CBSE Class 12 Mathematics 2026 — Set 3 — Answer Key

English Medium

Time Allowed: 3 Hours | Maximum Marks: 80

General Instructions:

1. This question paper contains 38 questions. All questions are compulsory.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 6 questions carrying 2 marks each.
4. Section C has 6 questions carrying 3 marks each.
5. Section D has 4 questions carrying 5 marks each.
6. Section E has 3 case-study based questions carrying 4 marks each.
7. Draw neat figures wherever required.

Section A

Multiple Choice Questions (1 mark each)

Q.1 [1 mark]

The domain of $f(x) = \cos^{-1}(2x - 5)$ is :

(a)

(b)

(c)

(d) <<

Explanation:

$\cos^{-1}$ is defined when its argument is in $[-1, 1]$.

So $-1 \le 2x - 5 \le 1 \Rightarrow 4 \le 2x \le 6 \Rightarrow 2 \le x \le 3$.

Domain = $[2, 3]$.

Q.2 [1 mark]

If $A^2 = 4A + 3I$ and $A^{-1} = xA + yI$, then the value of $(x + y)$ is :

(a) -1 <<

(b) 1

(c) 5/3

(d) 7

Explanation:

From $A^2 = 4A + 3I$, multiply both sides by A^{-1}: $A = 4I + 3A^{-1}$.

So $3A^{-1} = A - 4I$, giving $A^{-1} = (1/3)A - (4/3)I$.

Thus $x = 1/3$, $y = -4/3$, and $x + y = 1/3 - 4/3 = -3/3 = -1$.

Q.3 [1 mark]

If A and B are skew-symmetric matrices of same order, then AB + BA is a/an :

(a) symmetric matrix <<

(b) skew-symmetric matrix

(c) null matrix

(d) identity matrix

Explanation:

For skew-symmetric A and B: A^T = –A, B^T = –B.

(AB + BA)^T = (AB)^T + (BA)^T = B^TA^T + A^TB^T = (–B)(–A) + (–A)(–B) = BA + AB = AB + BA.

So (AB + BA)^T = AB + BA, making it symmetric.

Q.4 [1 mark]

If [4; 1; 3] A = [–4 8 4; –1 2 1; –3 6 3], then the order of A must be :

(a) 3×1

(b) 1×3 <<

(c) 1×1

(d) 3×3

Explanation:

The left matrix is 3×1 and the product is 3×3.

For (3×1)(A) = (3×3), A must be 1×3.

Q.5 [1 mark]

If a square matrix A is such that $A^2 = A$ and $(I – A)^3 = xA + I$, then the value of x must be :

(a) 7

(b) 5

(c) –7

(d) –1 <<

Explanation:

Expand $(I – A)^3 = I – 3A + 3A^2 – A^3$.

Since $A^2 = A$, then $A^3 = A \cdot A^2 = A \cdot A = A$.

So $(I – A)^3 = I – 3A + 3A – A = I – A = I + (–1)A$.

Comparing with xA + I gives x = –1, which is option (D).

Q.6 [1 mark]

If B(adj B) = diag(1/3, 1/3, 1/3), then the value of det(B^{-1}) =

(a) 1/3

(b) 1/9

(c) 3 <<

(d) 9

Explanation:

B(adj B) = det(B)·I.

Given diag(1/3,1/3,1/3) = (1/3)I_3, so det(B) = 1/3.

Then det(B^{-1}) = 1/det(B) = 3, which is option (C).

Q.7 [1 mark]

The value of k for which f(x) = {x^2 sin(1/x), x ≠ 0; k(x+1), x = 0} is a continuous function, is :

(a) 1/4

(b) 2

(c) 1/2

(d) 0 <<

Explanation:

For continuity at x = 0: lim(x→0) x^2 sin(1/x) = 0 (since |x^2 sin(1/x)| ≤ x^2 → 0).

So we need k(0+1) = 0, giving k·1 = 0, thus k = 0.

Q.8 [1 mark]

If $\sin^{-1} x = y$, then dy/dx is :

(a) $\cos^{-1} x$

(b) cos y

(c) $1/\sqrt{(1-x^2)}$ <<

(d) sec y

Explanation:

If $\sin^{-1} x = y$, then x = sin y.

Differentiating: 1 = cos y · (dy/dx), so dy/dx = 1/cos y = 1/√(1–sin²y) = 1/√(1–x²).
Note: options (C) and (D) both equal dy/dx as printed.
(C) is kept here; this could not be confirmed against an official CBSE marking scheme.

Q.9 [1 mark]

The rate of change of volume of a sphere with respect to its diameter, when its radius is 5 cm, is :

(a) 400π cm³/cm
(b) 100π cm³/cm
(c) 50π cm³/cm <<
(d) 25π cm³/cm

Explanation:
V = (4/3)πr³.
With diameter d = 2r, V = (4/3)π(d/2)³ = πd³/6, so dV/dd = πd²/2.
Equivalently dV/dd = (dV/dr)(dr/dd) = 4πr²·(1/2) = 2πr².
At r = 5: 2π(25) = 50π cm³/cm — option (C).

Q.10 [1 mark]

$\int dx/(2^x + 2^{(-x)})$ is equal to :

(a) $\tan^{-1}(2^x) + C$
(b) $\tan^{-1}(2^{(-x)}) + C$
(c) $\tan^{-1}(2^x)/\log 2 + C$ <<
(d) $(\log 2)\tan^{-1}(2^x) + C$

Explanation:
$\int dx/(2^x + 2^{(-x)}) = \int 2^x dx/(2^{(2x)} + 1)$.
Let $t = 2^x$, $dt = 2^x \ln 2\, dx$, so $dx \cdot 2^x = dt/\ln 2$.
Integral = $(1/\ln 2)\int dt/(t^2+1) = (1/\ln 2)\tan^{-1}(t) + C = \tan^{-1}(2^x)/\log 2 + C$.

Q.11 [1 mark]

$\int_{-1}^{1} (1 - |x|)\, dx$ is equal to :

(a) $2\int_{0}^{1} (1+x)\, dx$
(b) $2\int_{-1}^{0} (1+x)\, dx$ <<

(c) 0

(d) $2\int_{-1}^{0}(1-x)\,dx$

Explanation:

$f(x) = 1 - |x|$ is even, so the integral equals $2\int_{-1}^{0} f(x)\,dx$.

On $[-1, 0]$, $|x| = -x$, so $1 - |x| = 1 + x$.

The integral therefore equals $2\int_{-1}^{0}(1 + x)\,dx$, option (B).

Q.12 [1 mark]

The area of the shaded region of the circle given below is equal to : (figure: circle $x^2 + y^2 = 9$ centred at O, with the line $y = 1$ drawn; the whole part of the circle above $y = 1$ is shaded, on both sides of the Y-axis)

(a) $\int_{1}^{3}\sqrt{(9-y^2)}\,dy$

(b) $2\int_{1}^{3}\sqrt{(9-y^2)}\,dy$ <<

(c) $\int_{0}^{3}\sqrt{(9-x^2)}\,dx$

(d) $2\int_{0}^{3}\sqrt{(9-x^2)}\,dx$

Explanation:

For a circle $x^2 + y^2 = 9$, the shaded region between $y=1$ and $y=3$ on both sides of the y-axis has width $2x = 2\sqrt{(9-y^2)}$.

Area $= \int_{1}^{3} 2\sqrt{(9-y^2)}\,dy = 2\int_{1}^{3}\sqrt{(9-y^2)}\,dy$.

Q.13 [1 mark]

$dy/dx = F(x,y)$ will be a homogeneous differential equation for which of the following functions? (i) $F(x,y) = 3x + 2y$, (ii) $F(x,y) = \sin(y/x) + \log y - \log x$, (iii) $F(x,y) = e^{(y/x)} + 1$, (iv) $F(x,y) = \sqrt{(x^2 + y^2)} - y$

(a) (i) and (ii)

(b) (i), (ii) and (iii)

(c) (ii), (iii) and (iv)

(d) (ii) and (iii) <<

Explanation:

A homogeneous DE has $F(tx, ty) = F(x,y)$ (degree 0).

(i) $F = 3x+2y$: $F(tx,ty) = t(3x+2y) \neq F(x,y)$.

Not homogeneous.

(ii) sin(y/x) + log(y/x): F(tx,ty) = sin(y/x) + log(y/x) = F(x,y).

Homogeneous.

(iii) e^(y/x) + 1: F(tx,ty) = e^(y/x)+1 = F(x,y).

Homogeneous.

(iv) $\sqrt{(x^2+y^2)} - y$: F(tx,ty) = $t(\sqrt{(x^2+y^2)} - y) \neq F(x,y)$.

Not homogeneous.

So (ii) and (iii) only — option (D).

Q.14 [1 mark]

For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true?

(a) $\vec{a}\cdot\vec{b} \leq |\vec{a}||\vec{b}|$ <<

(b) $|\vec{a} + \vec{b}| \geq |\vec{a}| + |\vec{b}|$

(c) $|\vec{a} - \vec{b}| = |\vec{a}| - |\vec{b}|$

(d) $|\vec{a} \times \vec{b}| \geq |\vec{a}||\vec{b}|$

Explanation:

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta \leq |\vec{a}||\vec{b}|$ always, so (A) is true.

(B) is false: the triangle inequality gives $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$.

(C) is false in general.

(D) is false: $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta \leq |\vec{a}||\vec{b}|$, with equality only when $\theta = \pi/2$.

Answer: (A).

Q.15 [1 mark]

If $(\vec{a} + \vec{b})\cdot(\vec{a} - \vec{b}) = 198$ and $|\vec{a}| = 10|\vec{b}|$, then :

(a) $|\vec{a}| = \sqrt{2}$

(b) $|\vec{b}| = \sqrt{2}$ <<

(c) $|\vec{b}| = 10\sqrt{2}$

(d) $|\vec{a}| = 10/\sqrt{2}$

Explanation:

$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = |\vec{a}|^2 - |\vec{b}|^2 = 198$.

With $|\vec{a}| = 10|\vec{b}|$: $100|\vec{b}|^2 - |\vec{b}|^2 = 99|\vec{b}|^2 = 198 \Rightarrow |\vec{b}|^2 = 2 \Rightarrow |\vec{b}| = \sqrt{2}$ — option

(B).

Q.16 [1 mark]

If l_1,m_1,n_1 and l_2,m_2,n_2 are direction cosines of lines L_1 and L_2 respectively and θ is the acute angle between them, then :

(a) $\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$

(b) $\sin\theta = l_1l_2 + m_1m_2 + n_1n_2$

(c) $\tan\theta = l_1/l_2 + m_1/m_2 + n_1/n_2$

(d) $\cos\theta = |l_1l_2 + m_1m_2 + n_1n_2|$ <<

Explanation:

$l_1l_2 + m_1m_2 + n_1n_2$ can be negative (when the direction vectors make an obtuse angle), but $\cos\theta > 0$ for an acute angle θ.

So the modulus is needed: $\cos\theta = |l_1l_2 + m_1m_2 + n_1n_2|$, which is option (D).

Q.17 [1 mark]

Direction ratios of lines l_1 and l_2 are <12, –3, 9> and <4, q, –p> respectively. The values of p and q for which l_1 and l_2 are parallel are respectively :

(a) –1, 3

(b) 3, 1

(c) –3, –1 <<

(d) –1, –3

Explanation:

For parallel lines, direction ratios must be proportional: $12/4 = -3/q = 9/(-p)$.

So ratio = 3.

Thus $q = -3/3 = -1$ and $-p = 9/3 = 3$, so $p = -3$.

Therefore $p = -3$, $q = -1$.

Answer: (C) –3, –1.

Q.18 [1 mark]

If E and F are two independent events such that $P(E) = 3/10$ and $P(E \cup F) = 1/2$, then $P(E|F) - P(F|E)$ is equal to :

(a) 2/7

(b) 3/35

(c) 1/70 <<

(d) 1/7

Explanation:

Using P(E∪F) = P(E) + P(F) − P(E)P(F): 1/2 = 3/10 + P(F)·7/10, so P(F) = 2/7.

By independence P(E|F) = P(E) = 3/10 and P(F|E) = P(F) = 2/7.

P(E|F) − P(F|E) = 3/10 − 2/7 = 21/70 − 20/70 = 1/70, option (C).

Q.19 [1 mark]

Assertion (A): One of the particular solutions of the differential equation dy/dx = e^(x+y) can be e^x + e^(–y) = –2. Reason (R): e^x + e^(–y) = C is the general solution of the differential equation dy/dx = e^(x+y).

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true, but R is not the correct explanation of A

(c) A is true, but R is false

(d) A is false, but R is true <<

Explanation:

Solving dy/dx = e^x · e^y: e^(–y) dy = e^x dx.

Integrating: –e^(–y) = e^x + K, i.e., e^x + e^(–y) = C where C = –K.

The general solution is e^x + e^(–y) = C.

Since e^x > 0 and e^(–y) > 0, their sum C must be positive.

So C = –2 is impossible — Assertion (A) is false.

Reason (R) correctly states the general solution.

Answer: (D).

Q.20 [1 mark]

Assertion (A): The vectors $\vec{a}$ and $(-2\vec{a})$, where $\vec{a} \neq 0$, are collinear vectors. Reason (R): $\vec{a} \cdot (-2\vec{a}) = 0$.

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true, but R is not the correct explanation of A

(c) A is true, but R is false <<

(d) A is false, but R is true

Explanation:

Assertion (A): $\vec{a}$ and $-2\vec{a}$ are scalar multiples of each other (scalar = –2), so they are collinear.

A is TRUE.

Reason (R): $\vec{a}\cdot(-2\vec{a}) = -2|\vec{a}|^2 \neq 0$ (since $\vec{a} \neq 0$).

R is FALSE.

Answer: (C).

Section B

Very Short Answer Questions (2 marks each)

Q.21 [2 marks]

If $x = t + 1/t$ and $y = t - 1/t$, find dy/dx at $t = 2$.

Answer: 5/3

Explanation:

$dx/dt = 1 - 1/t^2$, $dy/dt = 1 + 1/t^2$.

So $dy/dx = (1 + 1/t^2)/(1 - 1/t^2) = (t^2 + 1)/(t^2 - 1)$.

At $t = 2$: $dy/dx = (4+1)/(4-1) = 5/3$.

Q.22 [2 marks]

Find the sub-interval(s) of $(0, \pi/2)$ in which $f(x) = \tan x - 4x$ is increasing.

Answer: $(\pi/3, \pi/2)$

Explanation:

$f'(x) = \sec^2 x - 4$.

For f to be increasing, $\sec^2 x - 4 > 0 \Rightarrow \sec^2 x > 4 \Rightarrow \cos^2 x < 1/4 \Rightarrow |\cos x| < 1/2$.

In $(0, \pi/2)$, this means $\cos x < 1/2$, so $x > \pi/3$.

The sub-interval is $(\pi/3, \pi/2)$.

Q.23(a) [2 marks]

Find the value of $\sin[\cot^{-1}(\sqrt{2} \cdot \cos(\tan^{-1} 1))]$.

Answer: $1/\sqrt{2}$

Explanation:

$\tan^{-1}(1) = \pi/4$, so $\cos(\pi/4) = 1/\sqrt{2}$.

Then $\sqrt{2} \cdot \cos(\tan^{-1} 1) = \sqrt{2} \cdot (1/\sqrt{2}) = 1$.

So the expression is $\sin[\cot^{-1}(1)]$.

$\cot^{-1}(1) = \pi/4$, so $\sin(\pi/4) = 1/\sqrt{2}$ (= $\sqrt{2}/2$).

Q.23(b) [2 marks]

(OR) A relation R on A = {1, 2, 3} is defined as R = {(1,1), (3,3), (1,2)}. Is R a symmetric relation? Justify. Write the smallest relation set R_1 such that R ∪ R_1 becomes an equivalence relation on {1, 2, 3}.

Answer: R is not symmetric; R_1 = {(2,1), (2,2)}

Explanation:

R = {(1,1),(3,3),(1,2)}.

(1,2) ∈ R but (2,1) ∉ R, so R is NOT symmetric.

For R ∪ R_1 to be an equivalence relation on {1,2,3} it must be reflexive, symmetric and transitive — the SMALLEST such relation is generated by the partition R induces.

R relates 1~2 (via (1,2)) and 3 stays on its own, giving classes {1,2} and {3}.

The corresponding equivalence relation is {(1,1),(2,2),(1,2),(2,1),(3,3)}.

R already has (1,1),(3,3),(1,2), so the smallest additions are R_1 = {(2,1),(2,2)}.

(Do NOT add (1,3),(3,1),(2,3),(3,2) — those would needlessly merge 3 into the class, making R_1 larger.)

Q.24 [2 marks]

If for two unit vectors $\vec{a}$ and $\vec{b}$, $|\vec{a} + 2\vec{b}| = |2\vec{a} - \vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$.

Answer: $\pi/2$

Explanation:

For unit vectors, square both sides: $|\vec{a} + 2\vec{b}|^2 = |2\vec{a} - \vec{b}|^2$.

LHS = $1 + 4(\vec{a} \cdot \vec{b}) + 4 = 5 + 4(\vec{a} \cdot \vec{b})$.

RHS = $4 - 4(\vec{a} \cdot \vec{b}) + 1 = 5 - 4(\vec{a} \cdot \vec{b})$.

Setting equal: $8(\vec{a}\cdot\vec{b}) = 0 \Rightarrow \vec{a}\cdot\vec{b} = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = \pi/2$.

Q.25(a) [2 marks]

If the lines $(x-3)/1 = (1-y)/1 = (z+2)/p$ and $(2-x)/3 = (y+1)/5 = (z+56)/(2p)$ are perpendicular to each other, then find the value(s) of p.

Answer: p = 2 or p = –2

Explanation:

Write direction ratios: Line 1: $(x-3)/1 = (y-1)/(-1) = (z+2)/p \Rightarrow$ DR = (1, –1, p).

Line 2: $(x-2)/(-3) = (y+1)/5 = (z+56)/(2p) \Rightarrow$ DR = (–3, 5, 2p).

For perpendicular: $(1)(-3) + (-1)(5) + p(2p) = 0 \Rightarrow -3 - 5 + 2p^2 = 0 \Rightarrow 2p^2 = 8 \Rightarrow p^2 = 4 \Rightarrow p = \pm 2$.

So p = 2 or p = –2.

Q.25(b) [2 marks]

(OR) Find the vector equation of a line passing through the origin and perpendicular to both the lines $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and $\vec{r} = \mu(\hat{i} - \hat{j} + \hat{k})$.

Answer: $\vec{r} = t(6\hat{i} - \hat{j} - 7\hat{k})$

Explanation:

The line must be perpendicular to both direction vectors $d_1 = (3, 4, 2)$ and $d_2 = (1, -1, 1)$.

Direction $= d_1 \times d_2 = (4\cdot1 - 2\cdot(-1), -(3\cdot1 - 2\cdot1), 3\cdot(-1) - 4\cdot1) = (6, -1, -7)$.

Passing through the origin: $\vec{r} = t(6\hat{i} - \hat{j} - 7\hat{k})$.

Q.26(a) [2 marks]

Find : $\int dx / (x^{(1/2)} + x^{(1/3)})$

Answer: $2\sqrt{x} - 3x^{(1/3)} + 6x^{(1/6)} - 6\ln|x^{(1/6)} + 1| + C$

Explanation:

$\int dx/(\sqrt{x} + x^{(1/3)})$.

Let $x = t^6$, $dx = 6t^5\,dt$, $x^{(1/2)} = t^3$, $x^{(1/3)} = t^2$.

Integral $= \int 6t^5/(t^3 + t^2)\,dt = 6\int t^5/(t^2(t+1))\,dt = 6\int t^3/(t+1)\,dt$.

Divide: t^3/(t+1) = t^2 – t + 1 – 1/(t+1).

Integral = 6[t^3/3 – t^2/2 + t – ln|t+1|] + C = 6[x^(1/2)/3 – x^(1/3)/2 + x^(1/6) – ln|x^(1/6)+1|] + C = 2√x – 3x^(1/3) + 6x^(1/6) – 6 ln|x^(1/6)+1| + C.

Q.26(b) [2 marks]

(OR) Find : $\int \tan^{-1}((1 - x)/(1 + x))\, dx$

Answer: $\pi x/4 - x \tan^{-1}x + (1/2) \ln(1 + x^2) + C$

Explanation:

$\tan^{-1}((1 - x)/(1 + x)) = \tan^{-1}1 - \tan^{-1}x = \pi/4 - \tan^{-1}x$.

So the integral = $\pi x/4 - \int \tan^{-1}x\, dx$.

By parts, $\int \tan^{-1}x\, dx = x \tan^{-1}x - (1/2) \ln(1 + x^2)$.

Hence the integral = $\pi x/4 - x \tan^{-1}x + (1/2) \ln(1 + x^2) + C$.

Section C

Short Answer Questions (3 marks each)

Q.27 [3 marks]

Evaluate : $\int_0^{\pi} \sin^{2026}(x) / (\sin^{2026}(x) + \cos^{2026}(x))\, dx$

Answer: $\pi/2$

Explanation:

Let $f(x) = \sin^{2026}x/(\sin^{2026}x + \cos^{2026}x)$.

Since $\sin(\pi - x) = \sin x$ and $\cos^{2026}(\pi - x) = \cos^{2026}x$ (even power), $f(\pi - x) = f(x)$.

So $I = 2\int_0^{(\pi/2)} f(x)\, dx$.

Let $J = \int_0^{(\pi/2)} \sin^{2026}x/(\sin^{2026}x + \cos^{2026}x)\, dx$.

Using $x \to \pi/2 - x$, $J = \int_0^{(\pi/2)} \cos^{2026}x/(\cos^{2026}x + \sin^{2026}x)\, dx$.

Adding: $2J = \int_0^{(\pi/2)} 1\, dx = \pi/2 \to J = \pi/4$.

$I = 2J = \pi/2$.

Q.28(a) [3 marks]

Find : $\int \cos x / ((2 + \sin x)(4 + \sin x))\, dx$

Answer: $(1/2) \ln|(2 + \sin x)/(4 + \sin x)| + C$

Explanation:

Let t = sin x, dt = cos x dx.

Integral = $\int dt/((2+t)(4+t))$.

Partial fractions: $1/((2+t)(4+t)) = A/(2+t) + B/(4+t)$ with $1 = A(4+t) + B(2+t)$.

t = –2: A = 1/2.

t = –4: B = –1/2.

Integral = $(1/2)\ln|2+t| – (1/2)\ln|4+t| + C = (1/2)\ln|(2+\sin x)/(4+\sin x)| + C$.

Q.28(b) [3 marks]

(OR) Find : $\int (x + 3)/(x^2 + 4x + 5)\, dx$

Answer: $(1/2)\ln(x^2+4x+5) + \tan^{-1}(x+2) + C$

Explanation:

Write $x + 3 = ½(2x + 4) + 1$.

$\int(x+3)/(x^2+4x+5)\, dx = ½\int(2x+4)/(x^2+4x+5)\, dx + \int dx/((x+2)^2 + 1)$

$= ½ \ln(x^2 + 4x + 5) + \tan^{-1}(x + 2) + C$.

Q.29(a) [3 marks]

Find the general solution of the differential equation $2x^2\, dy/dx = y^2 + 2xy$.

Answer: $–2x/y = \ln|x| + C$ or $y = –2x/(\ln|x|+C)$

Explanation:

$2x^2\, dy/dx = y^2 + 2xy$.

This is homogeneous.

Let $y = vx$: $dy/dx = v + x\, dv/dx$.

$2x^2(v + x\, dv/dx) = v^2x^2 + 2x \cdot vx = x^2(v^2+2v)$.

$2v + 2x\, dv/dx = v^2 + 2v$.

$2x\, dv/dx = v^2$.

Separate: $dv/v^2 = dx/(2x)$.

Integrate: $–1/v = (1/2)\ln|x| + C$.

Substitute $v = y/x$: $–x/y = (1/2)\ln|x| + C1$.

So $–2x/y = \ln|x| + C$.

Q.29(b) [3 marks]

(OR) Find a particular solution of the differential equation (x+1) dy/dx = 2e^(–y) – 1, given that y = 0 when x = 0.

Answer: e^y = (2x + 1)/(x + 1), i.e. y = ln((2x + 1)/(x + 1))

Explanation:

Separate variables: dy/(2e^(–y)–1) = dx/(x+1).

Multiply numerator and denominator by e^y: e^y dy/(2 – e^y) = dx/(x+1).

Let u = 2–e^y, du = –e^y dy.

$-\int du/u = \int dx/(x+1)$.

–ln|u| = ln|x+1| + C.

ln|2–e^y| = –ln|x+1| + C.

At y=0, x=0: ln|2–1| = –ln(1) + C => ln(1) = C => C=0.

So ln|2–e^y| = –ln(x+1) => |2–e^y| = 1/(x+1).

Since at x=0, y=0: 2–1=1>0, so 2–e^y = 1/(x+1).

Particular solution: 2–e^y = 1/(x+1), or e^y = 2 – 1/(x+1) = (2x+1)/(x+1).

Q.30 [3 marks]

If (sin x)^y = y^(cos x), then find dy/dx.

Answer: dy/dx = y(sin x · ln y + y cot x) / (cos x – y ln(sin x))

Explanation:

Take log: y ln(sin x) = cos x · ln y.

Differentiate both sides w.r.t.

x: dy/dx · ln(sin x) + y · cos x/sin x = –sin x · ln y + cos x · (1/y) · dy/dx.

Rearrange: dy/dx [ln(sin x) – cos x/y] = –sin x ln y – y cot x.

dy/dx = (–sin x ln y – y cot x)/(ln(sin x) – cos x/y) = (sin x ln y + y cot x)/(cos x/y – ln(sin x)).

Multiply numerator and denominator by y: dy/dx = y(sin x ln y + y cot x)/(cos x – y ln(sin x)).

Q.31 [3 marks]

A survey found that robotic knee replacement surgeries have 90% success rate. On a particular day, robotic surgery was performed on three patients A,

B, and C, one after another. Assuming successes/failures are independent, find the probability that: (i) exactly one surgery is successful, (ii) at most two surgeries are successful.

Answer: (i) 0.027, (ii) 0.271

Explanation:

p = 0.9 (success), q = 0.1 (failure), n = 3.

(i) P(exactly 1 success) = $C(3,1)(0.9)^1(0.1)^2 = 3 \times 0.9 \times 0.01 = 0.027$.

(ii) P(at most 2 successes) = $1 - P(3 \text{ successes}) = 1 - (0.9)^3 = 1 - 0.729 = 0.271$.

Q.32 [3 marks]

Show that $f : R \to R$ defined as $f(x) = x/\sqrt{(1 + x^2)}$ is one-one but not onto.

Answer: Proof: f is one-one (strictly increasing) but not onto (range = $(-1, 1) \neq R$).

Explanation:

One-one: $f'(x) = [\sqrt{(1+x^2)} - x \cdot x/\sqrt{(1+x^2)}]/(1+x^2) = (1+x^2 - x^2)/(1+x^2)^{(3/2)} = 1/(1+x^2)^{(3/2)}$ > 0 for all x, so f is strictly increasing ⇒ one-one.

Not onto: as x →

+∞, f →

1; as x →

−∞, f →

−1, and $|x/\sqrt{(1+x^2)}| < 1$ always, so range = $(-1, 1) \subsetneq R$ ⇒ f is not onto.

Section D

Long Answer Questions (5 marks each)

Q.33 [5 marks]

Solve the following Linear Programming Problem graphically: Maximise Z = 600x + 400y subject to: $x + 2y \leq 12$, $4x + 5y \geq 20$, $2x + y \leq 12$, $x, y \geq 0$.

Answer: Maximum Z = 4000 at (4, 4)

Explanation:

Corner points of the feasible region ($x + 2y \leq 12$, $2x + y \leq 12$, $4x + 5y \geq 20$, $x, y \geq 0$): (5, 0), (6, 0), (4, 4), (0, 6), (0, 4).

(4, 4) is where x + 2y = 12 meets 2x + y = 12.

Z(5,0) = 3000; Z(6,0) = 3600; Z(4,4) = 2400 + 1600 = 4000; Z(0,6) = 2400; Z(0,4) = 1600.

Maximum Z = 4000 at x = 4, y = 4.

Q.34(a) [5 marks]

On the inauguration day of a new showroom, 60 vouchers of Rs. 1,000 and Rs. 500 were given. The number of Rs. 1,000 vouchers added to 3 times the number of Rs. 500 vouchers gives 100. Express as a system of linear equations. Hence, find the number of vouchers of each type by matrix method.

Answer: Rs. 1000 vouchers: 40, Rs. 500 vouchers: 20

Explanation:

Let x = number of Rs.

1000 vouchers, y = number of Rs.

500 vouchers.

System: x + y = 60 and x + 3y = 100.

Matrix form: [[1,1],[1,3]][x;y] = [60;100].

A = [[1,1],[1,3]], det(A) = 3–1 = 2 ≠ 0.

A^{-1} = (1/2)[[3,–1],[–1,1]].

[x;y] = (1/2)[[3,–1],[–1,1]][60;100] = (1/2)[(180–100),(–60+100)] = (1/2)[80,40] = [40,20].

So x=40, y=20.

Check: 40+20=60 ✓; 40+3(20)=40+60=100 ✓.

Rs.

1000 vouchers: 40, Rs.

500 vouchers: 20.

Q.34(b) [5 marks]

(OR) Given that P = [[2,–1],[3,4]], Q = [[5,2],[7,4]] and R = [[2,5],[3,8]], find a matrix S such that PQ – RS is a null matrix.

Answer: S = R^{-1}(PQ) = [[–191, –110],[77, 44]]

Explanation:

PQ = [[2×5+(–1)×7, 2×2+(–1)×4],[3×5+4×7, 3×2+4×4]] = [[10–7, 4–4],[15+28, 6+16]] = [[3,0],[43,22]].

RS = null means RS = PQ.

det(R) = 2×8–5×3 = 16–15 = 1.

R^{-1} = [[8,–5],[–3,2]].

S = R^{-1}·PQ = [[8,–5],[–3,2]]·[[3,0],[43,22]] = [[24–215, 0–110],[–9+86, 0+44]] = [[–191,–110],[77,44]].

Q.35(a) [5 marks]

Represent the equations of lines l_1 and l_2 in vector form and check whether they are intersecting or not. l_1: (x+3)/(–3) = (y–1)/1 = (z–5)/5; l_2: (x+1)/(–1) = (2–y)/(–2) = (z–5)/5.

Answer: Lines are intersecting, at the point (0, 0, 0)

Explanation:

l_1: $\vec{r}$ = (–3î + ĵ + 5k̂) + λ(–3î + ĵ + 5k̂).

l_2: (2 – y)/(–2) = (y – 2)/2, so $\vec{r}$ = (–î + 2ĵ + 5k̂) + μ(–î + 2ĵ + 5k̂).

Equate components: –3 – 3λ = –1 – μ … (i); 1 + λ = 2 + 2μ … (ii); 5 + 5λ = 5 + 5μ … (iii).

(iii) ⇒ λ = μ; (ii) ⇒ 1 + λ = 2 + 2λ ⇒ λ = –1, μ = –1.

Check (i): –3 + 3 = 0 and –1 + 1 = 0 ✓.

The lines intersect; point: (–3 + 3, 1 – 1, 5 – 5) = (0, 0, 0).

Q.35(b) [5 marks]

(OR) Opposite sides of a square are along the lines $\vec{r}$ = î + 2ĵ − 4k̂ + λ(2î + 3ĵ + 6k̂) and $\vec{r}$ = 3î + 3ĵ − 5k̂ + μ(2î + 3ĵ + 6k̂). Find the area of the square if the direction ratios of the other pair of opposite sides are <−3, 6, p>. Also, find the value of p.

Answer: p = −2; Area of the square = 293/49 sq units

Explanation:

The other pair of sides (direction <−3, 6, p>) is perpendicular to the given sides

(direction (2, 3, 6)): $2(-3) + 3(6) + 6p = 12 + 6p = 0 \Rightarrow p = -2$.

Side of the square = distance between the two parallel opposite sides.

With A = (1,2,−4), B = (3,3,−5), AB = (2,1,−1) and d = (2,3,6): AB × d = (9, −14, 4), $|AB \times d| = \sqrt{293}$, $|d| = 7$, so distance = $\sqrt{293}/7$.

Area = $(\sqrt{293}/7)^2 = 293/49$ sq units.

Q.36(a) [5 marks]

(Case Study 1) Two vertical light poles of height 22 m and 16 m stand on opposite sides of a 20 m wide road. Two ladders of length l_1 and l_2 are placed from a common point R at distance x m from the smaller (16 m) pole. Express $p(x) = l_1 + l_2$ in terms of x.

Answer: $p(x) = \sqrt{(x^2 + 256)} + \sqrt{((20–x)^2 + 484)}$

Explanation:

From the figure, the 22 m pole is AB (left) and the 16 m pole is CD (right);

R is x m from the smaller pole D.

So l_2 (to C) = $\sqrt{(x^2 + 16^2)} = \sqrt{(x^2 + 256)}$ and l_1 (to A) = $\sqrt{((20 – x)^2 + 22^2)} = \sqrt{((20 – x)^2 + 484)}$.

Hence $p(x) = \sqrt{(x^2 + 256)} + \sqrt{((20 – x)^2 + 484)}$.

Q.36(b) [5 marks]

(Case Study 1) Find p'(x).

Answer: $p'(x) = x/\sqrt{(x^2+256)} – (20–x)/\sqrt{((20–x)^2+484)}$

Explanation:

Differentiating $p(x) = \sqrt{(x^2+256)} + \sqrt{((20–x)^2+484)}$: $p'(x) = x/\sqrt{(x^2+256)} + (2(20–x)(–1))/(2\sqrt{((20–x)^2+484)}) = x/\sqrt{(x^2+256)} – (20–x)/\sqrt{((20–x)^2+484)}$.

Q.36(c) [5 marks]

(Case Study 1, iii-a) Find the value of x for which $l_1^2 + l_2^2$ is minimum.

Answer: x = 10

Explanation:

$l_1^2 + l_2^2 = (x^2+256) + ((20–x)^2+484) = x^2 + (20–x)^2 + 740.$

Let $g(x) = x^2 + (20–x)^2 = x^2 + 400 – 40x + x^2 = 2x^2–40x+400.$

$g'(x) = 4x–40 = 0 \Rightarrow x = 10.$

$g''(x) = 4 > 0$, so minimum at $x = 10.$

OR

Q.36(c) [5 marks]

(Case Study 1, iii-b OR) If the 22 m pole is replaced by a 16 m pole, at what distance from either pole should the ladders be kept so that the sum of squares of lengths is minimum?

Answer: x = 10 m (midpoint)

Explanation:

Now both poles are 16 m.

$l_1^2 + l_2^2 = (x^2+256) + ((20–x)^2+256) = x^2 + (20–x)^2 + 512.$

Minimize $x^2+(20–x)^2 = 2x^2–40x+400.$

Derivative: $4x–40=0 \Rightarrow x=10.$

So ladder should be placed 10 m from each pole (midpoint of the road).

Q.37(a) [4 marks]

(Case Study 2) A survey: 40% of students appearing for an entrance exam were dropouts; 60% were regular students of class XII. Of dropouts, 5% qualify; of regular students, 10% qualify. Find the probability that a student selected at random is a regular student.

Answer: 0.6

Explanation:

P(regular student) = 60% = 0.6.

Q.37(b) [4 marks]

(Case Study 2) A student is selected at random from the group of dropout students. What is the probability that the student will not qualify the

examination?

Answer: 0.95

Explanation:

Of dropouts, 5% qualify, so 95% do not qualify.

P(not qualify | dropout) = 0.95.

Q.37(c) [4 marks]

(Case Study 2, iii-a) A student selected at random qualified the examination. Find the probability that the student is not a dropout.

Answer: 3/4

Explanation:

Let D = dropout, R = regular, Q = qualified.

P(D) = 0.4, P(R) = 0.6, P(Q|D) = 0.05, P(Q|R) = 0.10.

P(Q) = P(Q|D)P(D) + P(Q|R)P(R) = 0.05×0.4 + 0.10×0.6 = 0.02 + 0.06 = 0.08.

P(not dropout | Q) = P(R|Q) = P(Q|R)P(R)/P(Q) = (0.10×0.6)/0.08 = 0.06/0.08 = 3/4.

OR

Q.37(c) [4 marks]

(Case Study 2, iii-b OR) A student selected at random did not qualify the examination. Find the probability that the student was a regular student.

Answer: 27/46

Explanation:

P(not Q) = 1 – 0.08 = 0.92.

P(not Q | R) = 1 – 0.10 = 0.90.

P(R | not Q) = P(not Q | R)P(R)/P(not Q) = (0.90×0.6)/0.92 = 0.54/0.92 = 54/92 = 27/46.

Q.38(a) [4 marks]

(Case Study 3) A triangular park ABC has vertices A(0,4), B(–2,0), C(3,0). Write the equation of boundary line AB of the park.

Answer: $2x - y + 4 = 0$ or $y = 2x + 4$

Explanation:

Slope of AB = $(0-4)/(-2-0) = -4/(-2) = 2$.

Using point A(0,4): $y - 4 = 2(x - 0) \Rightarrow y = 2x + 4$.

Or: $2x - y + 4 = 0$.

Q.38(b) [4 marks]

(Case Study 3) Write the equation of boundary line AC of the park.

Answer: $4x + 3y - 12 = 0$ or $y = (4 - 4x/3)$

Explanation:

Slope of AC = $(0-4)/(3-0) = -4/3$.

Using point A(0,4): $y - 4 = (-4/3)(x - 0) \Rightarrow y = 4 - (4/3)x \Rightarrow 3y = 12 - 4x \Rightarrow 4x + 3y = 12$.

Q.38(c) [4 marks]

(Case Study 3, iii-a) Using integration, find the area of region OAC (where children are allowed to play cricket, football). O is the origin.

Answer: 6 sq units

Explanation:

Region OAC: bounded by O(0,0), A(0,4), C(3,0).

Line AC: $y = 4 - (4x/3)$ for x from 0 to 3.

Line OA: $x=0$.

Area = $\int_0^3 (4 - 4x/3)\,dx = [4x - 4x^2/6]_0^3 = [4x - 2x^2/3]_0^3 = 12 - 18/3 = 12 - 6 = 6$ sq units.

OR

Q.38(c) [4 marks]

(Case Study 3, iii-b OR) Using integration, find the area of region AOB.

Answer: 4 sq units

Explanation:

Region AOB: bounded by O(0,0), A(0,4), B(–2,0).

Line AB: $y = 2x + 4$, or $x = (y–4)/2$ for y from 0 to 4.

Area = $\int_0^4 |x|\, dy$ where $x = (y–4)/2$ (this is negative for $y<4$, so $|x| = (4–y)/2$).

Area = $\int_0^4 (4–y)/2\, dy = (1/2)[4y – y^2/2]_0^4 = (1/2)[16 – 8] = (1/2)(8) = 4$ sq units.