Case Study: A study revealed that 170 in 1000 males who smoke develop lung complications, while 120 out of 1000 females who smoke develop lung problems. In a colony, 50 smokers were found of which 30 are males. A person is selected at random.

(i) What is the probability that the selected person is a female?

Answer: 2/5

Explanation:

Total smokers = 50; females = 20. P(female) = 20/50 = 2/5.

Q.36(b) [5 marks]

Case Study (continued — same setup as q036a).

(ii) If a male person is selected, what is the probability that he will not be suffering from lung problems?

Answer: 83/100

Explanation:

P(lung | male) = 170/1000 = 17/100. P(no lung | male) = 1 - 17/100 = 83/100.

Q.36(c) [5 marks]

Case Study (continued — same setup as q036a).

(iii)(a) A person selected at random is detected with lung complications. Find the probability that the selected person is a female.

Answer: 8/25

Explanation:

P(male) = 30/50 = 3/5; P(female) = 2/5.

P(lung|male) = 17/100; P(lung|female) = 12/100 = 3/25.

P(lung) = (3/5)(17/100) + (2/5)(12/100) = 51/500 + 24/500 = 75/500 = 3/20.

P(female|lung) = P(female)*P(lung|female)/P(lung) = (2/5)(12/100)/(3/20) = (24/500)*(20/3) = 480/1500 = 8/25.

OR

Q.36(c) [5 marks]

Case Study (continued — same setup as q036a).

(iii)(b) A person selected at random is not having lung problems. Find the probability that the person is a male.

Answer: 249/425

Explanation:

P(no lung|male) = 83/100; P(no lung|female) = 88/100.

P(no lung) = (3/5)(83/100) + (2/5)(88/100) = 249/500 + 176/500 = 425/500 = 17/20.

P(male|no lung) = P(male)*P(no lung|male)/P(no lung) = (3/5)(83/100)/(17/20) = (249/500)*(20/17) = 4980/8500 = 249/425.

Q.37(a) [4 marks]

Case Study: A racing track is built around an elliptical ground with equation 9x^2 + 16y^2 = 144. The width of the track is 3 m.

(i) Express y as a function of x from the given equation of the ellipse.

Answer: y = (3/4)*sqrt(16 - x^2)

Explanation:

9x^2 + 16y^2 = 144 => 16y^2 = 144 - 9x^2 => y^2 = (144-9x^2)/16 => y = (3/4)*sqrt(16-x^2) (taking positive root for upper half of ellipse).

Q.37(b) [4 marks]

Case Study (continued — same setup as q037a).

(ii) Integrate the function obtained in (i) with respect to x.

Answer: (3x*sqrt(16-x^2))/8 + 6*sin^(-1)(x/4) + C

Explanation:

integral of (3/4)*sqrt(16-x^2) dx.

Use formula: integral of sqrt(a^2-x^2) dx = (x*sqrt(a^2-x^2))/2 + (a^2/2)*sin^(-1)(x/a) + C with a=4.

= (3/4)*[(x*sqrt(16-x^2))/2 + 8*sin^(-1)(x/4)] + C = (3x*sqrt(16-x^2))/8 + 6*sin^(-1)(x/4)

+ C.

Q.37(c) [4 marks]

Case Study (continued — same setup as q037a).

(iii)(a) Find the area of the region enclosed within the elliptical ground excluding the track using integration. (The track is 3 m wide outside the elliptical ground; inner ellipse is 9x^2 + 16y^2 = 144.)

Answer: 12*pi square metres

Explanation:

The elliptical ground is the region enclosed by 9x^2 + 16y^2 = 144, which has semi-axes a=4, b=3.

Area of ellipse = pi*a*b = pi*4*3 = 12*pi.

The track is outside this ellipse;

the region inside the elliptical ground (excluding the surrounding track) is just the area of the ellipse = 12*pi square metres.

OR

Q.37(c) [4 marks]

Case Study (continued — same setup as q037a).

(iii)(b) Write the co-ordinates of the points P and Q where the outer edge of the track cuts the x-axis and y-axis in the first quadrant (the outer ellipse has semi-axes a=7, b=6 since the track is 3 m wide). Find the area of the triangle formed by points P, O, Q using integration.

Answer: P = (7, 0), Q = (0, 6); Area of triangle POQ = 21 square metres

Explanation:

Outer ellipse (3m outside inner semi-axes a=4, b=3): semi-axes a=4+3=7, b=3+3=6.

P = (7,0) on x-axis, Q = (0,6) on y-axis.

Line PQ: x/7 + y/6 = 1 => y = 6(1 - x/7).

Area of triangle OPQ using integration: integral from 0 to 7 of 6(1-x/7) dx = [6x - 3x^2/7] from 0 to 7 = 42 - 21 = 21 sq m.

(Also = (1/2)*base*height = (1/2)*7*6 = 21 ✓.)

Q.38(a) [4 marks]

Case Study: The equation of a sports car racing track is $f(x) = x^4 – 4x^2 + 4$ for $0 \le x < 3$ and $f(x) = x^2 + 40$ for $x \ge 3$. (i) Find $f'(x)$ for $0 < x < 3$.

Answer: f'(x) = 4x^3 - 8x

Explanation:

For 0 < x < 3, f(x) = x^4 - 4x^2 + 4. f'(x) = 4x^3 - 8x.

Q.38(b) [4 marks]

Case Study (continued — same setup as q038a).

(ii) Find f'(4).

Answer: 8

Explanation:

For x > 3, f(x) = x^2 + 40. f'(x) = 2x. At x = 4: f'(4) = 2*4 = 8.

Q.38(c) [4 marks]

Case Study (continued — same setup as q038a).

(iii)(a) Test for continuity of f(x) at x = 3.

Answer: f is continuous at x = 3.

Explanation:

f(3) = 3^4 - 4*3^2 + 4 = 81 - 36 + 4 = 49.

lim(x->3^-) f(x) = 81 - 36 + 4 = 49.

lim(x->3^+) f(x) = 3^2 + 40 = 9 + 40 = 49.

Since f(3) = lim(x->3^-) f(x) = lim(x->3^+) f(x) = 49, f is continuous at x = 3.

OR

Q.38(c) [4 marks]

Case Study (continued — same setup as q038a).

(iii)(b) Test for differentiability of f(x) at x = 3.

Answer: f is NOT differentiable at x = 3.

Explanation:

LHD at x=3: f'(3^-) = 4*(3)^3 - 8*(3) = 108 - 24 = 84.

RHD at x=3: f'(3^+) = 2*(3) = 6.

Since LHD (84) ≠ RHD (6), f is not differentiable at x = 3 even though it is continuous there.